

# Geometry · Geometric Constructions · Worksheet Answer Key

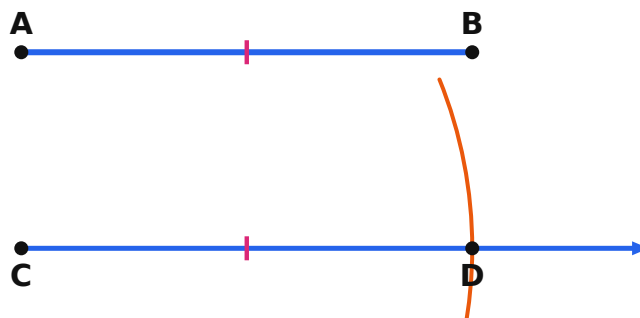
Common Core G-CO.D.12, G-CO.D.13 · mathovore.fr

Each answer lists the construction steps in order and explains why the construction works, usually with equal radii or congruent triangles. Your arcs may sit in different places than the figures, but the points you find must match. Check numeric answers against the calculations shown.

## GeometryCommon Core: G-CO.D.12 · G-CO.D.13

### Problem 1: Copy a segment

1. Draw a ray starting at point  $C$ .
2. Place the compass point on  $A$  and open it until the pencil is on  $B$ .
3. Keep that width, place the compass point on  $C$ , and draw an arc across the ray. Label the crossing  $D$ .

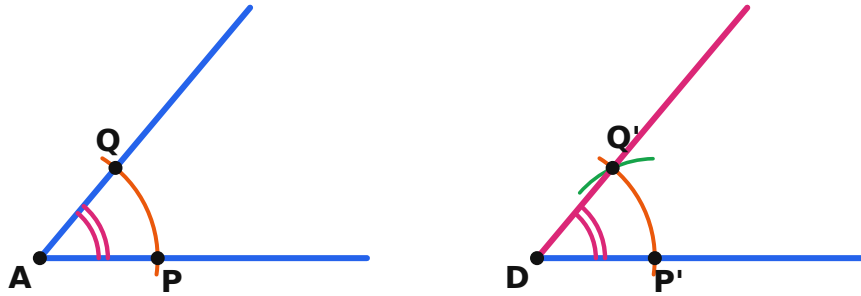


The arc centered at C with radius AB marks D, so  $CD = AB$ .

$$CD = AB$$

### Problem 2: Copy an angle

1. Draw an arc centered at the vertex that crosses both sides, at  $P$  and  $Q$ .
2. Draw a ray from the new vertex  $D$ . With the same width, draw an arc centered at  $D$  crossing the ray at  $P'$ .
3. Set the compass to  $PQ$ , and draw an arc centered at  $P'$  that crosses the first arc at  $Q'$ .
4. Draw ray  $DQ'$ .



The new angle at D is a copy of the angle at A.

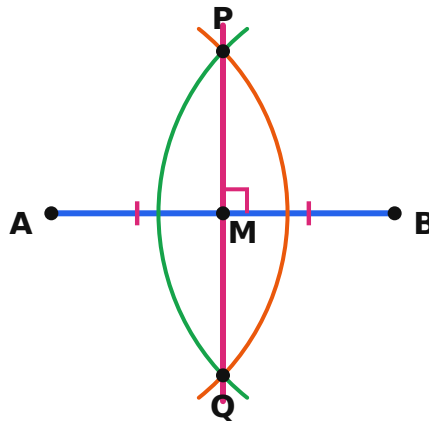
Triangles  $APQ$  and  $DP'Q'$  have three pairs of congruent sides (two radii and the copied width  $PQ$ ), so they are congruent by SSS and the angles at  $A$  and  $D$  are congruent.

### Congruent by SSS

### Problem 3: Bisect a segment

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1. Open the compass wider than half of  $AB$ .
2. Draw an arc centered at  $A$  above and below the segment, then draw an arc of the same radius centered at  $B$ .
3. Label the crossing points  $P$  and  $Q$ , and draw line  $PQ$ . It crosses  $AB$  at the midpoint  $M$ .



Point M is the midpoint of AB.

$$AM = MB$$

### Problem 4: Measurements on a bisector

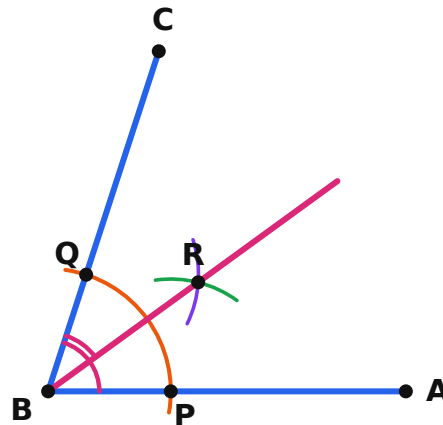
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The midpoint splits the segment into two congruent parts:  $AM = MB = \frac{14}{2} = 7$  in. A perpendicular bisector meets the segment at a right angle, so the angle at  $M$  measures  $90^\circ$ .

$AM = MB = 7$  in, angle  $90^\circ$

### Problem 5: Bisect an angle

1. Draw an arc centered at  $B$  that crosses the sides at  $P$  and  $Q$ .
2. Draw an arc centered at  $P$  and an arc of the same radius centered at  $Q$ . Label the crossing  $R$ .
3. Draw ray  $BR$ . It divides the angle into two congruent angles.



Ray  $BR$  bisects the angle.

Triangles  $BPR$  and  $BQR$  are congruent by SSS, so  $m\angle PBR = m\angle QBR$ .

**Ray  $BR$  is the angle bisector**

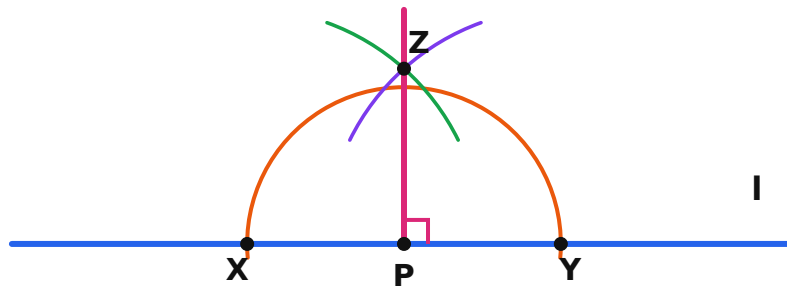
### Problem 6: Angle bisector with a variable

- a. The bisector gives two equal angles:  $m\angle PQS = \frac{84}{2} = 42^\circ$ .
- b.  $3x + 6 = 5x - 10$ , so  $16 = 2x$  and  $x = 8$ . Each half is  $3(8) + 6 = 30^\circ$ , so  $m\angle PQR = 2(30) = 60^\circ$ .  
Check:  $5(8) - 10 = 30$ .

**a)  $42^\circ$ ; b)  $x = 8$ ,  $m\angle PQR = 60^\circ$**

### Problem 7: Perpendicular through a point on a line

1. Draw an arc centered at  $P$  that crosses  $l$  at  $X$  and  $Y$  (so  $PX = PY$ ).
2. Open the compass wider than  $PX$ . Draw arcs of the same radius centered at  $X$  and  $Y$  that cross at  $Z$ .
3. Draw line  $PZ$ . It is the perpendicular bisector of  $XY$ , so it is perpendicular to  $l$  at  $P$ .



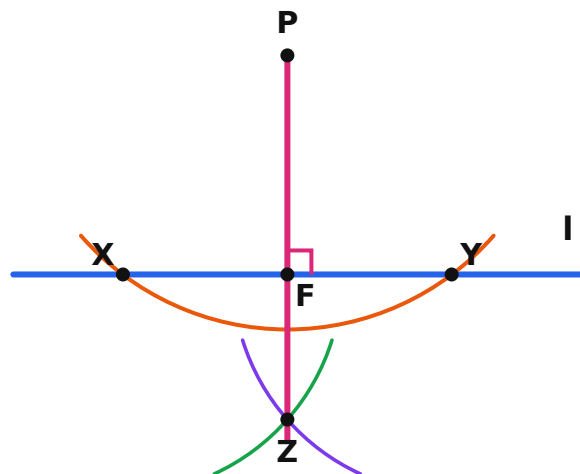
Line  $PZ$  is perpendicular to  $l$  at  $P$ .

**Line  $PZ$  is perpendicular to  $l$**

### Problem 8: Perpendicular from a point not on the line

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1. Draw an arc centered at  $P$  with a radius large enough to cross  $l$  twice, at  $X$  and  $Y$ .
2. With a radius greater than half of  $XY$ , draw arcs centered at  $X$  and  $Y$  that cross at  $Z$  on the other side of  $l$ .
3. Draw line  $PZ$ . Since  $PX = PY$  and  $ZX = ZY$ , it is the perpendicular bisector of  $XY$ .



Line  $PZ$  is perpendicular to  $l$ .

**Line  $PZ$  is perpendicular to  $l$**

### Problem 9: Distance between the arc crossings

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The midpoint  $M$  gives  $AM = 8$ . Triangle  $AMP$  is a right triangle with hypotenuse  $AP = 10$ , so  $PM^2 = 10^2 - 8^2 = 100 - 64 = 36$  and  $PM = 6$ . By symmetry  $QM = 6$ , so  $PQ = 6 + 6 = 12$  cm.

$$PQ = 12 \text{ cm}$$

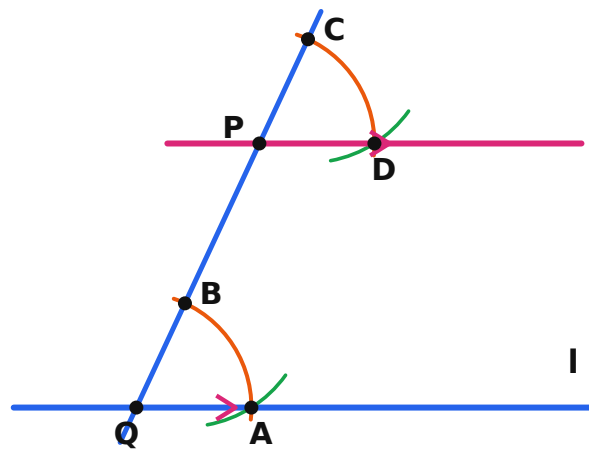
### Problem 10: Why the perpendicular bisector works

- $AP = AQ = BP = BQ$  because all four are radii of arcs with the same width. A quadrilateral with four congruent sides is a rhombus.
- $AB$  and  $PQ$  are the diagonals of the rhombus. The diagonals of a rhombus are perpendicular and bisect each other, so  $PQ$  is the perpendicular bisector of  $AB$ .

**Four congruent sides give a rhombus; its diagonals are perpendicular bisectors of each other**

### Problem 11: Parallel line through a point

- Draw a line through  $P$  that crosses  $l$  at  $Q$ .
- Draw an arc centered at  $Q$  crossing  $l$  at  $A$  and the transversal at  $B$ . Draw an arc of the same radius centered at  $P$  crossing the transversal at  $C$ .
- Set the compass to  $AB$ . Draw an arc centered at  $C$  that crosses the arc at  $D$ .
- Draw line  $PD$ .



Line PD is parallel to  $l$ .

**Line PD is parallel to  $l$**

### Problem 12: Why the lines are parallel

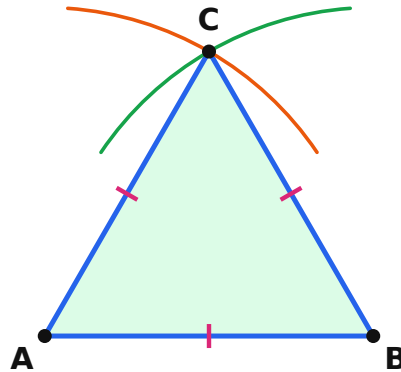
- The two angles are congruent corresponding angles, so the converse of the corresponding angles theorem proves the lines are parallel.
- A linear pair is supplementary:  $180^\circ - 65^\circ = 115^\circ$ .

**a) converse of the corresponding angles theorem; b)  $115^\circ$**

### Problem 13: Equilateral triangle on a segment

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1. Draw an arc centered at  $A$  with radius  $AB$ .
2. Draw an arc centered at  $B$  with radius  $AB$ . Label a crossing  $C$ .
3. Draw  $\overline{AC}$  and  $\overline{BC}$ .



Equilateral triangle ABC.

All sides are 7 cm, so the perimeter is  $3(7) = 21$  cm. The angles are equal and add to  $180^\circ$ , so each measures  $\frac{180}{3} = 60^\circ$ .

**Perimeter 21 cm; each angle  $60^\circ$**

### Problem 14: Why the triangle is equilateral

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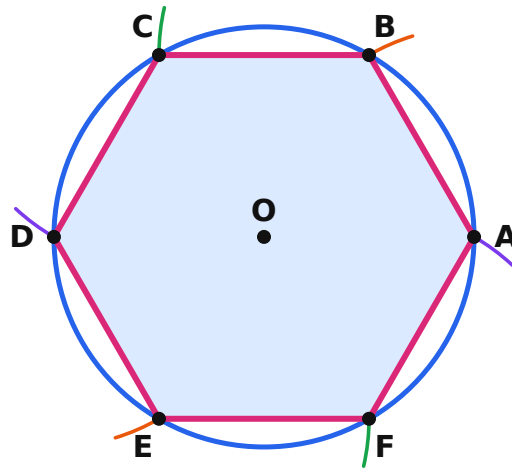
$C$  lies on the arc centered at  $A$  with radius  $AB$ , so  $AC = AB$ .  $C$  also lies on the arc centered at  $B$  with the same radius, so  $BC = AB$ . Therefore  $AC = AB = BC$ .

**All three sides equal the compass width  $AB$**

### Problem 15: Regular hexagon in a circle

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1. Mark a point  $A$  on the circle. With the compass set to the radius, draw an arc centered at  $A$  that crosses the circle at  $B$ .
2. Move to  $B$  and mark  $C$ , then  $D$ ,  $E$  and  $F$  in the same way.
3. Join the six points in order.



Regular hexagon ABCDEF inscribed in the circle.

Each side equals the radius, 4 cm. Perimeter:  $6(4) = 24$  cm.

**Side 4 cm; perimeter 24 cm**

### Problem 16: Why the radius steps around the circle

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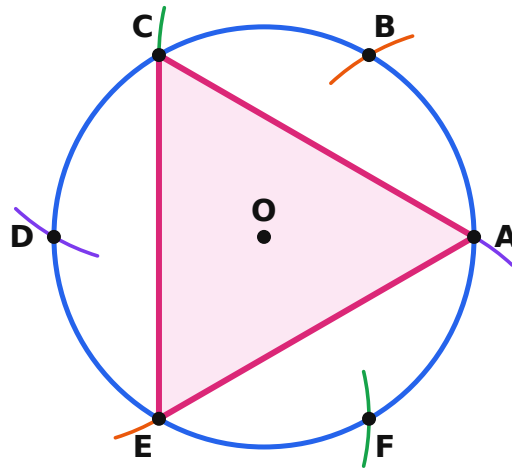
- $OA = OB = r$  and  $AB = r$  because the compass width stays at the radius. Triangle  $OAB$  is equilateral, so  $m\angle AOB = 60^\circ$ . Six of these angles make  $6(60) = 360^\circ$ .
- Each interior angle is made of two angles of equilateral triangles:  $60^\circ + 60^\circ = 120^\circ$ .

**a)  $60^\circ$ ; b)  $120^\circ$**

### Problem 17: Equilateral triangle in a circle

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- Construct the regular hexagon  $ABCDEF$  inscribed in the circle.
- Join every other vertex:  $A$ ,  $C$  and  $E$ .



Triangle ACE inscribed in the circle.

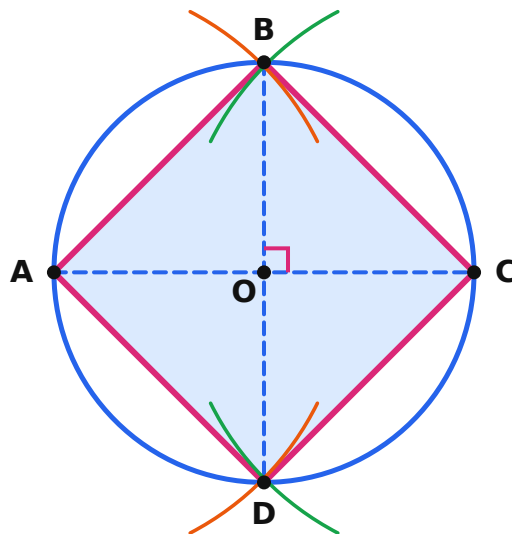
Each side of the triangle spans two sides of the hexagon, so its central angle is  $2(60) = 120^\circ$ .

**Join A, C, E; central angle  $120^\circ$**

### Problem 18: Square in a circle

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1. Draw a diameter  $\overline{AC}$  through the center  $O$ .
2. Construct the perpendicular bisector of  $\overline{AC}$ : draw arcs of the same radius, wider than  $OA$ , centered at  $A$  and  $C$ . It crosses the circle at  $B$  and  $D$ .
3. Join  $A$ ,  $B$ ,  $C$  and  $D$ .



Square ABCD inscribed in the circle.

The diagonals are diameters that bisect each other at right angles and are congruent, so

$ABCD$  is a square.

**Perpendicular diameters give the square**

### Problem 19: Square in a circle with a radius

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A diagonal is a diameter:  $d = 2(5) = 10$  cm. With side  $s$ ,  $s^2 + s^2 = 10^2$ , so  $2s^2 = 100$ ,  $s^2 = 50$  and  $s = \sqrt{50} = 5\sqrt{2} \approx 7.07$  cm. Perimeter:  $4(5\sqrt{2}) = 20\sqrt{2} \approx 28.28$  cm.

**Diagonal 10 cm; side  $5\sqrt{2} \approx 7.07$  cm; perimeter  $20\sqrt{2} \approx 28.28$  cm**

### Problem 20: Fire pit benches

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In an inscribed regular hexagon the side equals the radius, so each bench is 6 ft. Total:  $6(6) = 36$  ft. The angle between neighboring benches is an interior angle of the hexagon,  $120^\circ$ .

**6 ft each, 36 ft in total,  $120^\circ$**

### Problem 21: Splitting a board

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The first bisection gives two pieces of  $\frac{30}{2} = 15$  in. The second gives four pieces of  $\frac{15}{2} = 7.5$  in. Check:  $4(7.5) = 30$ .

**4 pieces of 7.5 in each**

### Problem 22: Which polygons fit around a point?

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The angles around a point add to  $360^\circ$ . Equilateral triangle:  $6(60) = 360$ . Square:  $4(90) = 360$ . Regular hexagon:  $3(120) = 360$ . Each interior angle divides evenly into  $360^\circ$ , so all three tile.

**All three: triangles (6 at a point), squares (4), hexagons (3)**

### Problem 23: Angles you can construct

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- $30^\circ$ : bisect  $60^\circ$ .
- $45^\circ$ : bisect  $90^\circ$ .
- $15^\circ$ : bisect  $30^\circ$ .
- $22.5^\circ$ : bisect  $45^\circ$ .
- $75^\circ$ : copy the  $60^\circ$  angle, then copy the  $15^\circ$  angle next to it:  $60^\circ + 15^\circ = 75^\circ$ .

**30, 45, 15 and 22.5 by bisecting; 75 by adding 60 and 15**

### Problem 24: Spotlights on a circular stage

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- a. The side of an inscribed regular hexagon equals the radius, so neighboring points are 9 ft

apart.

b. Every other point skips one 60 degree step:  $2(60) = 120^\circ$ .

c. The three lights form an equilateral triangle, so each angle is  $60^\circ$ .

**a) 9 ft; b)  $120^\circ$ ; c)  $60^\circ$**