

Geometry · Arc Length, Sector Area and Radians · Lesson

Common Core G-C.B.5 · mathovore.fr

Arc length and sector area are fractions of the whole circle. A central angle of θ degrees cuts off the fraction $\frac{\theta}{360}$ of the circumference and of the area. Radian measure is arc length divided by radius, so a full turn is 2π radians and 180 degrees equals π radians.

Geometry Common Core: G-C.B.5

Key facts:

- Arc length: $s = \frac{\theta}{360} \cdot 2\pi r$ with θ in degrees.
- Sector area: $A = \frac{\theta}{360} \cdot \pi r^2$ with θ in degrees.
- Radian measure: $\theta = \frac{s}{r}$, so $s = r\theta$ and $A = \frac{1}{2}r^2\theta$.
- $180^\circ = \pi$ radians, and one radian is about 57.3° .

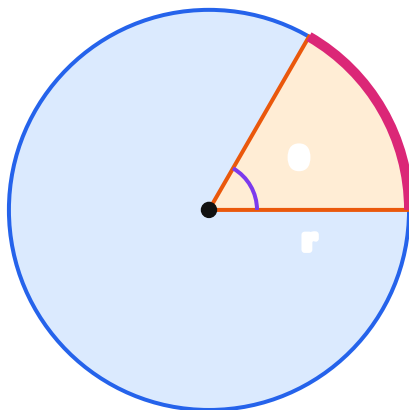
What are arc length and sector area?

A **central angle** has its vertex at the center of a circle. It cuts off an **arc** on the circle and a pie-slice region called a **sector**. The arc and the sector are fixed fractions of the whole circle, and that fraction is set by the angle alone.

Key terms

Arc length is a distance along the circle, measured in units such as inches. **Arc measure** is the central angle, measured in degrees or radians.

Sector area is the area enclosed by two radii and their arc, measured in square units.



A central angle θ cuts off the arc (pink) and the sector (orange).

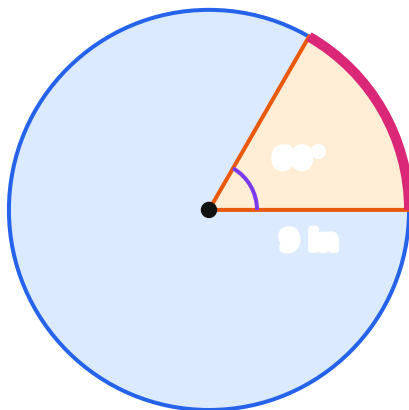
How do you find arc length and sector area?

Compare the angle to a full turn. Both measures scale with the fraction of the circle.

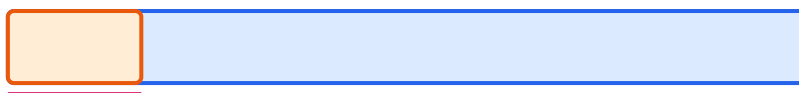
Quantity	Angle in degrees	Angle in radians
Arc length	$s = \frac{\theta}{360} \cdot 2\pi r$	$s = r\theta$
Sector area	$A = \frac{\theta}{360} \cdot \pi r^2$	$A = \frac{1}{2}r^2\theta$

1. Write the fraction of the circle: the angle divided by 360.
2. Find the whole circumference or the whole area.
3. Multiply the fraction by that whole.
4. Keep the exact answer with π , then round if a decimal is requested, and attach the units.

Worked example. A sector has radius 9 in and central angle 60° . The fraction is $\frac{60}{360} = \frac{1}{6}$. Arc length: $\frac{1}{6} \cdot 18\pi = 3\pi \approx 9.42$ in. Area: $\frac{1}{6} \cdot 81\pi = \frac{27\pi}{2} \approx 42.41$ in².



Worked example: one sixth of the circle gives one sixth of the circumference and one sixth of the area.



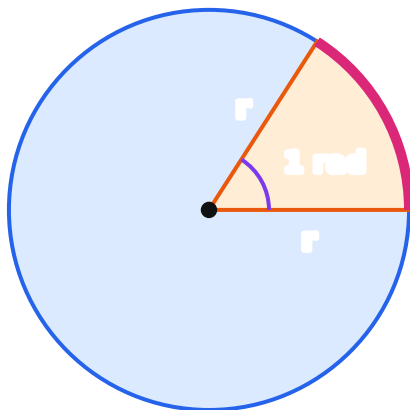
The arc is $60/360 = 1/6$ of the circumference.

What is a radian?

Radian

One **radian** is the measure of the central angle whose arc has the same length as the radius. In general, $\theta = \frac{s}{r}$, so the radian measure of an angle is its arc length divided by the radius.

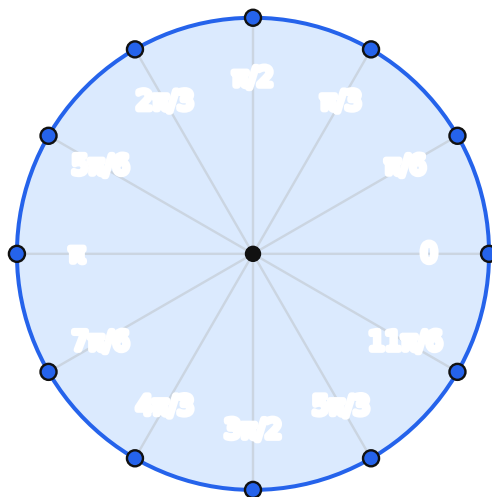
The whole circumference is $2\pi r$, so a full turn measures $\frac{2\pi r}{r} = 2\pi$ radians. Radian measure has no units: it is a ratio of two lengths.



One radian: the arc is exactly as long as the radius, about 57.3 degrees.

How do you convert between degrees and radians?

A half turn is 180° and also π radians, so $180^\circ = \pi$ radians. Multiply degrees by $\frac{\pi}{180}$ to get radians, and multiply radians by $\frac{180}{\pi}$ to get degrees. Example: $150 \cdot \frac{\pi}{180} = \frac{5\pi}{6}$ radians, and $\frac{3\pi}{4} \cdot \frac{180}{\pi} = 135^\circ$.



Common angles: degrees outside the circle, radians inside.

Arc Length, Sector Area and Radians: frequently asked questions

What is the formula for arc length?

With the central angle in degrees, arc length is $s = \frac{\theta}{360} \cdot 2\pi r$. With the angle in radians, it simplifies to $s = r\theta$. Both come from taking the same fraction of the circumference.

What is the difference between arc length and arc measure?

Arc measure is an angle, equal to the central angle, in degrees or radians. Arc length is a distance along the curve, in units like inches. Two arcs can have the same measure but different lengths when the circles have different radii.

How many degrees are in one radian?

One radian is $\frac{180}{\pi}$ degrees, which is about 57.30° . It is the angle whose arc is exactly as long as the radius.

Why is sector area a fraction of the circle's area?

A sector with central angle θ takes up $\frac{\theta}{360}$ of a full turn, and the area of a circle is spread evenly around the center. So the sector holds the same fraction of πr^2 .

When should I leave answers in terms of pi?

Leave π in the answer when the problem says exact or gives no rounding rule, because it is precise. Give a decimal such as 9.42 in only when the problem asks you to round.