

Geometry · Law of Sines, Law of Cosines and Triangle Area · Lesson

Common Core G-SRT.D.9, G-SRT.D.10, G-SRT.D.11 · mathovore.fr

The area of a triangle is one half the product of two sides and the sine of the included angle. The Law of Sines relates each side to the sine of its opposite angle, and the Law of Cosines extends the Pythagorean theorem. Together they let you solve any triangle, not just right triangles.

Geometry Common Core: G-SRT.D.9 · G-SRT.D.10 · G-SRT.D.11

Key facts:

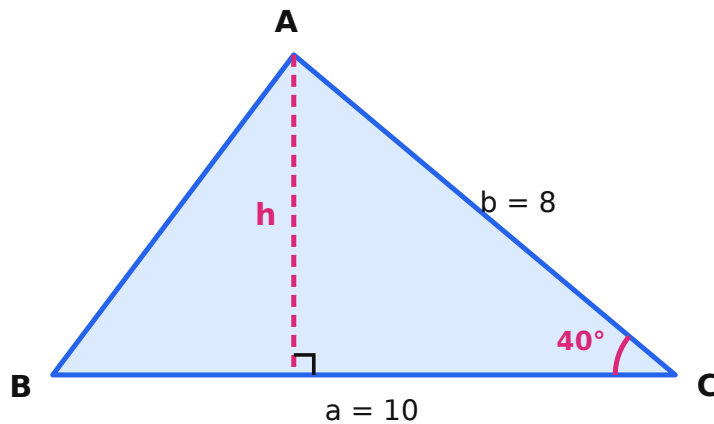
- Standards G-SRT.D.9, G-SRT.D.10 and G-SRT.D.11 cover the area formula, the two laws and applying them.
- Area = $(1/2)ab \sin C$, where C is the angle between sides a and b .
- Law of Sines: $\sin A / a = \sin B / b = \sin C / c$, for AAS, ASA and SSA.
- Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$, for SAS and SSS.
- SSA can give two triangles, so check the obtuse angle too.

How do you find the area of a triangle from two sides and an angle?

When you know two sides and the angle between them, you do not need the height. Draw the altitude h from A to side a . In the right triangle formed at C , $\sin C = \frac{h}{b}$, so $h = b \sin C$. Then the area $\frac{1}{2}ah$ becomes a formula that uses only sides and an angle.

Area of a triangle

Area = $\frac{1}{2}ab \sin C$, where C is the angle between sides a and b .



The altitude h equals $b \sin C$, so the area is one half $a b \sin C$.

For $a=10$, $b=8$ and $C=40^\circ$: $\text{Area} = \frac{1}{2}(10)(8)\sin 40^\circ \approx 25.7$ square units.

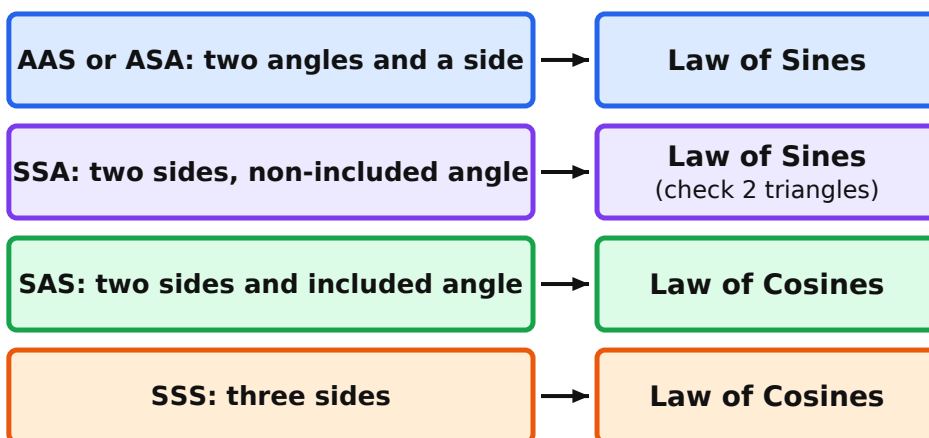
What are the Law of Sines and the Law of Cosines?

Two laws for any triangle

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$

The Law of Sines follows from writing the area three ways, $\frac{1}{2}ab \sin C = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B$, and dividing by $\frac{1}{2}abc$. The Law of Cosines extends the Pythagorean theorem: when $C=90^\circ$, $\cos C=0$ and the last term vanishes.

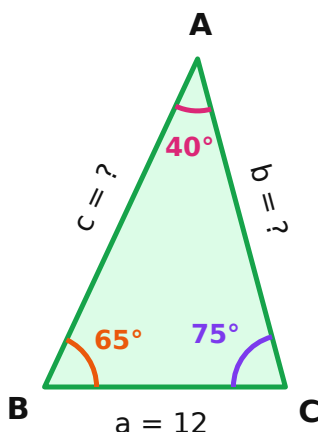


Choose the law from the information you are given.

How do you solve a triangle with the Law of Sines?

1. Find the third angle with $A+B+C=180^\circ$.
2. Write the proportion that contains one unknown side and a known side-angle pair.
3. Cross-multiply and divide, then round at the end.

Example. In the triangle, $A=40^\circ$, $B=65^\circ$ and $a=12$. Then $C=75^\circ$, $b=\frac{12\sin 65^\circ}{\sin 40^\circ} \approx 16.9$ and $c=\frac{12\sin 75^\circ}{\sin 40^\circ} \approx 18.0$.

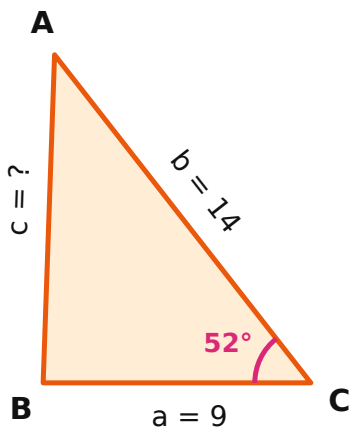


Given two angles and one side, the Law of Sines finds the other sides.

How do you use the Law of Cosines?

Use it for two sides and the included angle (SAS), or for three sides (SSS). With $a=9$, $b=14$ and $C=52^\circ$: $c^2=9^2+14^2-2(9)(14)\cos 52^\circ \approx 121.9$, so $c \approx 11.0$. For three sides, solve for an angle:

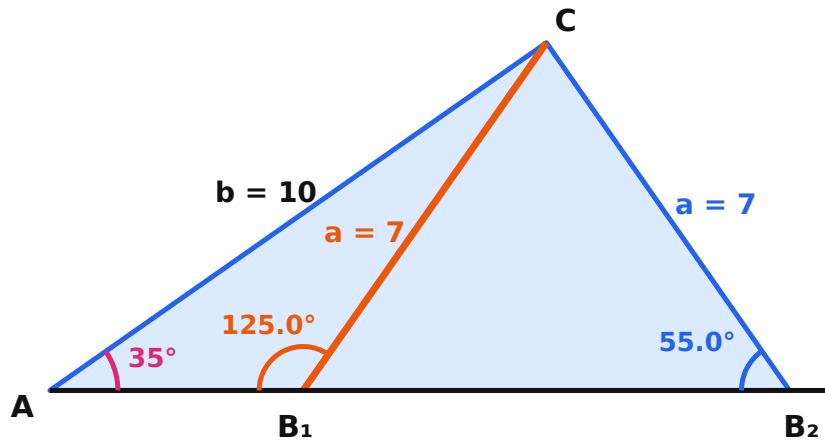
$$\cos C = \frac{a^2+b^2-c^2}{2ab}.$$



Two sides and the included angle: the Law of Cosines gives the third side.

What is the ambiguous case?

Given two sides and an angle not between them (SSA), the Law of Sines can describe two triangles. With $A = 35^\circ$, $b = 10$ and $a = 7$, $\sin B = \frac{10 \sin 35^\circ}{7} \approx 0.819$, so $B \approx 55.0^\circ$ or $B \approx 125.0^\circ$. Both fit.



Side a can swing to two positions, giving two different triangles.

Law of Sines, Law of Cosines and Triangle Area: frequently asked questions

What is the formula for the area of a triangle using sine?

The area is one half the product of two sides times the sine of the angle between them: $\frac{1}{2}ab\sin C$. It works for acute, right and obtuse triangles.

When do you use the Law of Sines instead of the Law of Cosines?

Use the Law of Sines when you know two angles and a side, or two sides and an angle that is not between them. Use the Law of Cosines for two sides and the included angle, or for three sides.

How do you find an angle with the Law of Cosines?

Rearrange to $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ and use the inverse cosine. For sides 7, 9 and 12, the angle opposite 12 has cosine $\frac{14}{126}$, so it measures about 96.4° .

What is the ambiguous case of the Law of Sines?

When you know two sides and a non-included angle, the sine of the unknown angle can match both an acute and an obtuse angle. Check whether the obtuse angle plus the given angle is less than 180° to see if a second triangle exists.

How is the Law of Cosines related to the Pythagorean theorem?

When the angle is 90° , its cosine is 0, so $c^2 = a^2 + b^2 - 2ab \cos C$ reduces to $c^2 = a^2 + b^2$. The Law of Cosines works for every triangle.