

Geometry · Parallelograms and Special Quadrilaterals · Worksheet Answer Key

Common Core G-CO.C.11 · mathovore.fr

Every solution below names the theorem it uses and shows each calculation. Compare your reasoning, not just your final numbers, and use the check step at the end of each solution to catch sign and arithmetic slips. Proof reasons are listed in full, so you can rewrite them in your own words.

Geometry Common Core: G-CO.C.11

Problem 1: Opposite sides

Opposite sides of a parallelogram are congruent. So $CD = AB = 12$ in and $DA = BC = 7$ in.

CD = 12 in, DA = 7 in

Problem 2: Opposite and consecutive angles

Opposite angles are congruent, so $m\angle C = m\angle A = 68^\circ$. Consecutive angles are supplementary, so $m\angle B = 180^\circ - 68^\circ = 112^\circ$. Then $m\angle D = m\angle B = 112^\circ$. Check: $68 + 112 + 68 + 112 = 360$.

$m\angle B = 112^\circ$, $m\angle C = 68^\circ$, $m\angle D = 112^\circ$

Problem 3: Diagonals bisect each other

The diagonals bisect each other, so E is the midpoint of each. $AE = \frac{18}{2} = 9$ and $BE = \frac{26}{2} = 13$.

$AE = 9$, $BE = 13$

Problem 4: Sides with a variable

Opposite sides are congruent: $3x + 5 = 5x - 9$. Subtract $3x$: $5 = 2x - 9$. Add 9: $14 = 2x$, so $x = 7$. Then $AB = 3(7) + 5 = 26$. Check: $CD = 5(7) - 9 = 26$.

$x = 7$, $AB = 26$ in

Problem 5: Opposite angles with a variable

Opposite angles are congruent: $4x + 8 = 6x - 12$. Then $20 = 2x$ and $x = 10$. So

$m\angle A = 4(10) + 8 = 48^\circ$ and $m\angle C = 6(10) - 12 = 48^\circ$. Consecutive angles are supplementary:
 $m\angle B = 180^\circ - 48^\circ = 132^\circ$.

$$x = 10, m\angle B = 132^\circ$$

Problem 6: Consecutive angles with a variable

$\angle B$ and $\angle C$ share side $\overline{BC}$, so they are consecutive and supplementary:
 $(3x+10) + (2x+20) = 180$. Then $5x+30=180$, $5x=150$ and $x=30$. So $m\angle B = 3(30)+10=100^\circ$ and $m\angle C = 2(30)+20=80^\circ$. Check: $100+80=180$.



All four angles: $A = 80$, $B = 100$, $C = 80$ and $D = 100$ degrees.

$$x = 30, m\angle B = 100^\circ, m\angle C = 80^\circ$$

Problem 7: Pieces of the diagonals

Each diagonal is bisected, so $AE = EC$: $2x+3=4x-7$, $10=2x$, $x=5$. Then $AE=13$ and $AC=2(13)=26$. Also $BE = ED$: $3y+1=y+13$, $2y=12$, $y=6$. Then $BE=19$ and $BD=2(19)=38$.

$$AC = 26, BD = 38$$

Problem 8: Parallelogram or not?

- a. Yes. Both pairs of opposite sides are congruent.
- b. No. Two pairs of congruent adjacent sides describe a kite, which is not a parallelogram in general.
- c. Yes. A quadrilateral whose diagonals bisect each other is a parallelogram.
- d. No. Exactly one pair of parallel sides describes a trapezoid.
- e. Yes. One pair of sides that is both parallel and congruent is enough.

a) yes, b) no, c) yes, d) no, e) yes

Problem 9: Perimeter with two variables

Opposite sides are congruent. From $AB = CD$: $2x+3=3x-2$, so $x=5$ and $AB=13$. From $BC = DA$: $4y-1=2y+7$, $2y=8$, so $y=4$ and $BC=15$. Perimeter $=2(13+15)=56$.

56 in

Problem 10: Diagonals of a rectangle

The diagonals of a rectangle are congruent: $5x-4=3x+14$. Then $2x=18$ and $x=9$. So $AC=5(9)-4=41$. Check: $BD=3(9)+14=41$.

$x=9$, $AC=41$

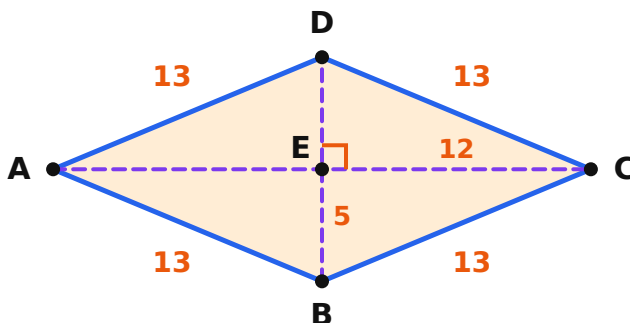
Problem 11: Length of a rectangle diagonal

A diagonal and two sides form a right triangle, so $d^2=8^2+15^2=64+225=289$. Then $d=\sqrt{289}=17$. The other diagonal is congruent, so it is also 17 cm.

17 cm

Problem 12: Rhombus perimeter and diagonals

The four sides are congruent, so $AB=\frac{52}{4}=13$. The diagonals bisect each other, so $AE=12$, and they are perpendicular, so triangle AEB is a right triangle. Then $BE^2=13^2-12^2=169-144=25$ and $BE=5$. So $BD=2(5)=10$.



Each side is 13 cm, $AE = 12$ cm and $BE = 5$ cm.

$BD=10$ cm

Problem 13: Diagonal bisects an angle of a rhombus

The two halves are equal: $3x+4=5x-12$, so $16=2x$ and $x=8$. Each half is $3(8)+4=28^\circ$, so $m\angle DAB=2(28)=56^\circ$. Consecutive angles are supplementary: $m\angle ABC=180^\circ-56^\circ=124^\circ$.

$$m\angle DAB = 56^\circ, m\angle ABC = 124^\circ$$

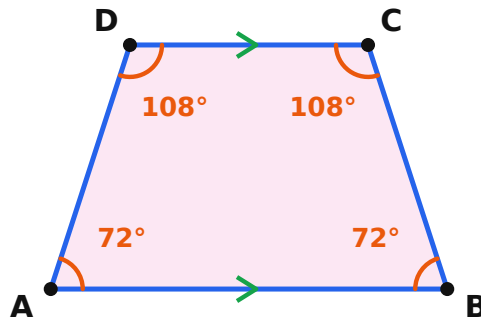
Problem 14: Diagonals of a square

A square is a rectangle, so its diagonals are congruent: $QS = 18$. They bisect each other, so $SE = \frac{18}{2} = 9$ cm. A square is also a rhombus, so the diagonals are perpendicular: $m\angle PEQ = 90^\circ$. Diagonal $\overline{PR}$ bisects the right angle at P , so $m\angle RPQ = 45^\circ$.

$$SE = 9 \text{ cm}, m\angle PEQ = 90^\circ, m\angle RPQ = 45^\circ$$

Problem 15: Base angles of an isosceles trapezoid

Base angles are congruent, so $m\angle B = 72^\circ$. Angles on the same leg between the parallel bases are supplementary, so $m\angle D = 180^\circ - 72^\circ = 108^\circ$. The other base angle matches: $m\angle C = 108^\circ$. Check: $72 + 72 + 108 + 108 = 360$.



The four angles are 72, 72, 108 and 108 degrees.

$$m\angle B = 72^\circ, m\angle C = 108^\circ, m\angle D = 108^\circ$$

Problem 16: Diagonals of an isosceles trapezoid

The diagonals of an isosceles trapezoid are congruent: $4x - 3 = 2x + 13$. Then $2x = 16$ and $x = 8$. So $AC = 4(8) - 3 = 29$. Check: $BD = 2(8) + 13 = 29$.

$$x = 8, AC = 29$$

Problem 17: Angles of a kite

The four angles of a quadrilateral add to 360° . So $m\angle Y = 360^\circ - 70^\circ - 118^\circ - 118^\circ = 54^\circ$.

$$m\angle Y = 54^\circ$$

Problem 18: Name every quadrilateral

- a. Square, rectangle, rhombus, parallelogram, quadrilateral.
- b. Rhombus, parallelogram, quadrilateral. Bisecting diagonals make it a parallelogram, perpendicular diagonals make it a rhombus, and unequal diagonals rule out a square.
- c. Trapezoid and quadrilateral (an isosceles trapezoid). It is not a parallelogram because it has only one pair of parallel sides.

a) square (also rectangle, rhombus, parallelogram), b) rhombus, c) isosceles trapezoid

Problem 19: Quilt patch

Opposite sides are congruent, so $CD=9$ and $DA=6$. Perimeter $=2(9+6)=30$ in. Opposite angles: $m\angle C=65^\circ$. Consecutive angles: $m\angle B=m\angle D=180^\circ-65^\circ=115^\circ$.

30 in of ribbon; $m\angle B=115^\circ$, $m\angle C=65^\circ$, $m\angle D=115^\circ$

Problem 20: Is the frame a rectangle?

- a. Both pairs of opposite sides are congruent (24 in and 24 in, 18 in and 18 in), so the frame is a parallelogram.
- b. A parallelogram with congruent diagonals is a rectangle, so all four corners are right angles.
- c. $18^2+24^2=324+576=900$ and $30^2=900$, so a right angle between the 18-inch and 24-inch sides is consistent with a 30-inch diagonal.

Parallelogram with congruent diagonals, so a rectangle

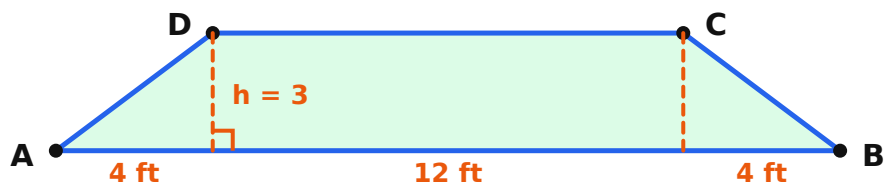
Problem 21: Flying kite frame

The horizontal spar is bisected, so each half is 12 in. The lower part of the vertical spar is $25-9=16$ in. Top sides: $\sqrt{9^2+12^2}=\sqrt{225}=15$ in. Bottom sides: $\sqrt{16^2+12^2}=\sqrt{400}=20$ in. The kite has two pairs of congruent adjacent sides, so the tape needed is $2(15+20)=70$ in.

70 in

Problem 22: Trapezoid garden bed

Perimeter: $20+12+5+5=42$ ft. Drop perpendicular segments from the ends of the short base. Each leg overhangs by $\frac{20-12}{2}=4$ ft, so $h^2+4^2=5^2$, $h^2=9$ and $h=3$ ft.



Each overhang is 4 ft, so the height is 3 ft.

Perimeter 42 ft; distance between bases 3 ft

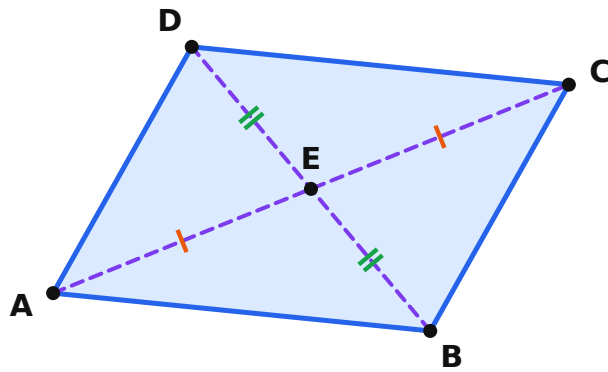
Problem 23: Rhombus floor tile

$AB = AD = 8$ because the sides of a rhombus are congruent, so triangle ABD is isosceles with vertex angle 60° . Its base angles are $\frac{180-60}{2} = 60^\circ$ each. All three angles are 60° , so the triangle is equilateral and $BD = 8$ in.

$BD = 8$ in; each angle of triangle ABD measures 60°

Problem 24: Proof: diagonals bisect each other

Statements	Reasons
1. $ABCD$ is a parallelogram	1. Given
2. $\overline{AB}$ is parallel to $\overline{DC}$	2. Definition of a parallelogram
3. $m\angle BAE = m\angle DCE$ and $m\angle ABE = m\angle CDE$	3. Alternate interior angles theorem (transversals $\overline{AC}$ and $\overline{BD}$)
4. $AB = CD$	4. Opposite sides of a parallelogram are congruent
5. Triangle ABE is congruent to triangle CDE	5. ASA (steps 3 and 4)
6. $AE = CE$ and $BE = DE$	6. Corresponding parts of congruent triangles are congruent (CPCTC)



Matching tick marks show the pieces proved congruent by CPCTC.

Reasons: Given; definition; alternate interior angles; opposite sides congruent; ASA; CPCTC