

# Geometry · Circles: Chords, Arcs and Inscribed Angles · Lesson

Common Core G-C.A.1, G-C.A.2 · mathovore.fr

Circle theorems connect angles, arcs, chords and tangents. A central angle equals its intercepted arc, an inscribed angle equals half of it, and an angle inscribed in a semicircle measures 90 degrees. A radius is perpendicular to a tangent at the point of tangency, and opposite angles of an inscribed quadrilateral add to 180 degrees.

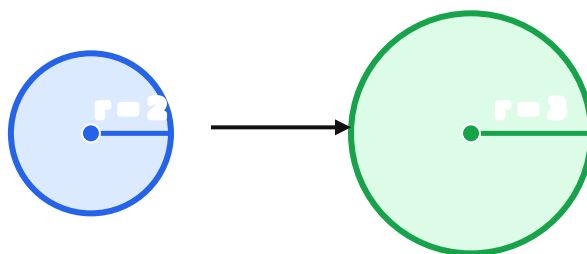
## GeometryCommon Core: G-C.A.1 · G-C.A.2

### Key facts:

- All circles are similar: a translation and a dilation with factor  $\frac{r_2}{r_1}$  map one onto another.
- A central angle equals its arc; an inscribed angle equals half its arc.
- An angle inscribed in a semicircle measures  $90^\circ$ .
- A radius is perpendicular to a tangent, and a circumscribed angle is  $180^\circ$  minus its minor arc.
- Opposite angles of an inscribed quadrilateral are supplementary.

## Why are all circles similar?

Translate circle A so that its center lands on the center of circle B. Then dilate by the scale factor  $\frac{r_B}{r_A}$ . Every point of A lands on B, so any two circles are similar. Lengths such as circumferences grow by that same factor.



Any two circles are similar: translate A onto B's center, then dilate by  $3/2$ .

## How do central and inscribed angles relate to arcs?

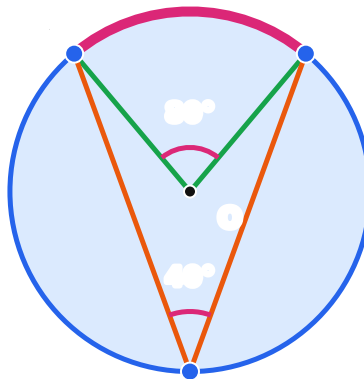
### Angle and arc measures

A **central angle** has its vertex at the center and equals its intercepted arc.

An **inscribed angle** has its vertex on the circle and equals half its intercepted arc.

1. Find the arc the angle intercepts.
2. For an inscribed angle, halve the arc; for a central angle, keep it.
3. State the result.

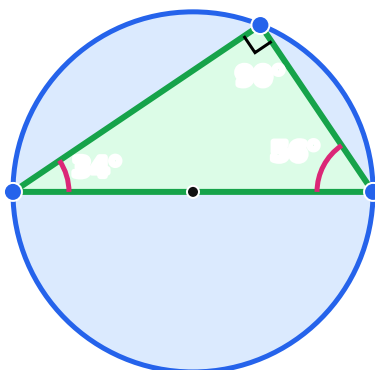
**Example.** Arc  $AB$  measures  $80^\circ$ . Then  $\angle AOB = 80^\circ$  and  $\angle ACB = \frac{1}{2}(80^\circ) = 40^\circ$ . Inscribed angles that intercept the same arc are equal.



The central angle measures  $80^\circ$ , the same as arc AB; the inscribed angle measures half,  $40^\circ$ .

## Why is an angle inscribed in a semicircle a right angle?

A diameter cuts off an arc of  $180^\circ$ , so any inscribed angle on it measures  $90^\circ$ . In the figure,  $\angle A = 34^\circ$  forces  $\angle B = 90^\circ - 34^\circ = 56^\circ$ .

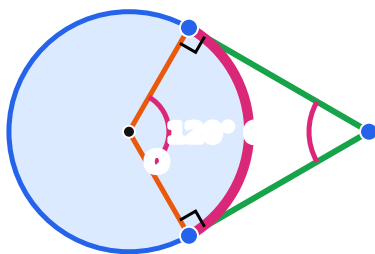


An angle inscribed in a semicircle is a right angle.

## What do tangents and circumscribed angles show?

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A radius drawn to the point of tangency is perpendicular to the tangent. Two tangents from an outside point  $P$  form a **circumscribed angle** with  $\angle P = 180^\circ - (\text{central angle})$ . If minor arc  $AB$  is  $120^\circ$ , then  $\angle P = 60^\circ$ .

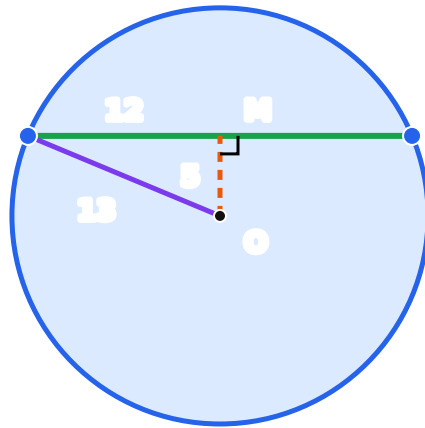


The tangent segments meet the radii at right angles, so angle  $P = 180^\circ - 120^\circ = 60^\circ$ .

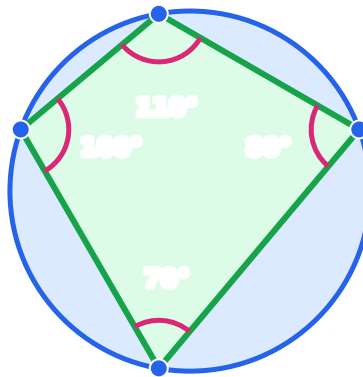
## How do chords and inscribed quadrilaterals work?

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A radius perpendicular to a chord bisects it, and equal chords cut equal arcs. For radius 13 and distance 5 from the center, half the chord is  $\sqrt{13^2 - 5^2} = 12$ , so the chord is 24. Opposite angles of an inscribed quadrilateral are supplementary:  $110^\circ$  and  $70^\circ$  sum to  $180^\circ$ .



The perpendicular from O bisects the chord:  $12^2 + 5^2 = 13^2$ .



Opposite angles  $110^\circ$  and  $70^\circ$ , and  $100^\circ$  and  $80^\circ$ , each sum to  $180^\circ$ .

## Circles: Chords, Arcs and Inscribed Angles: frequently asked questions

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### Why are all circles similar?

A translation moves one center onto the other, and a dilation with scale factor  $\frac{r_2}{r_1}$  then stretches or shrinks the circle to match. Because a similarity maps one circle exactly onto the other, any two circles are similar.

### What is the difference between a central angle and an inscribed angle?

A central angle has its vertex at the center and equals its intercepted arc. An inscribed angle has its vertex on the circle and equals half the same arc. An arc of  $80^\circ$  gives a central angle of  $80^\circ$  and an inscribed angle of  $40^\circ$ .

## Why is the angle in a semicircle always 90 degrees?

The diameter intercepts a semicircle, which is an arc of  $180^\circ$ . An inscribed angle is half its arc, so the angle measures  $90^\circ$ , no matter where the vertex is placed on the circle.

## How do you use a tangent line in a problem?

Draw the radius to the point of tangency. It is perpendicular to the tangent, which creates a right triangle. With radius 6 and tangent segment 8, the distance from the center to the outside point is  $\sqrt{6^2+8^2}=10$ .

## What is true about the angles of an inscribed quadrilateral?

Opposite angles are supplementary, so they add to  $180^\circ$ . If one angle is  $105^\circ$ , the opposite angle is  $75^\circ$ . All four angles together total  $360^\circ$ .