

Geometry · Dilations and Similarity Transformations · Lesson

Common Core G-SRT.A.1, G-SRT.A.2, G-SRT.A.3 · mathovore.fr

A dilation is a transformation that enlarges or reduces a figure from a fixed center by a scale factor k , moving every point P to P' on ray OP so that OP' equals k times OP . Dilations keep angle measures, multiply lengths by k , and send lines to parallel lines.

GeometryCommon Core: G-SRT.A.1 · G-SRT.A.2 · G-SRT.A.3

Key facts:

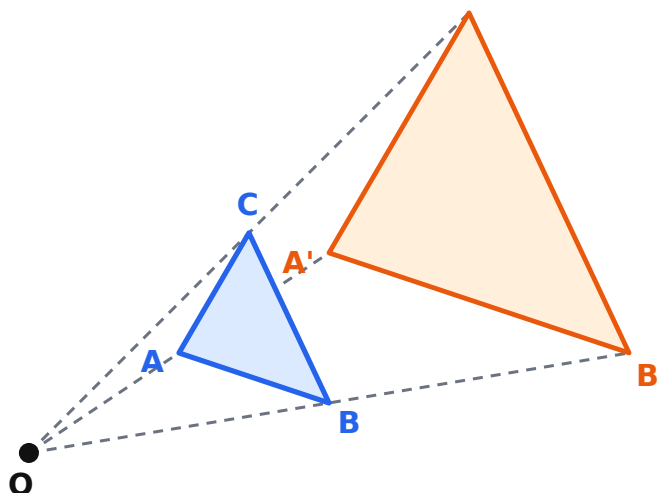
- A dilation needs a center and a positive scale factor k ; the center never moves (G-SRT.A.1).
- A line through the center maps to itself; any other line maps to a parallel line, and every segment is multiplied by k .
- About the origin, a dilation is (x, y) to (kx, ky) ; angle measures stay the same.
- Figures are similar when a sequence of rigid motions and dilations maps one onto the other (G-SRT.A.2).
- AA criterion: two pairs of congruent angles are enough to prove triangles similar (G-SRT.A.3).

What is a dilation?

A **dilation** stretches or shrinks a figure from a fixed point called the **center**. The number that says how much is the **scale factor**, written k .

Dilation with center O and scale factor k ($k > 0$)

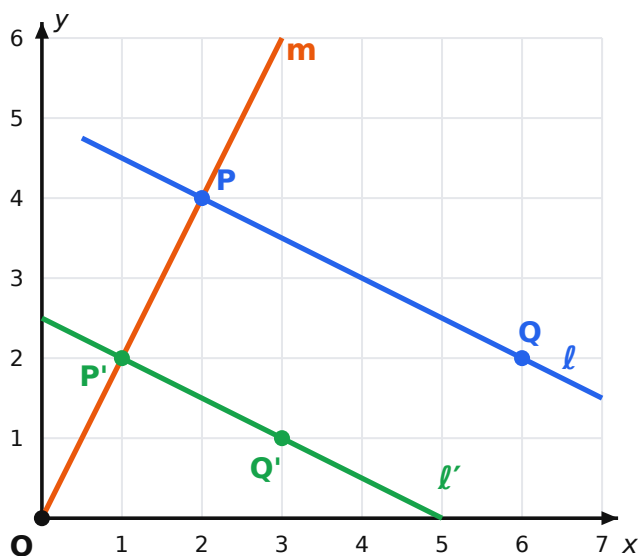
Every point P moves to the point P' on ray OP so that $OP' = k \cdot OP$. The center O does not move. If $k > 1$ the figure is enlarged, if $0 < k < 1$ it is reduced, and if $k = 1$ it keeps its size.



Each image point is twice as far from the center O as its original point: $OA' = 2 \cdot OA$.

A dilation has four properties you can rely on.

- A line through the center maps to itself.
- A line that misses the center maps to a parallel line.
- Every segment is multiplied by k : $A'B' = k \cdot AB$.
- Angle measures do not change.



Line m passes through the center, so it stays in place. The other line moves to a parallel line, half as far from O .

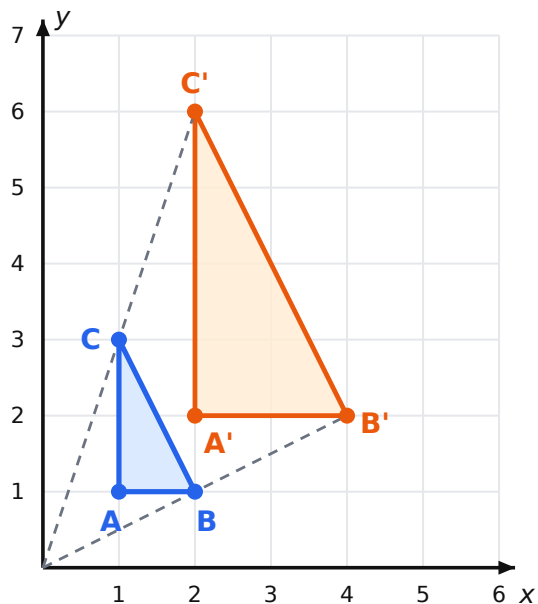
How do you dilate a figure on the coordinate plane?

When the center is the origin, multiply both coordinates by k : $(x,y) \rightarrow (kx,ky)$. For any other center C , measure each point's horizontal and vertical distance from C and multiply those distances by k .

1. Name the center and the scale factor.

2. Multiply each vertex's coordinates (or its distances from the center) by k .
3. Plot the new vertices and connect them in the same order.
4. Check that each image side is k times its original side.

Worked example. Dilate $A(1,1)$, $B(2,1)$, $C(1,3)$ about the origin with $k=2$. The images are $A'(2,2)$, $B'(4,2)$, $C'(2,6)$. Check: $AB=1$ and $A'B'=2$.



The rule $(x, y) \rightarrow (2x, 2y)$ sends each vertex of triangle ABC to the vertex of $A'B'C'$.

How do dilations show that two figures are similar?

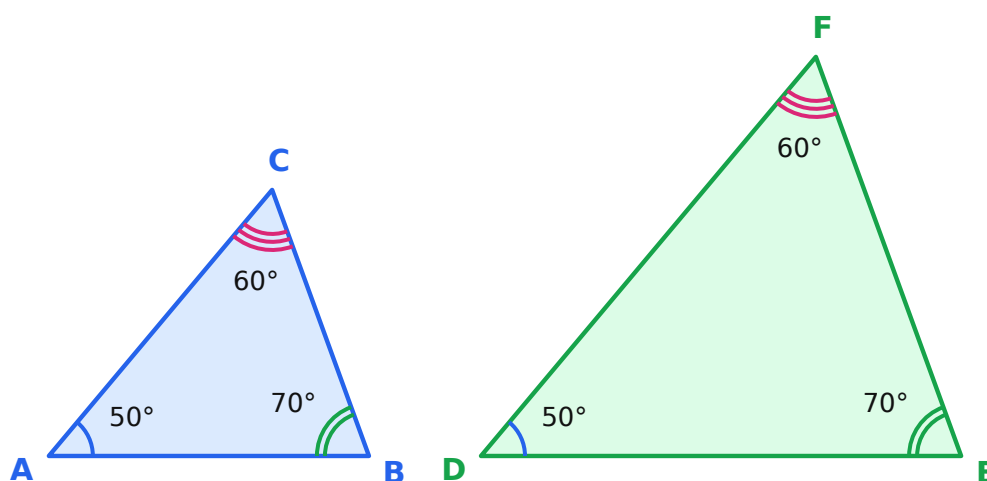
Two figures are **similar** when a sequence of rigid motions (translations, reflections, rotations) and dilations maps one onto the other. Rigid motions keep every length and angle, and a dilation multiplies every length by the same k . So similar figures have equal corresponding angles and corresponding sides in one constant ratio.

What is the AA criterion for similar triangles?

Angle-Angle (AA) criterion

If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar.

Why it works: the third angles must match, because each triangle's angles add to 180° . Dilate the first triangle by $k = \frac{DE}{AB}$. Its image has a side equal to DE with the same two angles at the ends, so the ASA criterion makes it congruent to the second triangle.



Two angle pairs match: 50° and 70° . The third angles must both be 60° .

Dilations and Similarity Transformations: frequently asked questions

How is a dilation different from a rigid motion?

A rigid motion (translation, reflection, rotation) keeps every length and angle. A dilation keeps angles but multiplies every length by the scale factor, so the image has the same shape and a different size unless k equals 1.

Why must the scale factor be positive in this chapter?

A scale factor of 0 would collapse the figure to the center, and the image of P must lie on ray OP , which needs a positive factor. Here k is always a positive number.

Does a dilation change angle measures?

No. Every angle in the image has the same measure as the matching angle in the original, which is why a dilated triangle always has the same angles as the first one.

How do you find the scale factor from a figure?

Divide any image length by the matching original length, or divide the distance from the center to an image point by the distance from the center to the original point. Both give the same k .

Why do two pairs of equal angles make triangles similar?

The third angles are forced to be equal because the angles add to 180 degrees. Then a dilation by the right ratio makes one triangle match the other side for side, so they are similar.