

Geometry · Law of Sines, Law of Cosines and Triangle Area · Worksheet Answer Key

Common Core G-SRT.D.9, G-SRT.D.10, G-SRT.D.11 · mathovore.fr

This answer key shows a full worked solution for every problem on the Law of Sines and Law of Cosines worksheet. Check the choice of law first, then the setup and rounding. Small differences in the last digit can come from rounding too early.

Geometry Common Core: G-SRT.D.9 · G-SRT.D.10 · G-SRT.D.11

Problem 1: Area from two sides and an angle

The angle is between the two given sides, so $\text{Area} = \frac{1}{2}ab\sin C$.

$$\text{Area} = \frac{1}{2}(6)(9)\sin 30^\circ = 27(0.5) = 13.5.$$

13.5 square units

Problem 2: Area, decimal answer

$$\text{Area} = \frac{1}{2}(12)(15)\sin 50^\circ = 90\sin 50^\circ \approx 68.9.$$

$$\approx 68.9 \text{ cm}^2$$

Problem 3: Area with an obtuse angle

The formula works for obtuse angles. $\text{Area} = \frac{1}{2}(7.5)(10)\sin 115^\circ = 37.5\sin 115^\circ \approx 34.0$.

$$\approx 34.0 \text{ in}^2$$

Problem 4: Angle from the area

$$60 = \frac{1}{2}(10)(15)\sin C = 75\sin C, \text{ so } \sin C = \frac{60}{75} = 0.8.$$

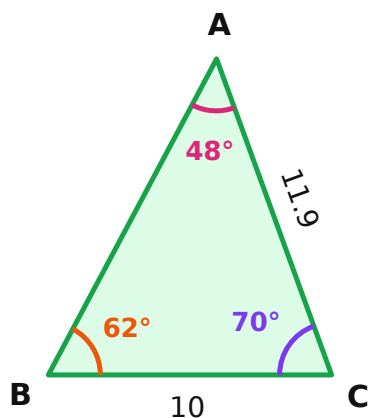
$$C = \sin^{-1}(0.8) \approx 53.1^\circ.$$

$$m\angle C \approx 53.1^\circ$$

Problem 5: Law of Sines for a side

$$\frac{b}{\sin 62^\circ} = \frac{10}{\sin 48^\circ}, \text{ so } b = \frac{10\sin 62^\circ}{\sin 48^\circ} \approx 11.9.$$

$$b \approx 11.9$$



Side b is about 11.9.

Problem 6: Third angle, then two sides

$$A = 180^\circ - 35^\circ - 80^\circ = 65^\circ.$$

$$a = \frac{24 \sin 65^\circ}{\sin 80^\circ} \approx 22.1 \text{ and } b = \frac{24 \sin 35^\circ}{\sin 80^\circ} \approx 14.0.$$

$$A = 65^\circ, a \approx 22.1, b \approx 14.0$$

Problem 7: Law of Sines for an angle

$$\frac{\sin B}{12} = \frac{\sin 70^\circ}{15}, \text{ so } \sin B = \frac{12 \sin 70^\circ}{15} \approx 0.7518.$$

$$B = \sin^{-1}(0.7518) \approx 48.7^\circ.$$

$$m \angle B \approx 48.7^\circ$$

Problem 8: Law of Cosines, SAS

$$c^2 = 7^2 + 12^2 - 2(7)(12)\cos 60^\circ = 49 + 144 - 168(0.5) = 109.$$

$$c = \sqrt{109} \approx 10.4.$$

$$c \approx 10.4$$

Problem 9: Law of Cosines, obtuse angle

$$a^2 = 13^2 + 9^2 - 2(13)(9)\cos 112^\circ. \text{ Since } \cos 112^\circ \approx -0.3746 \text{ is negative, subtracting it adds:}$$

$$a^2 \approx 250 + 87.66 \approx 337.7.$$

$$a \approx 18.4.$$

$$a \approx 18.4$$

Problem 10: Law of Cosines, SSS

The largest angle is opposite the longest side, 12.

$$\cos C = \frac{7^2 + 9^2 - 12^2}{2(7)(9)} = \frac{-14}{126} \approx -0.1111.$$

$$C = \cos^{-1}(-0.1111) \approx 96.4^\circ.$$

$$\approx 96.4^\circ$$

Problem 11: All three angles

Let $a=13$, $b=14$ and $c=15$.

$$\cos A = \frac{14^2 + 15^2 - 13^2}{2(14)(15)} = \frac{252}{420} = 0.6, \text{ so } A \approx 53.1^\circ.$$

$$\cos B = \frac{13^2 + 15^2 - 14^2}{2(13)(15)} = \frac{198}{390}, \text{ so } B \approx 59.5^\circ.$$

$$C = 180^\circ - 53.1^\circ - 59.5^\circ \approx 67.4^\circ \text{ (check with } \cos C = \frac{140}{364} \text{)}.$$

$$A \approx 53.1^\circ, B \approx 59.5^\circ, C \approx 67.4^\circ$$

Problem 12: Choose the law

- a. Two sides and the included angle (SAS): **Law of Cosines**, to find c .
- b. Two angles and a side (AAS): **Law of Sines**, since α and A are a known pair.
- c. Three sides (SSS): **Law of Cosines**, solved for an angle.
- d. Two sides and a non-included angle (SSA): **Law of Sines**, and check whether two triangles exist.

Problem 13: Cosines and the Pythagorean theorem

$$\text{Since } \cos 90^\circ = 0, c^2 = a^2 + b^2 - 2ab(0) = a^2 + b^2.$$

$$\text{With } a=6 \text{ and } b=8: c^2 = 36 + 64 = 100, \text{ so } c=10.$$

$$c^2 = a^2 + b^2$$

Problem 14: Proof of the area formula

The reasons are shown below.

Step	Statements	Reasons
1	Draw altitude $\overline{AD}$ to $\overline{BC}$, with $AD=h$	Through a point not on a line, there is exactly one perpendicular to the line

Step	Statements	Reasons
2	$\text{Area} = \frac{1}{2}ah$	Area of a triangle is one half base times height
3	$\sin C = \frac{h}{b}$	Definition of sine in right triangle ADC
4	$h = b\sin C$	Multiplication property of equality (multiply both sides by b)
5	$\text{Area} = \frac{1}{2}ab\sin C$	Substitution (replace h in step 2 with $b\sin C$)

Problem 15: Area after the Law of Sines

$C = 180^\circ - 40^\circ - 75^\circ = 65^\circ$. Find b with the Law of Sines: $b = \frac{12\sin 75^\circ}{\sin 40^\circ} \approx 18.03$.

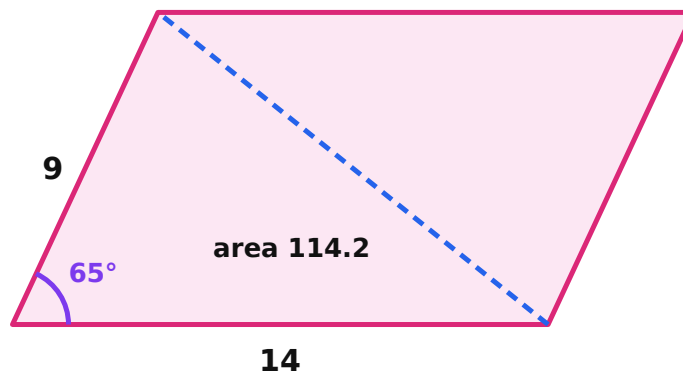
Sides a and b have included angle C : $\text{Area} = \frac{1}{2}(12)(18.03)\sin 65^\circ \approx 98.1$.

≈ 98.1 square units

Problem 16: Area of a parallelogram

Each triangle has area $\frac{1}{2}(14)(9)\sin 65^\circ$. The parallelogram is two triangles:
 $2 \cdot \frac{1}{2}(14)(9)\sin 65^\circ = 126\sin 65^\circ \approx 114.2$.

≈ 114.2 square units



Area is $126 \sin 65$ degrees, about 114.2.

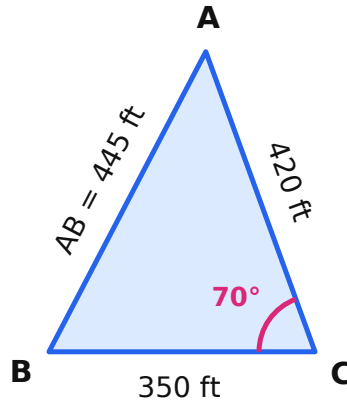
Problem 17: Distance across a pond

Two sides and the included angle are known, so use the Law of Cosines.

$$AB^2 = 420^2 + 350^2 - 2(420)(350)\cos 70^\circ \approx 298,900 - 100,553.9 \approx 198,346.$$

$$AB \approx 445.$$

About 445 ft



The pond is about 445 ft across.

Problem 18: Two boats leave a dock

The paths are two sides and the included angle of a triangle.

$$d^2 = 18^2 + 24^2 - 2(18)(24)\cos 70^\circ \approx 900 - 295.5 \approx 604.5.$$

$$d \approx 24.6.$$

$$\approx 24.6 \text{ mi}$$

Problem 19: Sod for a triangular lot

$$\text{Area} = \frac{1}{2}(120)(85)\sin 74^\circ = 5100\sin 74^\circ \approx 4902.4, \text{ which rounds to } 4902 \text{ square feet.}$$

$$\text{Cost: } 4902 \times 0.45 = 2205.90.$$

4902 ft²; \$2,205.90

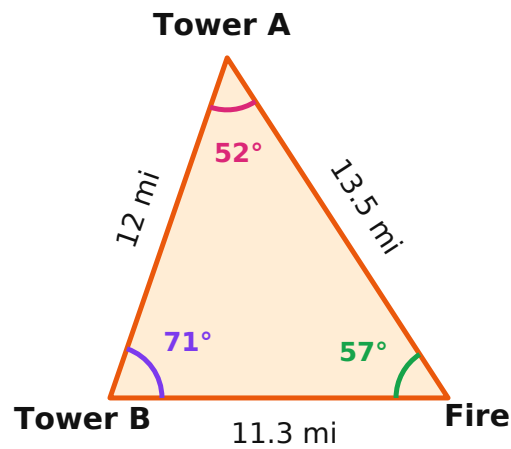
Problem 20: Locating a fire

The angle at the fire is $C = 180^\circ - 52^\circ - 71^\circ = 57^\circ$. Side $c = 12$ is opposite C .

$$\text{Distance from A (side } b, \text{ opposite } B): b = \frac{12\sin 71^\circ}{\sin 57^\circ} \approx 13.5.$$

$$\text{Distance from B (side } a): a = \frac{12\sin 52^\circ}{\sin 57^\circ} \approx 11.3.$$

≈ 13.5 mi from A, ≈ 11.3 mi from B



Distances from each tower to the fire.

Problem 21: Plane with a turn

The turn of 40° leaves an interior angle of $180^\circ - 40^\circ = 140^\circ$ in the triangle.

$$d^2 = 150^2 + 90^2 - 2(150)(90)\cos 140^\circ = 30,600 + 20683.2 \approx 51283.2.$$

$$d \approx 226.5.$$

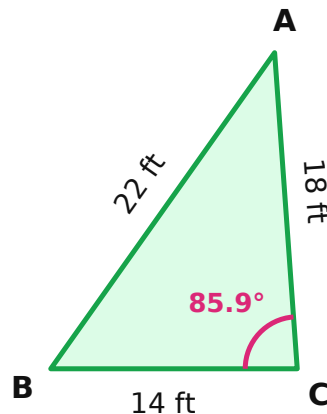
$$\approx 226.5 \text{ mi}$$

Problem 22: Garden angle and area

The largest angle is opposite 22 ft: $\cos C = \frac{14^2 + 18^2 - 22^2}{2(14)(18)} = \frac{36}{504} \approx 0.0714$, so $C \approx 85.9^\circ$.

The sides 14 and 18 enclose C : $\text{Area} = \frac{1}{2}(14)(18)\sin 85.9^\circ = 126\sin 85.9^\circ \approx 125.7$.

$$C \approx 85.9^\circ, \text{ area} \approx 125.7 \text{ ft}^2$$



The largest angle is opposite the 22 ft side.