

Geometry · Parabolas from Focus and Directrix, and Tangent Lines to a Circle · Worksheet

Common Core G-GPE.A.2, G-C.A.4 · mathovore.fr

Name: _____ Date: ____/____/____ Score: _____

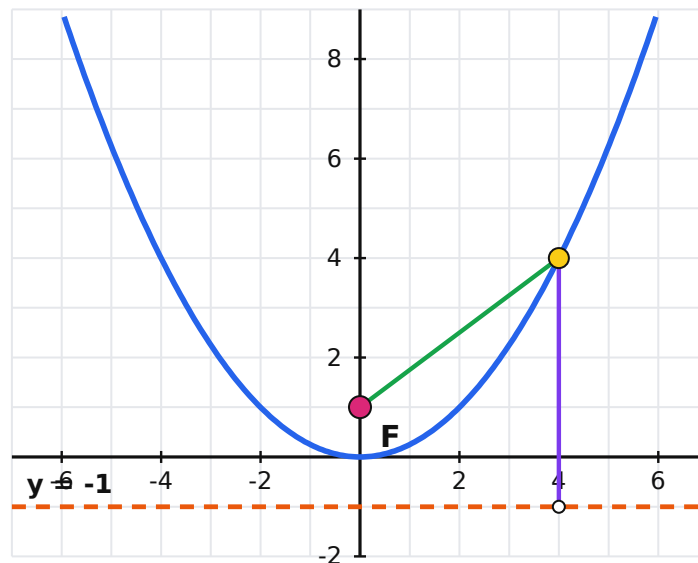
These 24 problems practice G-GPE.A.2 and G-C.A.4: finding the equation of a parabola from a focus and directrix, reading the focus and directrix from an equation, and working with tangent lines to a circle.

The last problems combine tangent constructions, coordinates and real settings such as a satellite dish and a headlight. Show distances, squares and checks.

GeometryCommon Core: G-GPE.A.2 · G-C.A.4

Problem 1: Checking a point against the definition

A parabola has focus $F(0,1)$ and directrix $y = -1$. (a) Show that $A(4,4)$ is on the parabola by comparing distances. (b) Decide whether $B(6,8)$ is on the parabola.



Point A(4, 4).

Problem 2: Solving for a missing ordinate

A parabola has focus $(3,0)$ and directrix $x = -3$. A point on it has x-coordinate 8. Find its y-coordinate(s) to the nearest hundredth.

Problem 3: Deriving an equation from the definition

Use the distance formula to derive the equation of the parabola with focus $(0,4)$ and directrix $y = -4$.

Problem 4: A parabola that opens downward

Write the equation of the parabola with focus $(0,-2)$ and directrix $y = 2$.

Problem 5: A parabola that opens to the right

Write the equation of the parabola with focus $(3,0)$ and directrix $x = -3$.

Problem 6: Focus and directrix from an equation

Find the focus and the directrix of $x^2 = 20y$.

Problem 7: Rewriting a quadratic function

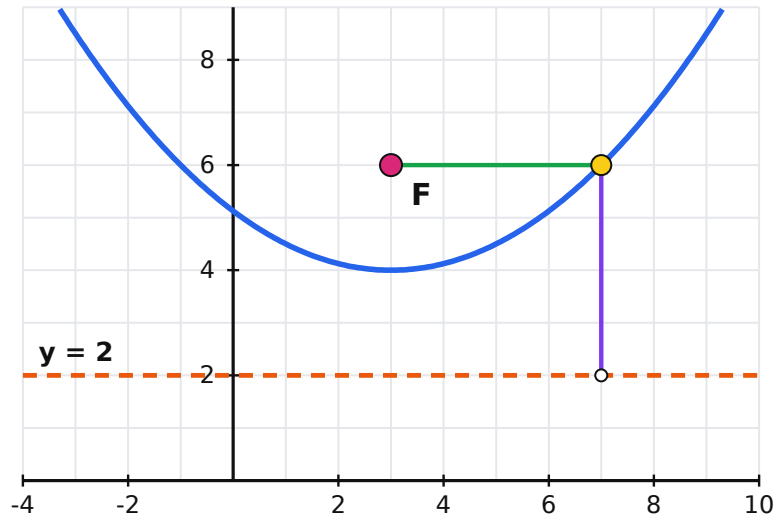
The graph of $y = \frac{1}{8}x^2$ is a parabola. Find its focus and directrix.

Problem 8: A sideways parabola

Find the focus and the directrix of $y^2 = -16x$, and state the direction in which it opens.

Problem 9: Vertex away from the origin

A parabola has focus $(3,6)$ and directrix $y = 2$. Find the vertex and p, write the equation, and verify that $(7,6)$ is on the parabola.



Focus $F(3, 6)$ and directrix $y = 2$.

Problem 10: Reading a shifted equation

Find the vertex, focus and directrix of $(x+2)^2 = 12(y-1)$.

Problem 11: Focus and a vertical directrix

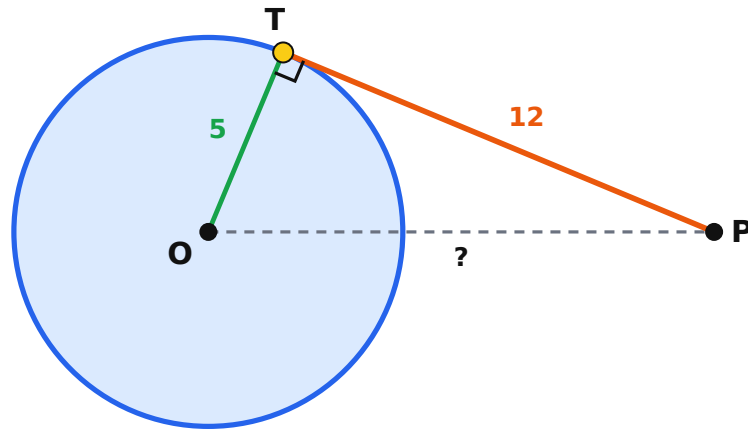
A parabola has focus $(-1, 3)$ and directrix $x = 5$. Write its equation in the form $(y-k)^2 = 4p(x-h)$.

Problem 12: Completing the square

Rewrite $x^2 - 6x - 8y + 1 = 0$ in the form $(x-h)^2 = 4p(y-k)$, then find the vertex, focus and directrix.

Problem 13: Tangent perpendicular to a radius

In the figure, $\overline{PT}$ is tangent to the circle with center O at T . The radius is 5 and $PT = 12$. Find OP .



Right angle at T.

Problem 14: Length of a tangent segment

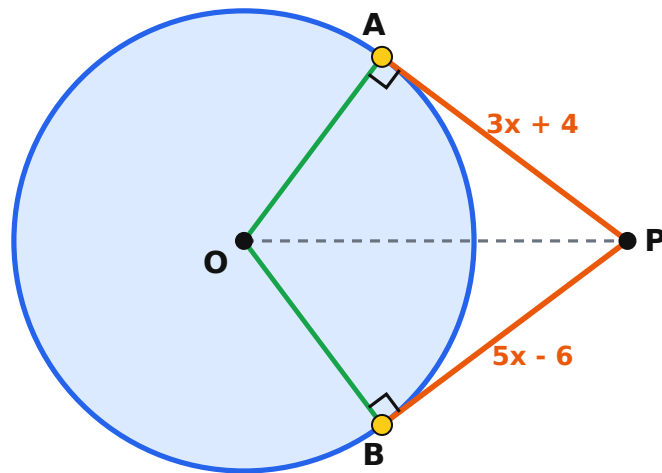
Point P is 17 units from the center O of a circle with radius 8. Find the length of a tangent segment from P to the circle.

Problem 15: Finding the radius

A tangent segment from P has length 24 and $OP = 25$. Find the radius of the circle.

Problem 16: Equal tangent segments

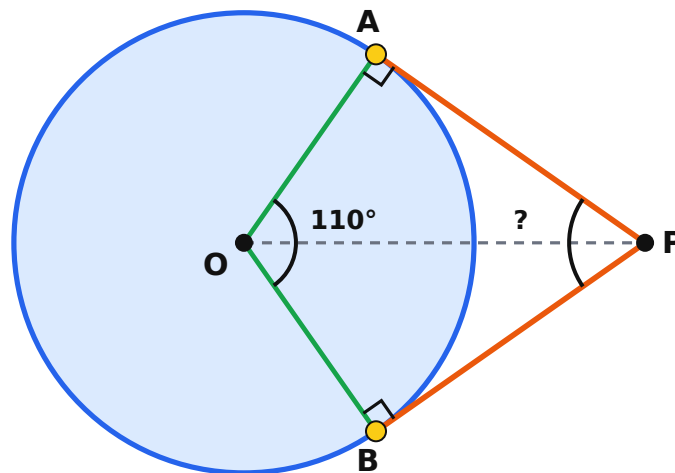
Tangent segments $\overline{PA}$ and $\overline{PB}$ are drawn from P, with $PA = 3x + 4$ and $PB = 5x - 6$. Find x and PA.



Two tangent segments from P.

Problem 17: Angles around two tangents

Tangent segments $\overline{PA}$ and $\overline{PB}$ touch a circle with center O at A and B. If $m\angle AOB = 110^\circ$, find $m\angle APB$.



Quadrilateral OAPB.

Problem 18: Is the line tangent?

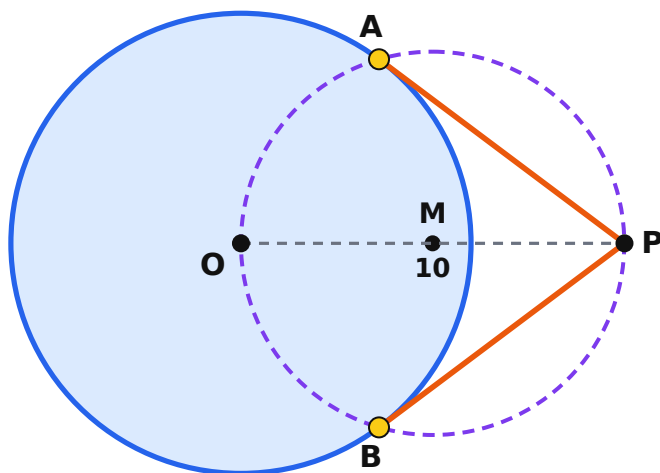
Decide whether $\overline{PT}$ is tangent to the circle at T. (a) $OT = 9$, $TP = 12$, $OP = 15$. (b) $OT = 7$, $TP = 24$, $OP = 26$.

Problem 19: Tangent line in the coordinate plane

The circle $x^2 + y^2 = 25$ passes through $T(3,4)$. Find the equation of the tangent line at T and the point where it crosses the x-axis.

Problem 20: Why the midpoint circle works

A tangent is constructed from P to a circle with center O and radius 6, where $OP = 10$. M is the midpoint of $\overline{OP}$, and the circle centered at M through O meets the given circle at A. (a) Find MO. (b) Explain why $\angle OAP = 90^\circ$. (c) Find PA.



Construction from P.

Problem 21: Cable to a water tank

A cable is stretched from a point P that is 20 ft from the center of a circular water tank to the tank, so that it just touches the side at one point. The tank has radius 12 ft. How long is the cable?

Problem 22: Satellite dish

A satellite dish is a paraboloid. Its cross-section has vertex at the origin and opens upward, with the receiver at the focus 1.5 ft above the vertex. (a) Write the equation. (b) The dish is 8 ft wide. Find its depth. (c) Find the distance from the receiver to the rim.

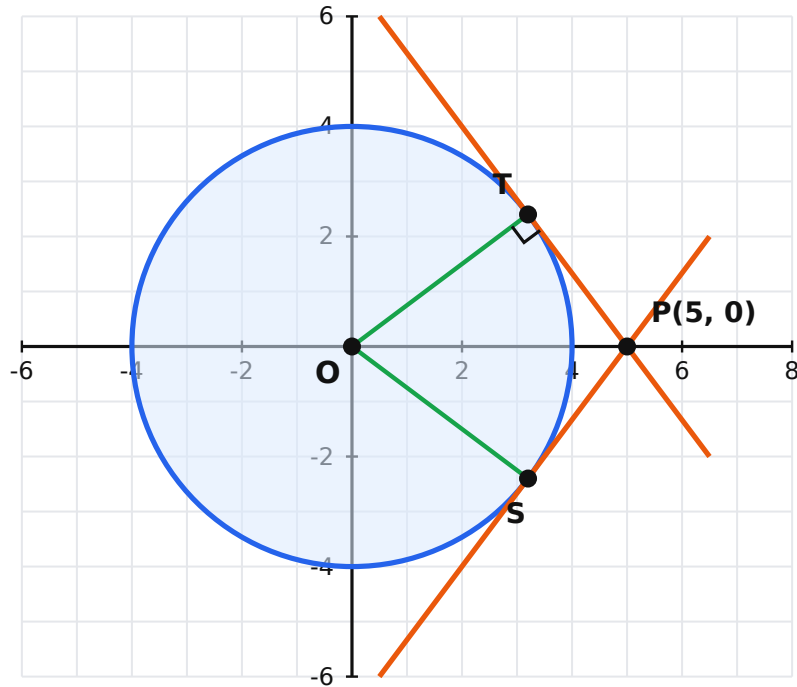
Problem 23: Headlight reflector

A headlight reflector has a parabolic cross-section with its vertex at the origin. The bulb is at the focus, 2.5 in. from the vertex, and the reflector opens upward. (a) Write the equation. (b)

The reflector is 10 in. wide at the opening. How deep is it? (c) How far is the bulb from the rim?

Problem 24: Tangents from a point on the axis

Point $P(5,0)$ lies outside the circle $x^2+y^2=16$. (a) Find the length of a tangent segment from P. (b) Find the coordinates of both points of tangency. (c) Write the equation of the tangent through the upper point of tangency.



Circle radius 4 and point $P(5, 0)$.