

Geometry · Arc Length, Sector Area and Radians · Worksheet Answer Key

Common Core G-C.B.5 · mathovore.fr

Every solution shows the fraction of the circle, the whole circumference or area, and the final answer in bold with units. Compare each step with your own work, and check that arc lengths are in units of length and areas in square units.

GeometryCommon Core: G-C.B.5

Problem 1: Fraction of a circle

$$\text{Fraction} = \text{angle} \div 360^\circ : \frac{150}{360} = \frac{5}{12}.$$

$\frac{5}{12}$ of the circumference

Problem 2: Quarter-circle arc

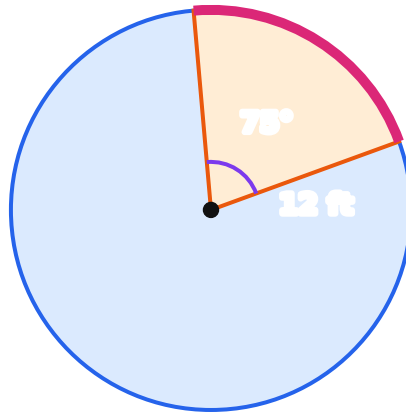
$$\text{Fraction of the circle: } \frac{90}{360} = \frac{1}{4}.$$

$$\text{Circumference: } 2\pi r = 2\pi(14) = 28\pi \text{ yd.}$$

$$\text{Arc length: } \frac{1}{4} \cdot 28\pi = 7\pi \text{ yd} \approx 21.99 \text{ yd.}$$

$$\text{Arc length} = 7\pi \text{ yd} \approx 21.99 \text{ yd}$$

Problem 3: Arc length of a sector



Result: $75/360 = 5/24$ of the circumference 24π ft.

Fraction of the circle: $\frac{75}{360} = \frac{5}{24}$.

Circumference: $2\pi r = 2\pi(12) = 24\pi$ ft.

Arc length: $\frac{5}{24} \cdot 24\pi = 5\pi$ ft ≈ 15.71 ft.

Arc length = 5π ft ≈ 15.71 ft

Problem 4: Arc with an obtuse angle

Fraction of the circle: $\frac{135}{360} = \frac{3}{8}$.

Circumference: $2\pi r = 2\pi(10) = 20\pi$ in.

Arc length: $\frac{3}{8} \cdot 20\pi = \frac{15\pi}{2}$ in ≈ 23.56 in.

Arc length = $\frac{15\pi}{2}$ in ≈ 23.56 in

Problem 5: Area of a sector

Fraction of the circle: $\frac{60}{360} = \frac{1}{6}$.

Area of the whole circle: $\pi r^2 = \pi(6)^2 = 36\pi$ in².

Sector area: $\frac{1}{6} \cdot 36\pi = 6\pi$ in² ≈ 18.85 in².

Sector area = 6π in² ≈ 18.85 in²

Problem 6: Area with an obtuse angle

Fraction of the circle: $\frac{135}{360} = \frac{3}{8}$.

Area of the whole circle: $\pi r^2 = \pi(10)^2 = 100\pi \text{ in}^2$.

Sector area: $\frac{3}{8} \cdot 100\pi = \frac{75\pi}{2} \text{ in}^2 \approx 117.81 \text{ in}^2$.

Sector area = $\frac{75\pi}{2} \text{ in}^2 \approx 117.81 \text{ in}^2$

Problem 7: Central angle from arc length

Circumference: $2\pi(8) = 16\pi \text{ in}$.

Fraction of the circle: $\frac{4\pi}{16\pi} = \frac{1}{4}$.

Angle: $\frac{1}{4} \cdot 360^\circ = 90^\circ$.

$\theta = 90^\circ$

Problem 8: Radius from arc length

Fraction: $\frac{72}{360} = \frac{1}{5}$. So $\frac{1}{5} \cdot 2\pi r = 6\pi$.

Multiply both sides by 5: $2\pi r = 30\pi$. Divide by 2π : $r = 15$.

Check: $\frac{1}{5} \cdot 2\pi(15) = 6\pi$. ✓

$r = 15 \text{ cm}$

Problem 9: Radius from sector area

Fraction: $\frac{120}{360} = \frac{1}{3}$. So $\frac{1}{3} \pi r^2 = 12\pi$.

Multiply by 3: $\pi r^2 = 36\pi$. Divide by π : $r^2 = 36$, so $r = 6$.

Check: $\frac{1}{3} \pi(6)^2 = 12\pi$. ✓

$r = 6 \text{ ft}$

Problem 10: Central angle from sector area

Area of the whole circle: $\pi(5)^2 = 25\pi \text{ in}^2$.

Fraction: $\frac{5\pi}{25\pi} = \frac{1}{5}$.

Angle: $\frac{1}{5} \cdot 360^\circ = 72^\circ$.

$$72^\circ$$

Problem 11: Meaning of a radian

Radian measure = arc length $\div$ radius: $\theta = \frac{s}{r} = \frac{15}{5} = 3$.

3 radians

Problem 12: Radians from arc and radius

$$\theta = \frac{s}{r} = \frac{10}{4} = \frac{5}{2} = 2.5.$$

$\frac{5}{2}$ radians (2.5 radians)

Problem 13: Degrees to radians

Multiply by $\frac{\pi}{180}$: $150 \cdot \frac{\pi}{180} = \frac{150\pi}{180} = \frac{5\pi}{6}$.

$\frac{5\pi}{6}$ radians

Problem 14: Reflex angle in radians

$$225 \cdot \frac{\pi}{180} = \frac{225\pi}{180} = \frac{5\pi}{4} \text{ (divide top and bottom by 45).}$$

$\frac{5\pi}{4}$ radians

Problem 15: Radians to degrees

$$\text{Multiply by } \frac{180}{\pi}: \frac{7\pi}{6} \cdot \frac{180}{\pi} = \frac{7 \cdot 180}{6} = 7 \cdot 30 = 210^\circ.$$

$$210^\circ$$

Problem 16: Radians to degrees, again

$$\frac{5\pi}{3} \cdot \frac{180}{\pi} = 5 \cdot 60 = 300^\circ.$$

$$300^\circ$$

Problem 17: A radian without pi

$$2 \cdot \frac{180}{\pi} = \frac{360}{\pi} \approx 114.59^\circ.$$

$$\approx 114.59^\circ$$

Problem 18: Arc length with radians

With radians, $s = r\theta$: $s = 7 \cdot \frac{\pi}{3} = \frac{7\pi}{3} \approx 7.33$ in.

$$\frac{7\pi}{3} \text{ in} \approx 7.33 \text{ in}$$

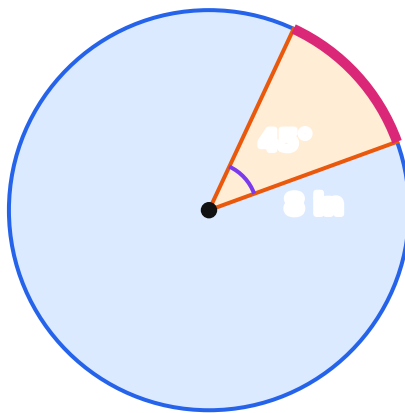
Problem 19: Sector area with radians

With radians, $A = \frac{1}{2}r^2\theta$: $A = \frac{1}{2}(6)^2 \cdot \frac{\pi}{4} = 18 \cdot \frac{\pi}{4} = \frac{9\pi}{2} \approx 14.14 \text{ ft}^2$.

Check with degrees: $\frac{\pi}{4} = 45^\circ$, and $\frac{45}{360} \cdot 36\pi = \frac{9\pi}{2}$. ✓

$$\frac{9\pi}{2} \text{ ft}^2 \approx 14.14 \text{ ft}^2$$

Problem 20: Pizza slice



Each slice is $\frac{1}{8}$ of the pizza: crust 2π in, area 8π square inches.

The diameter is 16 in, so the radius is 8 in. Each slice has central angle $360^\circ \div 8 = 45^\circ$, which is $\frac{1}{8}$ of the pizza.

(a) Crust: $\frac{1}{8} \cdot 2\pi(8) = \frac{1}{8} \cdot 16\pi = 2\pi \approx 6.28 \text{ in}$.

(b) Area: $\frac{1}{8} \cdot \pi(8)^2 = \frac{1}{8} \cdot 64\pi = 8\pi \approx 25.13 \text{ in}^2$.

(a) $2\pi \approx 6.28 \text{ in}$; **(b)** $8\pi \approx 25.13 \text{ in}^2$

Problem 21: Lawn sprinkler

Fraction of the circle: $\frac{80}{360} = \frac{2}{9}$.

Area of the whole circle: $\pi r^2 = \pi(25)^2 = 625\pi \text{ ft}^2$.

Sector area: $\frac{2}{9} \cdot 625\pi = \frac{1250\pi}{9} \text{ ft}^2 \approx 436.33 \text{ ft}^2$.

$$\text{Sector area} = \frac{1250\pi}{9} \text{ ft}^2 \approx 436.33 \text{ ft}^2$$

Rounded: **about 436 square feet.**

Problem 22: Running the curve

$$\text{Fraction of the circle: } \frac{200}{360} = \frac{5}{9}.$$

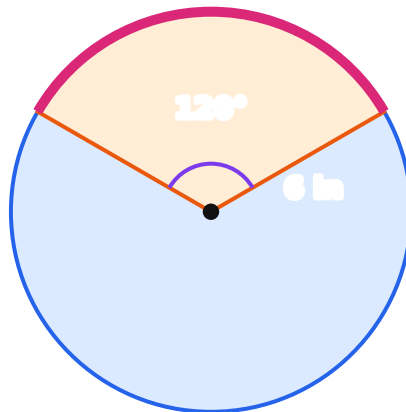
$$\text{Circumference: } 2\pi r = 2\pi(30) = 60\pi \text{ yd.}$$

$$\text{Arc length: } \frac{5}{9} \cdot 60\pi = \frac{100\pi}{3} \text{ yd} \approx 104.72 \text{ yd.}$$

$$\text{Arc length} = \frac{100\pi}{3} \text{ yd} \approx 104.72 \text{ yd}$$

Rounded to the nearest tenth: **104.7 yd.**

Problem 23: Clock hand



The tip travels one third of the circle: 4π in.

The tip moves along an arc of radius 6 in. Fraction: $\frac{120}{360} = \frac{1}{3}$ (also $\frac{20}{60}$ of an hour).

$$\text{Arc length: } \frac{1}{3} \cdot 2\pi(6) = \frac{1}{3} \cdot 12\pi = 4\pi \approx 12.57 \text{ in.}$$

$$4\pi \approx 12.57 \text{ in}$$

Problem 24: Pie chart slice

$$(a) 0.30 \cdot 360^\circ = 108^\circ.$$

$$(b) \text{Arc: } 0.30 \cdot 2\pi(5) = 0.30 \cdot 10\pi = 3\pi \approx 9.42 \text{ in.}$$

(c) Area: $0.30 \cdot \pi(5)^2 = 0.30 \cdot 25\pi = \frac{15\pi}{2} \approx 23.56 \text{ in}^2$.

(a) 108° ; **(b)** $3\pi \approx 9.42 \text{ in}$; **(c)** $\frac{15\pi}{2} \approx 23.56 \text{ in}^2$

Problem 25: Ferris wheel

(a) $\theta = \frac{s}{r} = \frac{100}{40} = 2.5 \text{ radians}$.

(b) $2.5 \cdot \frac{180}{\pi} = \frac{450}{\pi} \approx 143.24^\circ$.

(a) 2.5 radians; **(b)** $\approx 143.24^\circ$