

Geometry · Congruent Triangles: SSS, SAS, ASA, AAS and HL · Worksheet Answer Key

Common Core G-CO.B.6, G-CO.B.7, G-CO.B.8, G-SRT.B.5 · mathovore.fr

Every answer shows the matching of vertices, the criterion used, and the algebra. Check that your reasons name a property or theorem, not just a picture, and that proofs list the congruent parts in the same order as the triangle names.

Geometry Common Core: G-CO.B.6 · G-CO.B.7 · G-CO.B.8 · G-SRT.B.5

Problem 1: Name corresponding parts

The letters match in order: A with D, B with E, C with F.

Sides: $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$.

Angles: $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$.

Problem 2: Read a congruence statement

P matches X, Q matches Y, and R matches Z.

$\overline{QR}$ corresponds to $\overline{YZ}$. $\angle R$ corresponds to $\angle Z$. $\overline{PR}$ is congruent to $\overline{XZ}$.

Problem 3: Find missing measures

Corresponding parts of congruent triangles are congruent. A matches D, B matches E, C matches F.

$DE = AB = 9$ in and $EF = BC = 12$ in.

$m\angle D = m\angle A = 48^\circ$ and $m\angle F = m\angle C = 75^\circ$.

Answer: $DE = 9$ in, $EF = 12$ in, $m\angle D = 48^\circ$, $m\angle F = 75^\circ$.

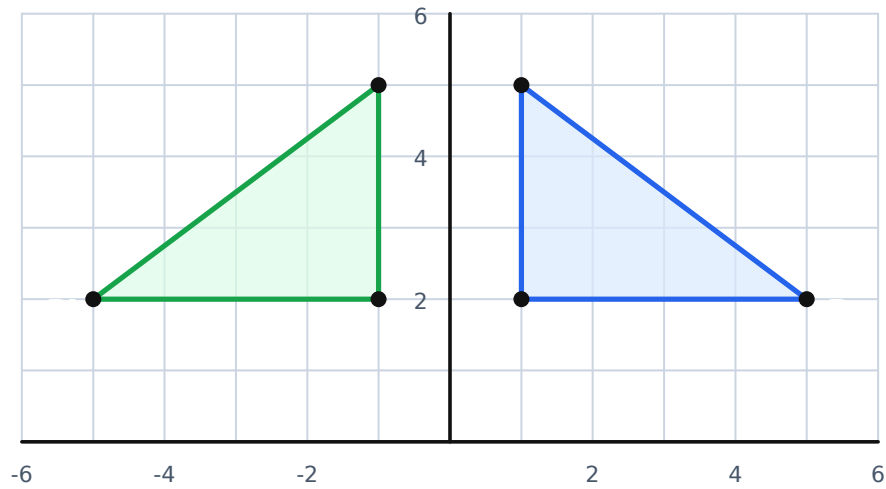
Problem 4: Reflect a triangle

A reflection across the y -axis changes (x,y) to $(-x,y)$: $A'(-1,2)$, $B'(-5,2)$, $C'(-1,5)$.

Original: $AB = 4$, $AC = 3$, and $BC = \sqrt{4^2 + 3^2} = 5$. Image: $A'B' = 4$, $A'C' = 3$, $B'C' = 5$.

All three pairs match, so the reflection produced a congruent triangle:

$$\triangle ABC \cong \triangle A'B'C'.$$



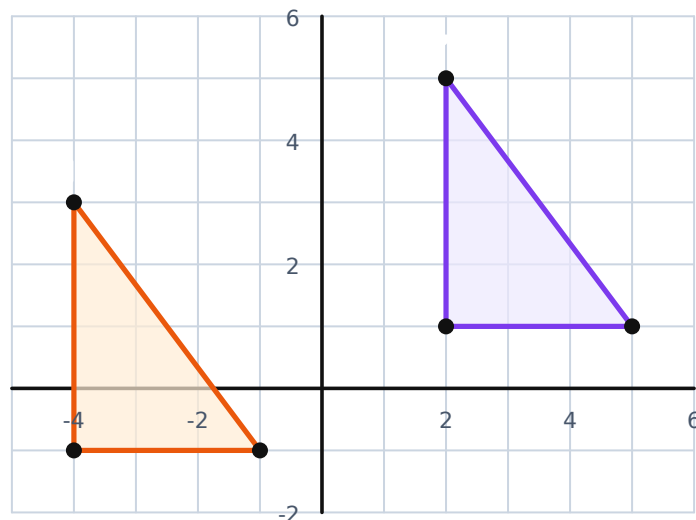
Triangle ABC and its reflection across the y-axis.

Problem 5: Translate a triangle

Add 6 to each x and 2 to each y : $L'(2,1)$, $M'(5,1)$, $N'(2,5)$.

Original: $LM=3$, $LN=4$, $MN=\sqrt{3^2+4^2}=5$. Image: $L'M'=3$, $L'N'=4$, $M'N'=5$.

A translation is a rigid motion and every side matches, so $\triangle LMN \cong \triangle L'M'N'$.



Triangle LMN and its image after the slide.

Problem 6: Name the criterion: pair 1

All three pairs of sides are marked congruent, so the criterion is **SSS**.

Problem 7: Name the criterion: pair 2

Sides $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$ are marked, and the angle between them, $\angle A \cong \angle D$, is marked. The criterion is **SAS**.

Problem 8: Name the criterion: pair 3

Angles $\angle A \cong \angle D$ and $\angle B \cong \angle E$ are marked, and the side between them, $\overline{AB} \cong \overline{DE}$, is marked. The criterion is **ASA**.

Problem 9: Name the criterion: pair 4

Angles $\angle A \cong \angle D$ and $\angle B \cong \angle E$ are marked, but the marked side $\overline{BC} \cong \overline{EF}$ is not between them. The criterion is **AAS**.

Problem 10: Name the criterion: pair 5

Each triangle has a right angle at C and F. The hypotenuses $\overline{AB}$ and $\overline{DE}$ are congruent, and the legs $\overline{AC}$ and $\overline{DF}$ are congruent. The criterion is **HL**.

Problem 11: Name the criterion: pair 6

$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\angle A \cong \angle D$ are marked, but the angle is not between the two sides (SSA). The third sides clearly differ, so the answer is **not enough information**.

Problem 12: Solve for x with sides

Corresponding sides are equal: $3x+2=5x-10$.

Subtract $3x$: $2=2x-10$. Add 10: $12=2x$. Divide by 2: $x=6$.

$AB=3(6)+2=20$ and $DE=5(6)-10=20$. Answer: $x=6$, $AB=20$ in.

Problem 13: Solve for x with angles

Corresponding angles are equal: $4x+7=6x-13$.

Subtract $4x$: $7=2x-13$. Add 13: $20=2x$. Divide: $x=10$.

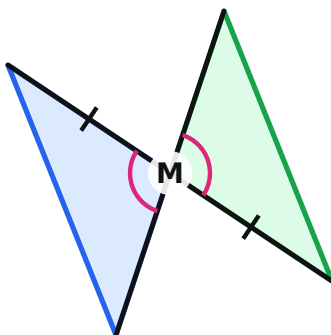
$m\angle B=4(10)+7=47^\circ$ and $m\angle E=6(10)-13=47^\circ$. Answer: $x=10$, $m\angle B=47^\circ$.

Problem 14: Triangles in a kite

- The shared side is $\overline{AC}$; $\overline{AC} \cong \overline{AC}$ by the reflexive property.
- $\overline{AB} \cong \overline{AD}$, $\overline{CB} \cong \overline{CD}$, and $\overline{AC} \cong \overline{AC}$ give three pairs of congruent sides: **SSS**.
- B corresponds to D in $\triangle ABC \cong \triangle ADC$, so $\angle B \cong \angle D$ by CPCTC.

Problem 15: Proof with parallel segments

Statements	Reasons
$\overline{AC} \parallel \overline{BD}$	Given
$\angle CAM \cong \angle DBM$	Alternate interior angles are congruent
M is the midpoint of $\overline{AB}$	Given
$\overline{AM} \cong \overline{BM}$	Definition of midpoint
$\angle AMC \cong \angle BMD$	Vertical angles are congruent
$\triangle AMC \cong \triangle BMD$	ASA (steps 2, 4, 5)



Marked parts used in the ASA proof.

Problem 16: Why SSA fails

Both triangles share angle A, the side $\overline{AB}$, and a side of 9 inches opposite the angle. These are two sides and an angle that is *not* between them (SSA).

The third sides are 13.8 in and 4.6 in, which are different, so the triangles are not congruent.

SSA is not a congruence criterion: the given parts fit two different triangles.

Problem 17: Points on a perpendicular bisector

Statements	Reasons
M is the midpoint of $\overline{AB}$	Given
$\overline{AM} \cong \overline{BM}$	Definition of midpoint
$\ell \perp \overline{AB}$ at M	Given
$\angle PMA$ and $\angle PMB$ are right angles, so $\angle PMA \cong \angle PMB$	Perpendicular lines form right angles; right angles are congruent
$\overline{PM} \cong \overline{PM}$	Reflexive property
$\triangle PMA \cong \triangle PMB$	SAS (steps 2, 4, 5)
$\overline{PA} \cong \overline{PB}$	CPCTC

Problem 18: Base angles of an isosceles triangle

Statements	Reasons
$\overline{AB} \cong \overline{AC}$	Given
$\overline{AD}$ bisects $\angle BAC$	Given
$\angle BAD \cong \angle CAD$	Definition of angle bisector
$\overline{AD} \cong \overline{AD}$	Reflexive property
$\triangle ABD \cong \triangle ACD$	SAS (steps 1, 3, 4)
$\angle B \cong \angle C$	CPCTC

Problem 19: Flower beds

- All three pairs of sides match (12 with 12, 16 with 16, 20 with 20), so the beds are congruent by **SSS**.
- $12^2 + 16^2 = 144 + 256 = 400 = 20^2$, so the converse of the Pythagorean theorem gives a right angle between the 12 ft and 16 ft sides. Area $= \frac{1}{2}(12)(16) = 96$ square feet.
- Both beds: $2 \times 96 = 192$ square feet. Cost: $192 \times 0.75 = 144$. The mulch costs **\$144**.

Problem 20: Truss braces

- Both triangles are right triangles. The hypotenuses $\overline{AB} \cong \overline{CB}$ (13 ft each) and the leg $\overline{BD}$ is shared, so **HL**.
- By CPCTC, $DC = AD = 5$ ft.
- $BD^2 = 13^2 - 5^2 = 169 - 25 = 144$, so $BD = 12$ ft. The base is $AC = 5 + 5 = 10$, so $AC = 10$ ft.

Problem 21: Distance across a pond

In $\triangle ACB$ and $\triangle DCE$: $\overline{AC} \cong \overline{DC}$ (by construction), $\angle ACB \cong \angle DCE$ (vertical angles), and $\overline{BC} \cong \overline{EC}$ (by construction).

That is SAS, so $\triangle ACB \cong \triangle DCE$. By CPCTC, $\overline{AB} \cong \overline{DE}$.

Therefore $AB = 84$ yd.

Problem 22: Perimeter and angle from congruence

- a. $2x+3=4x-9$ gives $12=2x$, so $x=6$. $5y-4=3y+10$ gives $2y=14$, so $y=7$.
 $3z+9=5z-21$ gives $30=2z$, so $z=15$. Answer: $x=6$, $y=7$, $z=15$.
- b. $DE=4(6)-9=15$, $EF=3(7)+10=31$, and $DF=AC=22$. Perimeter $=15+31+22=68$: **68 in.**
- c. $m\angle A=3(15)+9=54^\circ$. Check: $m\angle D=5(15)-21=54^\circ$. Answer: 54° .

Problem 23: Congruent triangles on a grid

- a. Each point moves 2 units right and 1 unit up: (0,0) moves to (2,1), (6,0) to (8,1), and (0,8) to (2,9). A translation is a rigid motion, so $\triangle ABC \cong \triangle DEF$.
- b. In $\triangle ABC$: $AB=6$ is horizontal and $AC=8$ is vertical, so the angle at A is a right angle. In $\triangle GHI$: $GH=8-2=6$ is vertical and $GI=5-(-3)=8$ is horizontal, so the angle at G is a right angle. Two sides and the included angle match (A with G): **SAS**.
- c. $BC=\sqrt{6^2+8^2}=10$, $EF=\sqrt{6^2+8^2}=10$, and $HI=\sqrt{8^2+6^2}=10$. Each hypotenuse is **10 units**.

Problem 24: Ethan's claim about equal angles

- a. Each triangle is equilateral: $3 \times 4 = 12$ ft and $3 \times 6 = 18$ ft.
- b. Equal angles (AAA) give triangles of the same shape but not necessarily the same size. The perimeters 12 ft and 18 ft differ, so the triangles are **not congruent**.
- c. If one pair of corresponding sides were congruent (for example both sides 4 ft), the triangles would be congruent by **AAS**.