

Geometry · Parabolas from Focus and Directrix, and Tangent Lines to a Circle · Worksheet Answer Key

Common Core G-GPE.A.2, G-C.A.4 · mathovore.fr

This answer key shows a full worked solution for each problem, including the distance comparisons, the algebra for every equation and the right triangles for every tangent. Check each answer against the stated check value.

Geometry Common Core: G-GPE.A.2 · G-C.A.4

Problem 1: Checking a point against the definition

(a) Distance to the focus: $\sqrt{4^2 + (4-1)^2} = \sqrt{25} = 5$. Distance to the directrix: $4 - (-1) = 5$. The distances are equal, so A is on the parabola.

(b) Distance to the focus: $\sqrt{6^2 + 7^2} = \sqrt{85} \approx 9.22$. Distance to the directrix: $8 + 1 = 9$. They differ, so B is not on it. **Answer: A is on the parabola; B is not**

Problem 2: Solving for a missing ordinate

The distance to the directrix is $8 + 3 = 11$. The distance to the focus is $\sqrt{(8-3)^2 + y^2} = \sqrt{25 + y^2}$.

Set $\sqrt{25 + y^2} = 11$: $25 + y^2 = 121$, so $y^2 = 96$ and $y = \pm 4\sqrt{6} \approx \pm 9.80$. Check with $y^2 = 12x = 96$.

Answer: $y \approx 9.80$ or $y \approx -9.80$

Problem 3: Deriving an equation from the definition

For a point (x, y) : $\sqrt{x^2 + (y-4)^2} = |y+4|$.

Square both sides: $x^2 + y^2 - 8y + 16 = y^2 + 8y + 16$. Subtract $y^2 + 16$ from both sides and add $8y$: $x^2 = 16y$. **Answer: $x^2 = 16y$**

Problem 4: A parabola that opens downward

The vertex is the midpoint $(0, 0)$, and the focus is below it, so $p = -2$. Then $x^2 = 4py = 4(-2)y$.

Check with the point $(4, -2)$: distance to the focus 4, distance to the directrix $|-2 - 2| = 4$, and $4^2 = 16 = -8(-2)$. **Answer: $x^2 = -8y$**

Problem 5: A parabola that opens to the right

Set $\sqrt{(x-3)^2+y^2}=|x+3|$ and square: $x^2-6x+9+y^2=x^2+6x+9$. Then $y^2=12x$, which matches $y^2=4px$ with $p=3$. **Answer:** $y^2=12x$

Problem 6: Focus and directrix from an equation

Compare with $x^2=4py$: $4p=20$, so $p=5$. The focus is $(0,p)=(0,5)$ and the directrix is $y=-p=-5$. **Answer: focus** $(0,5)$; **directrix** $y=-5$

Problem 7: Rewriting a quadratic function

Multiply by 8: $x^2=8y$. Then $4p=8$ and $p=2$. The focus is $(0,2)$ and the directrix is $y=-2$. **Answer: focus** $(0,2)$; **directrix** $y=-2$

Problem 8: A sideways parabola

Compare with $y^2=4px$: $4p=-16$, so $p=-4$. The focus is $(p,0)=(-4,0)$ and the directrix is $x=-p=4$. Since $p<0$, it opens to the left. **Answer: focus** $(-4,0)$; **directrix** $x=4$; **opens left**

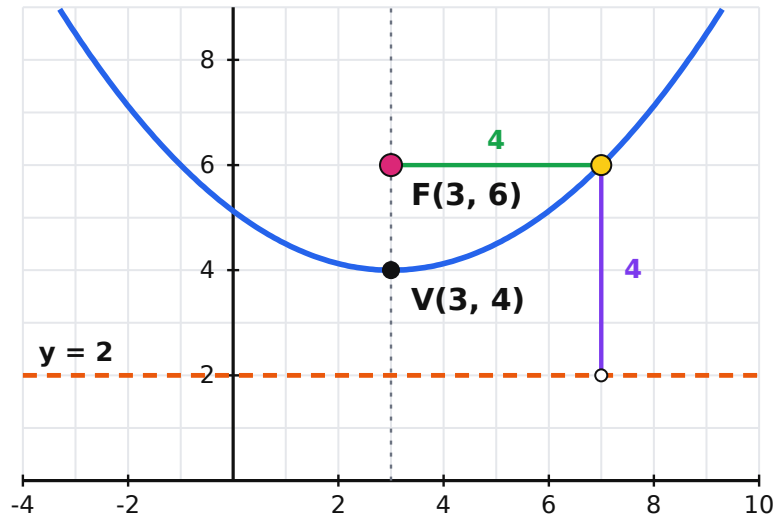
Problem 9: Vertex away from the origin

The vertex is the midpoint of $(3,6)$ and the point $(3,2)$ on the directrix: $(3,4)$. The focus is 2 above it, so $p=2$.

Equation: $(x-3)^2=4(2)(y-4)=8(y-4)$.

Check $(7,6)$: $(7-3)^2=16$ and $8(6-4)=16$. Distances: to the focus 4, to the directrix $6-2=4$.

Answer: vertex $(3,4)$, $p=2$, $(x-3)^2=8(y-4)$



Vertex $V(3, 4)$, $p = 2$.

Problem 10: Reading a shifted equation

Compare with $(x-h)^2 = 4p(y-k)$: $h = -2$, $k = 1$, $4p = 12$, $p = 3$.

Vertex $(-2, 1)$; focus $(h, k+p) = (-2, 4)$; directrix $y = k-p = -2$. **Answer: vertex $(-2, 1)$; focus $(-2, 4)$; directrix $y = -2$**

Problem 11: Focus and a vertical directrix

The vertex is halfway between $x = -1$ and $x = 5$ at $y = 3$: $(2, 3)$. The focus is 3 units to the left of the vertex, so $p = -3$.

$(y-3)^2 = 4(-3)(x-2) = -12(x-2)$. Check the point $(-1, 9)$: $(9-3)^2 = 36 = -12(-3)$, and the distances to the focus and to $x = 5$ are both 6. **Answer: $(y-3)^2 = -12(x-2)$**

Problem 12: Completing the square

Isolate the x-terms: $x^2 - 6x = 8y - 1$. Add 9 to both sides: $(x-3)^2 = 8y + 8 = 8(y+1)$.

So $h = 3$, $k = -1$, $p = 2$. Vertex $(3, -1)$; focus $(3, 1)$; directrix $y = -3$. **Answer: $(x-3)^2 = 8(y+1)$; vertex $(3, -1)$; focus $(3, 1)$; directrix $y = -3$**

Problem 13: Tangent perpendicular to a radius

The tangent is perpendicular to the radius, so triangle OTP has a right angle at T:

$OP^2 = 5^2 + 12^2 = 169$, so $OP = 13$. **Answer: $OP = 13$**

Problem 14: Length of a tangent segment

The tangent segment is a leg of a right triangle with hypotenuse $OP = 17$ and other leg 8:
 $PT = \sqrt{17^2 - 8^2} = \sqrt{225} = 15$. **Answer: 15 units**

Problem 15: Finding the radius

$r^2 + 24^2 = 25^2$, so $r^2 = 625 - 576 = 49$ and $r = 7$. **Answer: 7**

Problem 16: Equal tangent segments

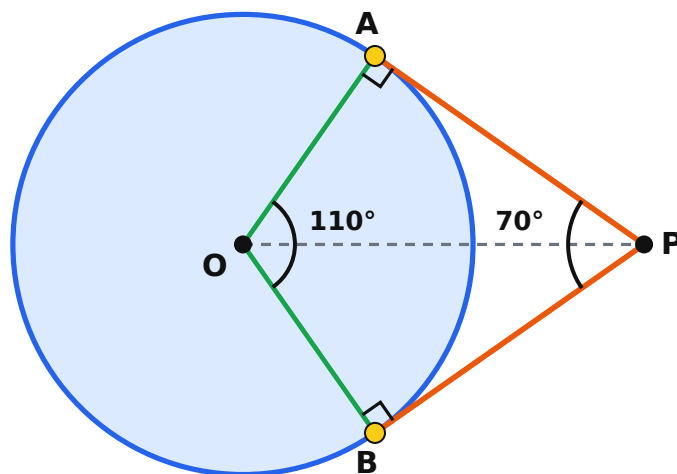
Tangent segments from one point are congruent: $3x + 4 = 5x - 6$. Then $10 = 2x$, so $x = 5$.

$PA = 3(5) + 4 = 19$ and $PB = 5(5) - 6 = 19$. **Answer: $x = 5$; $PA = 19$**

Problem 17: Angles around two tangents

The radii OA and OB are perpendicular to the tangents, so $\angle OAP = \angle OBP = 90^\circ$. The angles of quadrilateral $OAPB$ add to 360° :

$m\angle APB = 360^\circ - 90^\circ - 90^\circ - 110^\circ = 70^\circ$. **Answer: 70°**



$$90 + 90 + 110 + 70 = 360.$$

Problem 18: Is the line tangent?

A line is tangent at T exactly when $OT \perp PT$, so test the Pythagorean relationship.

(a) $9^2 + 12^2 = 81 + 144 = 225 = 15^2$: right angle at T, so tangent.

(b) $7^2 + 24^2 = 49 + 576 = 625$, but $26^2 = 676$: no right angle, so not tangent. **Answer: (a) tangent; (b) not tangent**

Problem 19: Tangent line in the coordinate plane

The radius from $O(0,0)$ to T has slope $\frac{4}{3}$, so the tangent has slope $-\frac{3}{4}$.

$y - 4 = -\frac{3}{4}(x - 3)$, so $4y - 16 = -3x + 9$ and $3x + 4y = 25$.

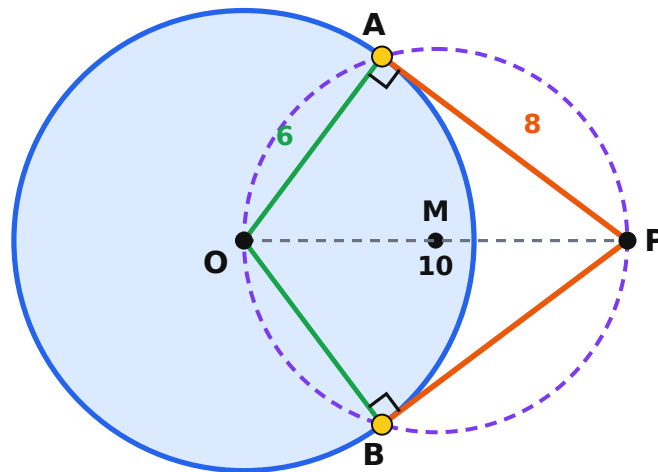
At $y = 0$: $x = \frac{25}{3}$. **Answer: $3x + 4y = 25$; crosses the x-axis at $(\frac{25}{3}, 0)$**

Problem 20: Why the midpoint circle works

(a) $MO = \frac{1}{2} \cdot 10 = 5$.

(b) OP is a diameter of the dashed circle and A is on that circle, so $\angle OAP$ is inscribed in a semicircle and equals 90° . Then $PA \perp OA$, so PA is tangent.

(c) $PA = \sqrt{10^2 - 6^2} = \sqrt{64} = 8$. **Answer: $MO = 5$; right angle by the semicircle; $PA = 8$**



Completed construction: 6, 8, 10.

Problem 21: Cable to a water tank

The cable is a tangent segment, perpendicular to the radius at the touching point:
 $\sqrt{20^2 - 12^2} = \sqrt{256} = 16$. **Answer: 16 ft**

Problem 22: Satellite dish

(a) $p = 1.5$, so $x^2 = 4(1.5)y = 6y$.

(b) At the rim $x=4$: $y=\frac{16}{6}=\frac{8}{3}\approx 2.67$ ft.

(c) The rim is $\frac{8}{3}+\frac{3}{2}=\frac{25}{6}$ ft from the directrix, hence from the focus: $\sqrt{16+\left(\frac{8}{3}-\frac{3}{2}\right)^2}=\sqrt{\frac{625}{36}}=\frac{25}{6}\approx 4.17$ ft. **Answer:** $x^2=6y$; **about 2.67 ft; about 4.17 ft**

Problem 23: Headlight reflector

(a) $p=2.5$, so $x^2=10y$.

(b) At $x=5$: $y=\frac{25}{10}=2.5$ in.

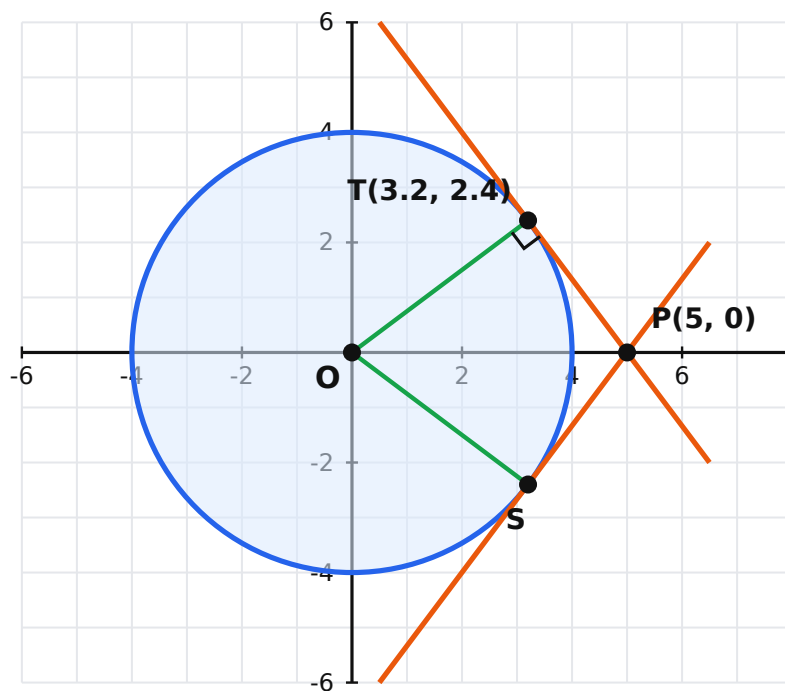
(c) The rim is at $(5,2.5)$ and the bulb at $(0,2.5)$, 5 in. apart. Check: the rim is $2.5+2.5=5$ in. from the directrix. **Answer:** $x^2=10y$; **2.5 in.; 5 in.**

Problem 24: Tangents from a point on the axis

(a) $OP=5$, $r=4$: $PT=\sqrt{25-16}=3$.

(b) For a point $T(x,y)$ of tangency, $OT\perp PT$: $x(x-5)+y^2=0$. With $x^2+y^2=16$ this gives $16-5x=0$, so $x=3.2$ and $y^2=16-10.24=5.76$, $y=\pm 2.4$. Check: $\sqrt{(5-3.2)^2+2.4^2}=3$.

(c) Slope of PT : $\frac{2.4-0}{3.2-5}=-\frac{4}{3}$. Then $y=-\frac{4}{3}(x-5)$, or $4x+3y=20$. Check T : $12.8+7.2=20$.
Answer: 3; (3.2, 2.4) and (3.2, -2.4); $4x+3y=20$



Upper tangent: $4x + 3y = 20$.