

Geometry · Cross Sections, Solids of Rotation and Geometric Modeling · Worksheet Answer Key

Common Core G-GMD.B.4, G-MG.A.1, G-MG.A.2, G-MG.A.3 · mathovore.fr

Each worked solution names the shape or formula, shows the substitution and ends with the final answer in bold. Use the answers to check units, and compare your reasoning for the design problems step by step.

GeometryCommon Core: G-GMD.B.4 · G-MG.A.1 · G-MG.A.2 · G-MG.A.3

Problem 1: Slicing a cube

A cut parallel to a face of a cube is a square the same size as the face.

The area is $6 \times 6 = 36 \text{ cm}^2$.

Answer: **a square with area 36 cm^2 .**

Problem 2: Rectangle from a cylinder

The cross section is a rectangle. Its width is the diameter, $2 \times 4 = 8 \text{ cm}$, and its height is 10 cm .

Area $= 8 \times 10 = 80 \text{ cm}^2$.

Answer: **a rectangle, 8 cm by 10 cm , area 80 cm^2 .**

Problem 3: Circle from a cylinder

The cross section is a circle with the same radius, 5 cm .

Area $= \pi(5)^2 = 25\pi \approx 78.54 \text{ cm}^2$. Circumference $= 2\pi(5) = 10\pi \approx 31.42 \text{ cm}$.

Answer: **a circle; $25\pi \text{ cm}^2$; $10\pi \text{ cm}$.**

Problem 4: Cutting a cone

(a) A circle. (b) An isosceles triangle with base $2 \times 3 = 6 \text{ in}$ and height 4 in .

(c) Area $= \frac{1}{2}(6)(4) = 12 \text{ in}^2$.

Answers: **circle; triangle; 12 in^2 .**

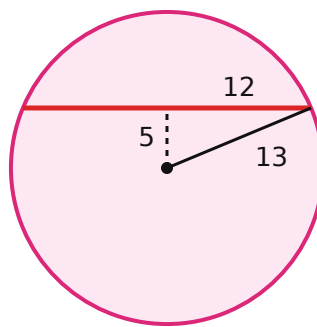
Problem 5: Slicing a sphere

The radius of the sphere, the distance 5, and the slice radius form a right triangle with hypotenuse 13.

$$x^2 + 5^2 = 13^2, \text{ so } x^2 = 144 \text{ and } x = 12 \text{ in.}$$

$$\text{Area} = \pi(12)^2 = 144\pi \approx 452.39 \text{ in}^2.$$

Answer: **radius 12 in ; area $144\pi \text{ in}^2$.**



Right triangle 5, 12, 13.

Problem 6: Slicing a pyramid

Halfway up, the cross section is a smaller square similar to the base with scale factor $\frac{1}{2}$.

Side $= \frac{1}{2}(10) = 5 \text{ cm}$. Area $= 5^2 = 25 \text{ cm}^2$, which is $\frac{1}{4}$ of the base area 100 cm^2 .

Answer: **a square with side 5 cm and area 25 cm^2 .**

Problem 7: Slices of a prism

A cut parallel to a face is a rectangle congruent to that face.

The 6 by 4 faces give 24 cm^2 . The 6 by 3 faces give 18 cm^2 . The 4 by 3 faces give 12 cm^2 .

Answer: 24 cm^2 , 18 cm^2 , 12 cm^2 .

Problem 8: Rotating a rectangle

The solid is a cylinder with radius 3 cm and height 7 cm .

$$V = \pi(3)^2(7) = 63\pi \approx 197.92 \text{ cm}^3.$$

Answer: **cylinder**, $r = 3 \text{ cm}$, $h = 7 \text{ cm}$, $V = 63\pi \text{ cm}^3$.

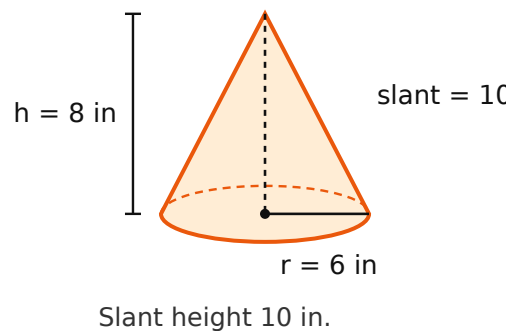
Problem 9: Rotating a right triangle

The solid is a cone with radius 6 in and height 8 in.

$$\text{Slant height} = \sqrt{6^2 + 8^2} = \sqrt{100} = 10 \text{ in.}$$

$$V = \frac{1}{3}\pi(6)^2(8) = 96\pi \approx 301.59 \text{ in}^3.$$

Answer: **cone**; **slant height** 10 in; $V = 96\pi \text{ in}^3$.



Problem 10: Rotating about the other leg

Now the radius is 8 in and the height is 6 in: $V = \frac{1}{3}\pi(8)^2(6) = 128\pi \approx 402.12 \text{ in}^3$.

$$\text{Difference: } 128\pi - 96\pi = 32\pi \approx 100.53 \text{ in}^3.$$

Answer: $128\pi \text{ in}^3$; $32\pi \text{ in}^3$ **more**.

Problem 11: Rotating a semicircle

The solid is a sphere with radius 5 cm.

$$V = \frac{4}{3}\pi(5)^3 = \frac{500\pi}{3} \approx 523.60 \text{ cm}^3.$$

Answer: **sphere**; $\frac{500\pi}{3} \approx 523.60 \text{ cm}^3$.

Problem 12: Rotating a triangle about its altitude

The solid is a cone. The base of the triangle becomes the circle, so $r = 8 \div 2 = 4 \text{ in}$, and $h = 5 \text{ in}$.

$$V = \frac{1}{3}\pi(4)^2(5) = \frac{80\pi}{3} \approx 83.78 \text{ in}^3.$$

Answer: **cone**; $\frac{80\pi}{3} \approx 83.78 \text{ in}^3$.

Problem 13: Modeling a tree trunk

The radius is 1 ft. $V = \pi(1)^2(30) = 30\pi \approx 94.25 \text{ ft}^3$.

Answer: **about** 94.25 ft^3 .

Problem 14: Population density

Density $= 45,000 \div 30 = 1,500$.

Answer: **1,500 people per square mile**.

Problem 15: Density of a county

Area $= 12 \times 9 = 108 \text{ mi}^2$.

Density $= 54,000 \div 108 = 500$.

Answer: 108 mi^2 ; **500 people per square mile**.

Problem 16: Heating a room

Volume $= 12 \times 10 \times 8 = 960 \text{ ft}^3$.

BTUs $= 960 \times 6 = 5,760$.

Answer: **5,760 BTUs**.

Problem 17: Density of a metal block

Volume $= 5 \times 4 \times 3 = 60 \text{ cm}^3$.

Density $= 162 \div 60 = 2.7$.

Answer: 2.7 g/cm^3 .

Problem 18: Comparing towns

Town A: $9,600 \div 12 = 800$ people per mi^2 . Town B: $7,350 \div 7 = 1,050$ people per mi^2 .

$1,050 - 800 = 250$.

Answer: **Town B, by 250 people per square mile**.

Problem 19: Fencing a pen

The length and width add to $48 \div 2 = 24$ ft. Test pairs: $8 \times 16 = 128$, $10 \times 14 = 140$, $11 \times 13 = 143$, $12 \times 12 = 144$, and the areas then decrease.

The best pen is 12 ft by 12 ft with area 144 ft^2 . Compared with 128 ft^2 , it adds 16 ft^2 .

Answer: 12 ft **by** 12 ft; 16 ft^2 **more**.

Problem 20: Two soup cans

Volumes: A: $\pi(9)(8) = 72\pi$. B: $\pi(16)(4.5) = 72\pi$.

Surface area $= 2\pi r^2 + 2\pi rh$. A: $18\pi + 48\pi = 66\pi \approx 207.35 \text{ cm}^2$. B: $32\pi + 36\pi = 68\pi \approx 213.63 \text{ cm}^2$.

Answer: **Can A needs less metal ($66\pi \text{ cm}^2$ versus $68\pi \text{ cm}^2$)**.

Problem 21: Sizing a water tank

Volume needed $= 1000 \div 7.5 \approx 133.33 \text{ ft}^3$.

$\pi(3)^2 h \geq 133.33$, so $h \geq \frac{133.33}{9\pi} \approx 4.72$.

Rounding up to the next tenth so the tank holds enough: 4.8 ft.

Problem 22: Choosing a shipping box

Volumes: $10 \times 8 \times 6 = 480 \text{ in}^3$ and $12 \times 8 \times 5 = 480 \text{ in}^3$.

Surface areas: Box 1: $2(80 + 60 + 48) = 376 \text{ in}^2$. Box 2: $2(96 + 60 + 40) = 392 \text{ in}^2$.

Costs: $376 \times 0.02 = \$7.52$ and $392 \times 0.02 = \$7.84$.

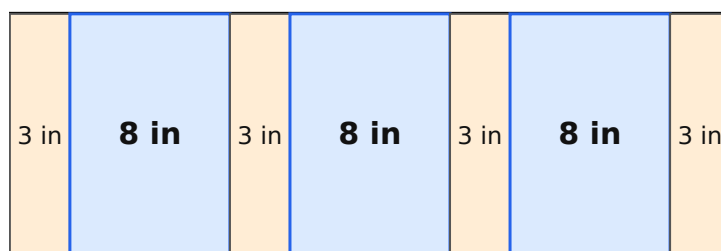
Answer: **Box 1 is cheaper by \$0.32**.

Problem 23: Banner grid

Margins use $2 \times 3 = 6$ in and gutters use $2 \times 3 = 6$ in, so $36 - 6 - 6 = 24$ in remain for the columns.

Each column is $24 \div 3 = 8$ in.

Answer: 8 in.



Banner width: 36 in

Grid for a 36-inch banner.

Problem 24: Crowded field

$$\text{Area} = 120 \times 80 = 9,600 \text{ yd}^2.$$

$$(a) 14,400 \div 9,600 = 1.5 \text{ people per yd}^2.$$

$$(b) \text{Maximum} = 2 \times 9,600 = 19,200 \text{ people, so } 19,200 - 14,400 = 4,800 \text{ more.}$$

Answers: **1.5 people per square yard; 4,800 more people.**