

Geometry · Circles: Chords, Arcs and Inscribed Angles · Worksheet Answer Key

Common Core G-C.A.1, G-C.A.2 · mathovore.fr

This answer key gives a complete worked solution for each problem on the circles worksheet. Check each step against your work, especially which arc an angle intercepts, and compare your figures with the diagrams shown for the harder problems.

Geometry Common Core: G-C.A.1 · G-C.A.2

Problem 1: Similar circles

(a) Translate A so its center lands on the center of B. Then dilate about that center by the scale factor $\frac{10}{4} = \frac{5}{2} = 2.5$.

(b) $\frac{2\pi(10)}{2\pi(4)} = \frac{20\pi}{8\pi} = 2.5$.

(a) Translation, then dilation with scale factor 2.5. (b) The ratio is 2.5.

Problem 2: Ratio of circumferences

$C_P = 2\pi(3) = 6\pi$ and $C_Q = 2\pi(12) = 24\pi$. The ratio is $\frac{24\pi}{6\pi} = 4$, the same as the ratio of the radii, $\frac{12}{3} = 4$.

$C_P = 6\pi$ in, $C_Q = 24\pi$ in, ratio 4.

Problem 3: Central angle and arc

A central angle equals its intercepted arc, so minor arc AB is 72° . The full circle is 360° , so major arc AB is $360^\circ - 72^\circ = 288^\circ$.

Minor arc 72° ; major arc 288° .

Problem 4: Inscribed angle from an arc

An inscribed angle is half its intercepted arc: $\frac{1}{2}(110^\circ) = 55^\circ$.

$m\angle ACB = 55^\circ$

Problem 5: Arc from an inscribed angle

The arc is twice the inscribed angle: $2(38^\circ) = 76^\circ$. A central angle equals the arc.

Arc 76° ; central angle 76° .

Problem 6: Inscribed angle algebra

The angle is half the arc: $3x+5 = \frac{1}{2}(8x-10)$, so $3x+5 = 4x-5$ and $x=10$.

Angle: $3(10)+5 = 35^\circ$. Arc: $8(10)-10 = 70^\circ$. Check: $\frac{70}{2} = 35$.

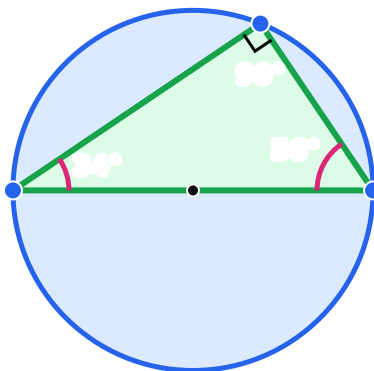
$x=10$, angle 35° , arc 70° .

Problem 7: Triangle in a semicircle

$\angle ACB$ is inscribed in a semicircle, so it is 90° . The angles of a triangle sum to 180° :

$\angle B = 180^\circ - 90^\circ - 34^\circ = 56^\circ$.

$\angle ACB = 90^\circ$ and $\angle B = 56^\circ$.



The angle at C is inscribed in a semicircle.

Problem 8: Semicircle algebra

$\angle ACB = 90^\circ$, so the other two angles sum to 90° : $(2x+4) + (3x+6) = 90$, $5x+10 = 90$, $x=16$.

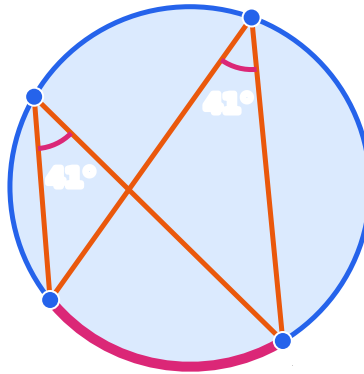
$2(16)+4 = 36^\circ$ and $3(16)+6 = 54^\circ$. Check: $36+54 = 90$.

$x=16$; the angles are 36° and 54° .

Problem 9: Angles on the same arc

Both inscribed angles intercept arc AB , so each is half of 82° : $\frac{1}{2}(82^\circ) = 41^\circ$.

$$m\angle ACB = m\angle ADB = 41^\circ$$



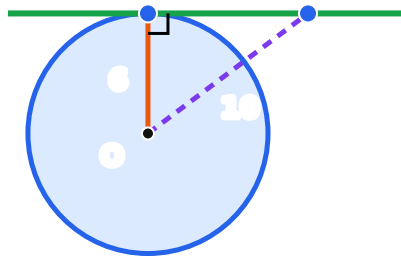
Both inscribed angles intercept arc AB, so each measures half of $82^\circ = 41^\circ$.

Problem 10: Radius and tangent

The radius OA is perpendicular to the tangent, so triangle OAP has a right angle at A .

$$OP = \sqrt{6^2 + 8^2} = \sqrt{100} = 10.$$

$$OP = 10$$



OA is a radius and AP lies on the tangent, so the angle at A is a right angle.

Problem 11: Angle at a tangent

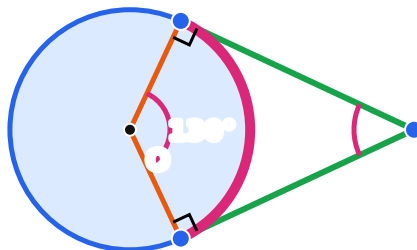
Since $\overline{OA} \perp \ell$, $\angle OAP = 90^\circ$. Then $m\angle APO = 180^\circ - 90^\circ - 52^\circ = 38^\circ$.

$$m\angle APO = 38^\circ$$

Problem 12: Two tangents

$\angle AOB$ is a central angle, so it is 130° . The radii meet the tangents at right angles, so quadrilateral $OAPB$ has angles 90° , 90° and 130° at A , B and O . The angles of a quadrilateral sum to 360° : $m\angle APB = 360^\circ - 90^\circ - 90^\circ - 130^\circ = 50^\circ$.

$m\angle AOB = 130^\circ$ and $m\angle APB = 50^\circ$.



The tangent segments meet the radii at right angles, so angle P = $180^\circ - 130^\circ = 50^\circ$.

Problem 13: Equal tangent segments

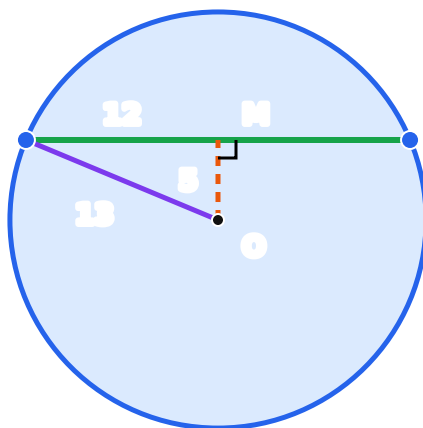
Triangle OAP has a right angle at A : $PA = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$. Triangle OBP is congruent, so $PB = 12$.

$$PA = PB = 12$$

Problem 14: Distance to a chord

The perpendicular from the center bisects the chord, so half the chord is 12. The distance d satisfies $d^2 + 12^2 = 13^2$, so $d = \sqrt{169 - 144} = 5$.

The distance is 5 units.



The perpendicular from O bisects the chord: $12^2 + 5^2 = 13^2$.

Problem 15: Chord from a distance

Half the chord is $\sqrt{17^2 - 8^2} = \sqrt{225} = 15$, so the chord is $2(15) = 30$.

The chord is 30 units long.

Problem 16: Equal chords

Equal chords cut equal arcs: $4x+6=6x-14$, so $20=2x$ and $x=10$.

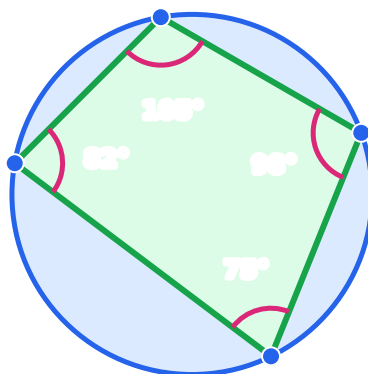
Arc AB : $4(10)+6=46^\circ$. Arc CD : $6(10)-14=46^\circ$.

$x=10$; **each arc is 46° .**

Problem 17: Inscribed quadrilateral

Opposite angles of an inscribed quadrilateral are supplementary. $m\angle C = 180^\circ - 105^\circ = 75^\circ$ and $m\angle D = 180^\circ - 82^\circ = 98^\circ$. Check: $105+82+75+98=360$.

$m\angle C = 75^\circ$ **and** $m\angle D = 98^\circ$.



Opposite angles 105° and 75° , and 82° and 98° , are supplementary.

Problem 18: Inscribed quadrilateral algebra

Opposite angles sum to 180° : $(4x+10)+(2x+20)=180$, $6x+30=180$, $x=25$.

$\angle A = 4(25)+10=110^\circ$ and $\angle C = 2(25)+20=70^\circ$.

$x=25$, $\angle A=110^\circ$, $\angle C=70^\circ$.

Problem 19: Ferris wheel seats

(a) $\frac{360^\circ}{12} = 30^\circ$.

(b) Three gaps: $3(30^\circ) = 90^\circ$.

(c) Her angle is inscribed and intercepts the 90° arc, so it is $\frac{1}{2}(90^\circ) = 45^\circ$.

(a) 30° . (b) 90° . (c) 45° .

Problem 20: Pizza cut

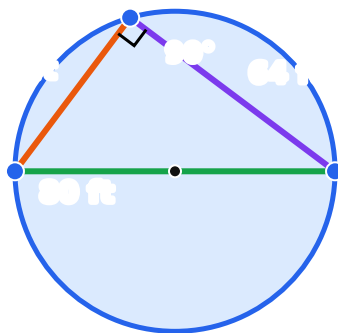
The radius is 10 inches and half the chord is 6 inches. The distance from the center is $\sqrt{10^2 - 6^2} = \sqrt{64} = 8$ inches.

The cut is 8 inches from the center.

Problem 21: Plaza lights

Angle ALB is inscribed in a semicircle, so $m\angle ALB = 90^\circ$. Triangle ALB is right with hypotenuse 80: $LB = \sqrt{80^2 - 48^2} = \sqrt{6400 - 2304} = \sqrt{4096} = 64$.

$m\angle ALB = 90^\circ$; **Liam is 64 feet from pole B .**

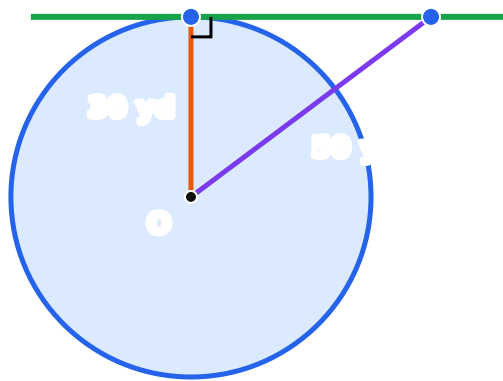


AB is a diameter, so the angle at L is 90° and $48^2 + 64^2 = 80^2$.

Problem 22: Pond path

The path is tangent, so $OT \perp TP$. $OP = \sqrt{30^2 + 40^2} = 50$ yards. The nearest point of the water is on segment OP , 30 yards from O , so Mei is $50 - 30 = 20$ yards from the water.

$OP = 50$ yards; the shortest distance to the water is 20 yards.



The path is tangent at T, so OT is perpendicular to the path and $OP = 50$ yd.

Problem 23: Central and inscribed algebra

The central angle is twice the inscribed angle: $12x - 20 = 2(5x - 5)$, so $12x - 20 = 10x - 10$, $2x = 10$ and $x = 5$.

$$\angle AOB = 12(5) - 20 = 40^\circ \text{ and } \angle ACB = 5(5) - 5 = 20^\circ.$$

$$x = 5, \angle AOB = 40^\circ, \angle ACB = 20^\circ.$$

Problem 24: Tangents and arcs algebra

The angle at P and the minor arc are supplementary: $(4x - 2) + (6x + 2) = 180$, so $10x = 180$ and $x = 18$.

$$\angle P = 4(18) - 2 = 70^\circ \text{ and arc } AB = 6(18) + 2 = 110^\circ.$$

$$x = 18, \angle P = 70^\circ, \text{ arc } AB = 110^\circ.$$