

Geometry · Dilations and Similarity Transformations · Worksheet Answer Key

Common Core G-SRT.A.1, G-SRT.A.2, G-SRT.A.3 · mathovore.fr

Every solution below shows the scale factor, the coordinate rule or the angle sum that gives the answer, and the final result is bold. Check your own work by testing one image length against the scale factor.

GeometryCommon Core: G-SRT.A.1 · G-SRT.A.2 · G-SRT.A.3

Problem 1: Scale factor from distances

The scale factor is the ratio of the distances from the center: $k = \frac{OP'}{OP} = \frac{15}{6} = \frac{5}{2}$.

Since $k > 1$, the dilation is an **enlargement with $k = 5/2$ (2.5)**.

Problem 2: Find the image length

Every length is multiplied by k : $A'B' = \frac{3}{4} \cdot 12 = 9$.

The image is **9 inches** long.

Problem 3: Find the original length

Image length = $k \cdot$ original length, so original = $\frac{20}{2.5} = 8$.

The original segment was **8 centimeters** long. Check: $2.5 \cdot 8 = 20$.

Problem 4: Enlargement or reduction

Compare each factor with 1.

- a. $\frac{7}{4} = 1.75 > 1$: **enlargement**
- b. $0.6 < 1$: **reduction**
- c. 1: **no change** (same size)
- d. $1.2 > 1$: **enlargement**
- e. $\frac{5}{9} < 1$: **reduction**

Problem 5: Dilate points about the origin

Use $(x, y) \rightarrow (kx, ky)$.

- $(3 \cdot 3, 3 \cdot (-2))$ gives $P'(9, -6)$
- $(\frac{1}{2} \cdot (-4), \frac{1}{2} \cdot 6)$ gives $Q'(-2, 3)$
- $(4 \cdot 5, 4 \cdot 0)$ gives $R'(20, 0)$
- $(\frac{3}{7} \cdot 0, \frac{3}{7} \cdot (-7))$ gives $S'(0, -3)$

Problem 6: Scale factor from coordinates

Divide an image coordinate by the matching original coordinate, then confirm with the other coordinate.

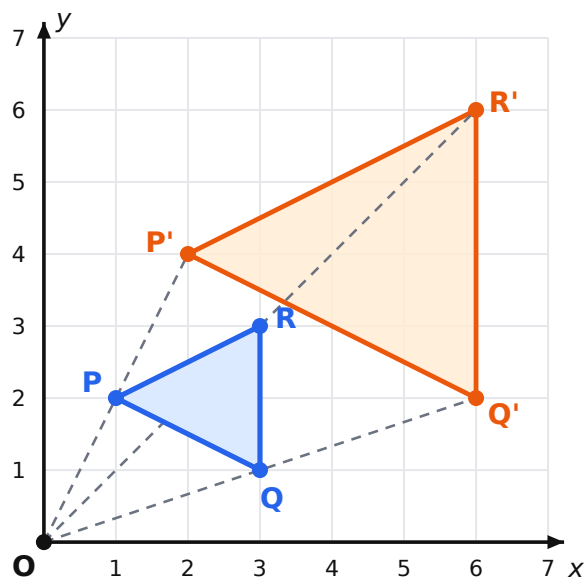
- $\frac{5}{2} = 2.5$ and $\frac{10}{4} = 2.5$: $k = \frac{5}{2}$
- $\frac{2}{6} = \frac{1}{3}$ and $\frac{-3}{-9} = \frac{1}{3}$: $k = \frac{1}{3}$

Problem 7: Dilate a triangle

The vertices are $P(1,2)$, $Q(3,1)$, $R(3,3)$. Multiply each coordinate by 2.

$P'(2,4)$, $Q'(6,2)$, $R'(6,6)$

Check one side: $PQ = \sqrt{5}$ and $P'Q' = \sqrt{20} = 2\sqrt{5}$, which is twice as long. **Image: $P'(2, 4)$, $Q'(6, 2)$, $R'(6, 6)$.**



With $k = 2$, each vertex moves to twice its coordinates: $P'(2, 4)$, $Q'(6, 2)$, $R'(6, 6)$.

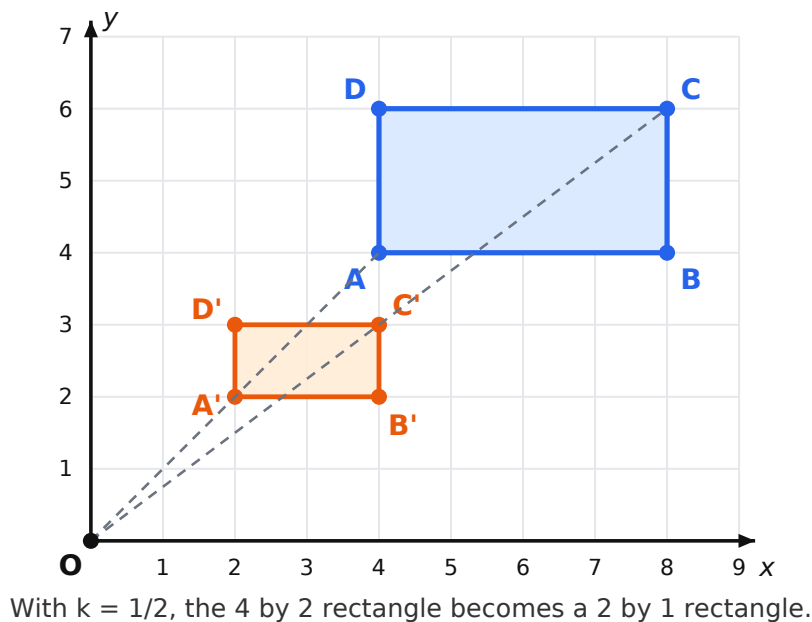
Problem 8: Shrink a rectangle

The vertices are $A(4,4)$, $B(8,4)$, $C(8,6)$, $D(4,6)$. Multiply each coordinate by $\frac{1}{2}$.

$A'(2,2)$, $B'(4,2)$, $C'(4,3)$, $D'(2,3)$

The original rectangle is 4 by 2, so the image is $\frac{1}{2} \cdot 4 = 2$ by $\frac{1}{2} \cdot 2 = 1$. **The new rectangle is 2**

units by 1 unit.

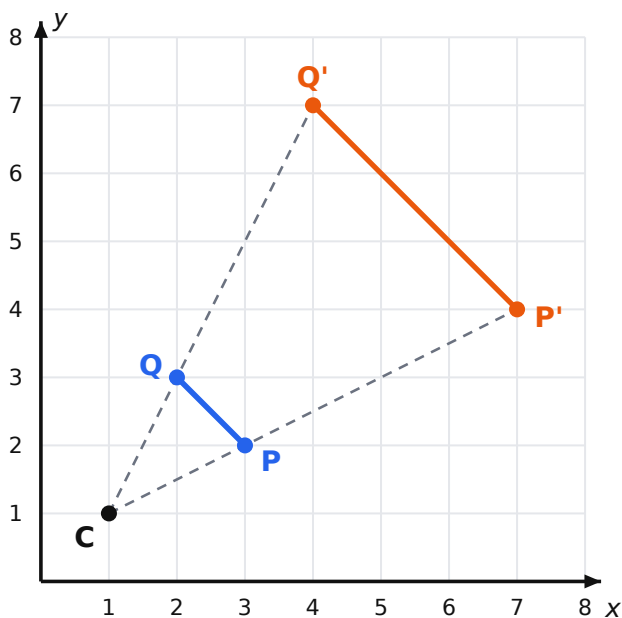


Problem 9: Center not at the origin

From C to P: 2 right and 1 up. Multiply by 3: 6 right and 3 up from C, so $P' = (1+6, 1+3) = (7, 4)$.

From C to Q: 1 right and 2 up. Multiply by 3: 3 right and 6 up, so $Q' = (1+3, 1+6) = (4, 7)$.

$PQ = \sqrt{1^2 + 1^2} = \sqrt{2}$ and $P'Q' = \sqrt{3^2 + 3^2} = 3\sqrt{2}$. **$P'(7, 4)$, $Q'(4, 7)$; the image segment is 3 times as long.**



With center C and $k = 3$, $P' = (7, 4)$ and $Q' = (4, 7)$; the segment triples in length.

Problem 10: Dilate a line

Pick two points on the line: $(0, -2)$ and $(1, 1)$. Their images are $(0, -4)$ and $(2, 2)$.

Slope: $\frac{2 - (-4)}{2 - 0} = 3$, and the y -intercept is -4 .

$y = 3x - 4$ The image line is parallel to the original and has $2 \cdot (-2) = -4$ as its intercept.

Problem 11: Which lines stay put

A line through the origin maps to itself. Any other line maps to a parallel line whose intercept is multiplied by 3.

- a. It passes through the origin: **same line**, $y = -3x$
- b. Intercept $6 \cdot 3 = 18$: $y = -3x + 18$
- c. It passes through the origin: **same line**, $y = \frac{1}{2}x$
- d. Intercept $-1 \cdot 3 = -3$: $y = \frac{1}{2}x - 3$

Problem 12: Perimeter after a dilation

Multiply each side by 2.5: $7 \cdot 2.5 = 17.5$, $9 \cdot 2.5 = 22.5$, $12 \cdot 2.5 = 30$.

Image perimeter: $17.5 + 22.5 + 30 = 70$ in. The original perimeter is $7 + 9 + 12 = 28$ in., and $2.5 \cdot 28 = 70$.

Sides 17.5 in., 22.5 in., 30 in.; perimeter 70 in., which is 2.5 times the original.

Problem 13: Angles after a dilation

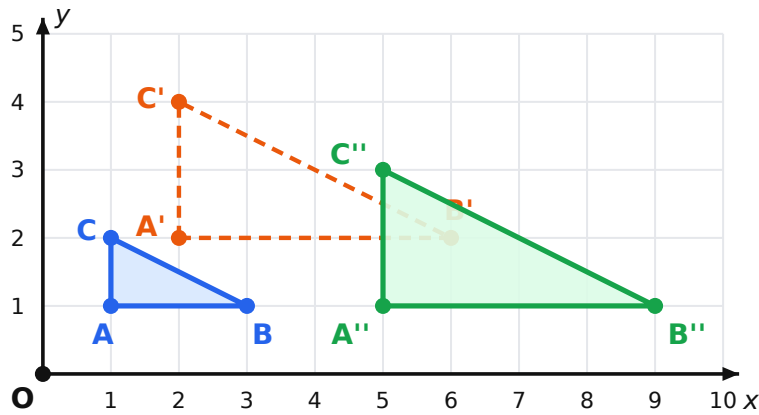
- a. $m\angle C = 180^\circ - 48^\circ - 67^\circ = 65^\circ$
- b. A dilation preserves angle measure, so 48° , 67° , 65° for A' , B' , C' respectively.

Problem 14: Dilation then translation

Step 1, dilate: $A'(2, 2)$, $B'(6, 2)$, $C'(2, 4)$.

Step 2, translate $(x, y) \rightarrow (x + 3, y - 1)$: $A''(5, 1)$, $B''(9, 1)$, $C''(5, 3)$.

Check: $AB = 2$ and $A''B'' = 4$. **Final vertices: (5, 1), (9, 1), (5, 3). The triangles are similar with ratio 2.**



Step 2: translate 3 right and 1 down. The final triangle is shaded green.

Problem 15: Two dilations in a row

First dilation: $(2 \cdot 4, 2 \cdot 6) = (8, 12)$.

Second dilation: $(\frac{3}{4} \cdot 8, \frac{3}{4} \cdot 12) = (6, 9)$.

The factors multiply: $2 \cdot \frac{3}{4} = \frac{3}{2}$, and $\frac{3}{2} \cdot (4, 6) = (6, 9)$. **Final image (6, 9); single scale factor 3/2.**

Problem 16: Third angles and similarity

$m\angle R = 180^\circ - 52^\circ - 71^\circ = 57^\circ$ and $m\angle Z = 180^\circ - 71^\circ - 57^\circ = 52^\circ$.

Matching angles: $P = 52^\circ = Z$, $Q = 71^\circ = X$, $R = 57^\circ = Y$. Two pairs already match, so AA applies.

The triangles are similar: $PQR \sim ZXY$ (P matches Z, Q matches X, R matches Y).

Problem 17: Similar or not

- Third angles: $180^\circ - 35^\circ - 80^\circ = 65^\circ$ and $180^\circ - 80^\circ - 65^\circ = 35^\circ$. Both triangles have 35° , 65° , 80° . **Similar (AA).**
- Triangle 3 has a third angle of 90° , with angles 30° , 60° , 90° . Triangle 4 has angles 45° , 45° , 90° . Only the right angle matches. **Not similar.**
- Triangle 5 has base angles $\frac{180^\circ - 40^\circ}{2} = 70^\circ$. Triangle 6 has base angles 70° and a vertex angle $180^\circ - 2 \cdot 70^\circ = 40^\circ$. **Similar (AA).**

Problem 18: Slope and length after dilation

$P'(4, 2)$ and $Q'(10, 6)$.

Slopes: $\frac{3-1}{5-2} = \frac{2}{3}$ and $\frac{6-2}{10-4} = \frac{4}{6} = \frac{2}{3}$. They are equal, so the segments are parallel.

Lengths: $PQ = \sqrt{3^2 + 2^2} = \sqrt{13}$ and $P'Q' = \sqrt{6^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$. **Same slope $\frac{2}{3}$; the image is**

twice as long.

Problem 19: Photo enlargement

Dimensions: $4 \cdot 2.5 = 10$ in. and $6 \cdot 2.5 = 15$ in.

Perimeters: $2(4+6) = 20$ in. and $2(10+15) = 50$ in., and $2.5 \cdot 20 = 50$.

The enlargement is 10 in. by 15 in.; its perimeter, 50 in., is 2.5 times the original 20 in.

Problem 20: Projector screen

Convert to one unit: $6 \text{ ft} = 72 \text{ in.}$ Scale factor $k = \frac{72}{8} = 9$.

Height: $6 \cdot 9 = 54 \text{ in.} = \frac{54}{12} = 4.5 \text{ ft.}$

Scale factor 9; the projected image is 4.5 feet tall.

Problem 21: Copier reduction

Scale factor $60\% = 0.6$. Dimensions: $11 \cdot 0.6 = 6.6$ in. and $8.5 \cdot 0.6 = 5.1$ in.

To undo a dilation of 0.6 use the reciprocal: $\frac{1}{0.6} = \frac{5}{3} \approx 1.667$, about 167%.

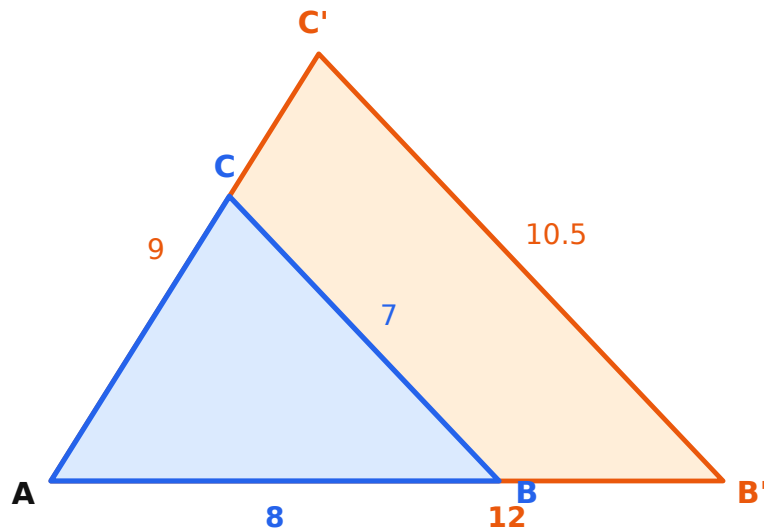
The copy is 6.6 in. by 5.1 in.; a setting of about 167% restores the original size.

Problem 22: Dilation about a vertex

Multiply each side by 1.5: $AB' = 1.5 \cdot 8 = 12$, $AC' = 1.5 \cdot 6 = 9$, $B'C' = 1.5 \cdot 7 = 10.5$.

Perimeters: $8+6+7 = 21$ and $12+9+10.5 = 31.5$, and $1.5 \cdot 21 = 31.5$.

$AB' = 12$, $AC' = 9$, $B'C' = 10.5$; perimeters 21 and 31.5.



Every side of $A'B'C'$ is 1.5 times the matching side of ABC .

Problem 23: Find the center

If the center is C , then $P' = C + 2(P - C) = 2P - C$, so $C = 2P - P'$.

$$C = (2 \cdot 3 - 5, 2 \cdot 2 - 4) = (1, 0).$$

$$\text{For } Q: (1 + 2(4 - 1), 0 + 2(5 - 0)) = (7, 10).$$

Center $(1, 0)$; $Q' = (7, 10)$.