

Geometry · Parabolas from Focus and Directrix, and Tangent Lines to a Circle · Test Answer Key

Common Core G-GPE.A.2, G-C.A.4 · mathovore.fr

This answer key gives the worked solution and a scoring guide for each question of the parabola and tangent test. Award points as described, give partial credit for a correct setup, and total the score out of 20 points.

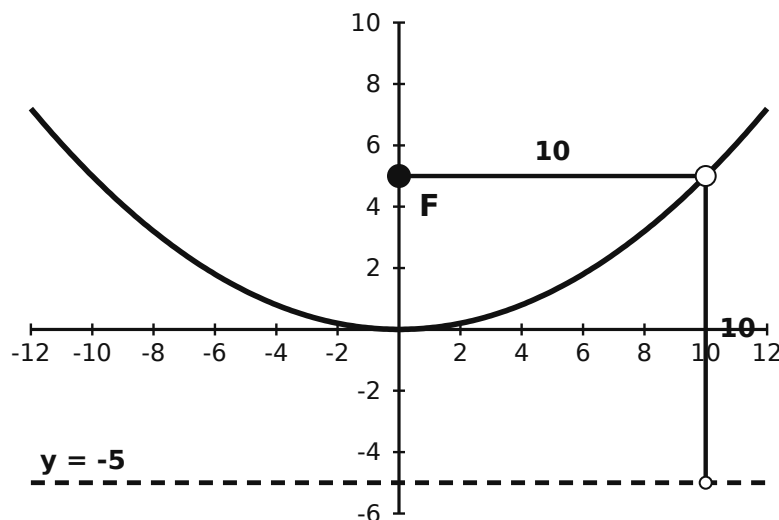
GeometryCommon Core: G-GPE.A.2 · G-C.A.4

Question 1: Equation from a focus and a directrix (2 points)

(a) $p=5$ and the vertex is the origin: $x^2=20y$.

(b) $PF=10$ and the distance to the directrix is $5+5=10$; also $10^2=20\cdot5$.

Scoring: 1 point for $x^2=20y$; 1 point for equal distances (or substitution) with the numbers shown.



Both distances are 10.

Question 2: Focus and directrix from an equation (2 points)

$4p=24$, so $p=6$. Focus $(0,6)$; directrix $y=-6$.

Scoring: 1 point for the focus, 1 point for the directrix.

Question 3: A sideways parabola (2 points)

The vertex is $(0,0)$ and $p=-3$, so $y^2=4(-3)x=-12x$. Check $(-3,6)$: distance to the focus 6, to the directrix $|-3-3|=6$, and $36=-12(-3)$.

Scoring: 1 point for the vertex and $p=-3$; 1 point for $y^2=-12x$.

Question 4: Parabola with a shifted vertex (3 points)

(a) The vertex is the midpoint of $(1,7)$ and $(1,3)$: $(1,5)$, and $p=2$. The equation is $(x-1)^2=8(y-5)$.

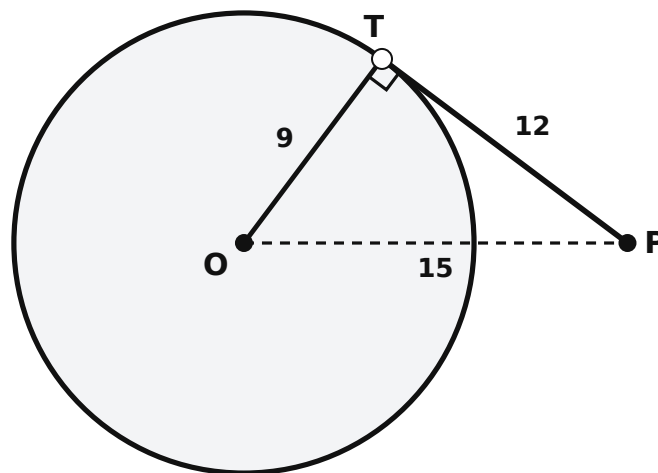
(b) For $x=5$: $16=8(y-5)$, so $y=7$.

Scoring: (a) 1 point for the vertex and p , 1 point for the equation; (b) 1 point for $y=7$.

Question 5: Length of a tangent (2 points)

$OT \perp PT$, so $PT = \sqrt{15^2 - 9^2} = \sqrt{144} = 12$.

Scoring: 1 point for identifying the right angle at T (or writing $9^2 + PT^2 = 15^2$); 1 point for 12.



Right triangle 9-12-15.

Question 6: Two tangent segments (2 points)

$2x+7=4x-5$, so $12=2x$ and $x=6$. Then $PA=2(6)+7=19$ (and $PB=4(6)-5=19$).

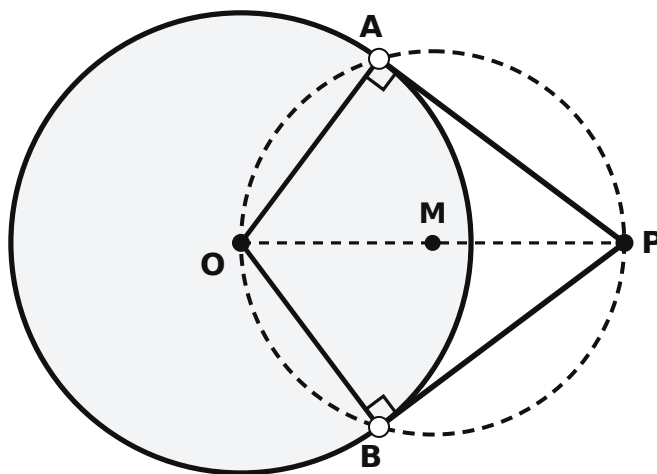
Scoring: 1 point for the equation from congruent tangent segments and $x=6$; 1 point for $PA=19$.

Question 7: Constructing a tangent (3 points)

(a) Draw OP ; construct its midpoint M ; draw the circle with center M and radius MO ; it meets the given circle at A and B ; draw lines PA and PB .

(b) $\angle OAP$ is inscribed in a semicircle with diameter OP , so it is 90° . A line perpendicular to a radius at its end on the circle is tangent.

Scoring: (a) 1 point for the midpoint and the circle centered at M through O , 1 point for the intersection points and the two lines; (b) 1 point for the semicircle (right angle) reason.



Completed construction.

Question 8: Angles formed by two tangents (2 points)

(a) Quadrilateral OAPB has right angles at A and B: $m\angle AOB = 360^\circ - 90^\circ - 90^\circ - 48^\circ = 132^\circ$.

(b) OP bisects $\angle APB$, so $m\angle OPA = 48^\circ \div 2 = 24^\circ$.

Scoring: 1 point for each part.

Question 9: A satellite dish (2 points)

(a) $p=2$, so $x^2=8y$.

(b) At the rim $x=5$: $y=\frac{25}{8}=3.125$ ft.

Scoring: 1 point per part.

Total
20 points