

AP Statistics · Sampling Distributions · Lesson

Common Core AP Statistics Unit 2 (2026-27) · mathovore.fr

This ap statistics sampling distributions lesson explains sampling distributions in one printable page, with definitions, a method and a worked example.

A sampling distribution shows how a statistic, such as a sample proportion or sample mean, varies across all random samples of the same size. Its center equals the parameter, its standard deviation shrinks as the sample size grows, and by the Central Limit Theorem it becomes approximately normal for large samples.

AP StatisticsCommon Core: AP Statistics Unit 2 (2026-27)

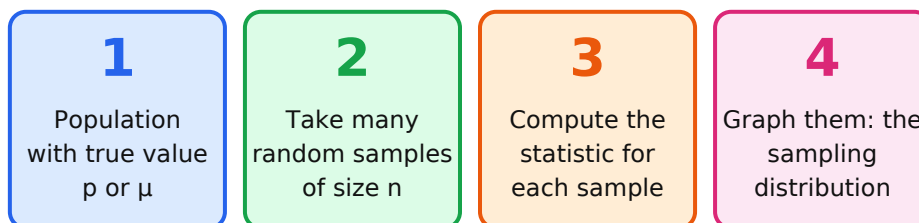
AP Statistics Sampling Distributions Lesson: key facts

- Sample proportion: $\mu_{\hat{p}} = p$, $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$; normal when $np \geq 10$ and $n(1-p) \geq 10$.
- Sample mean: $\mu_{\bar{x}} = \mu$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$; approximately normal if the population is normal or $n \geq 30$ (Central Limit Theorem).
- The standard-deviation formulas need independence: $n \leq 10\%$ of the population when sampling without replacement.
- Random sampling gives unbiased statistics; a larger n reduces variability, and quadrupling n halves the standard deviation.
- For differences, the means subtract but the variances add.

What is a sampling distribution?

A **parameter** describes a population, such as the true proportion p or the true mean μ . A **statistic**, such as $\hat{p}$ or $\bar{x}$, describes one sample. The **sampling distribution** of a statistic shows every value it takes over all possible samples of the same size n , and how often each occurs.

How a sampling distribution is built



A sampling distribution describes how a statistic varies from sample to sample.

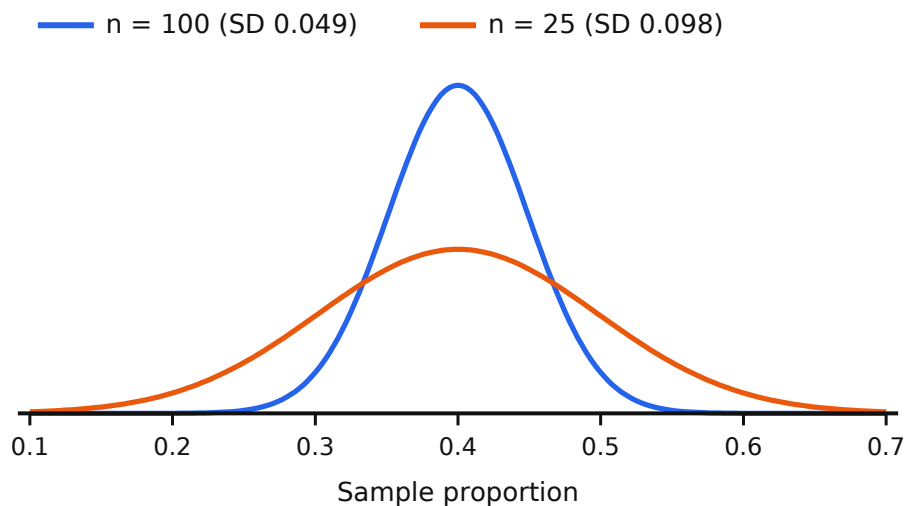
What is the sampling distribution of a sample proportion?

Sample proportion $\hat{p}$ from a random sample of size n

$$\mu_{\hat{p}} = p \quad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Needs $n \leq 10\%$ of the population. The shape is approximately normal when $np \geq 10$ and $n(1-p) \geq 10$.

Example. If 40% of students bike to school, samples of $n=100$ give $\mu_{\hat{p}}=0.40$ and $\sigma_{\hat{p}} = \sqrt{\frac{0.4(0.6)}{100}} \approx 0.0490$. With $n=25$ the standard deviation grows to 0.0980.

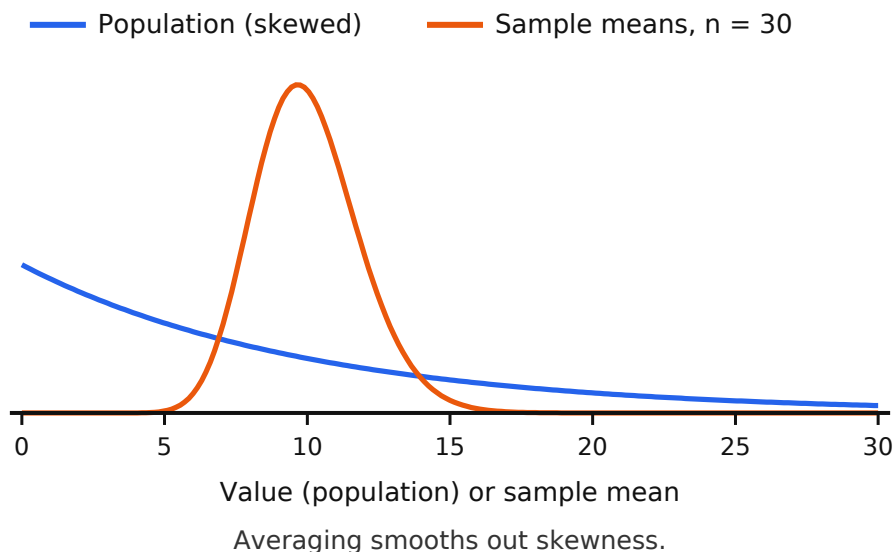


Larger samples give a narrower sampling distribution.

What does the Central Limit Theorem say about sample means?

For a random sample of size n from a population with mean μ and standard deviation σ , $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ (again with $n \leq 10\%$ of the population). The shape of the sampling distribution is normal if the population is normal. By the **Central Limit Theorem**, it is also approximately normal for any population shape when $n \geq 30$.

Take a skewed population with $\mu=10$ and $\sigma=10$, and $n=30$. Then $\sigma_{\bar{x}} = \frac{10}{\sqrt{30}} \approx 1.826$, and $P(\bar{x} > 12) \approx P(z > 1.10) = 0.1357$.



What are bias and variability?

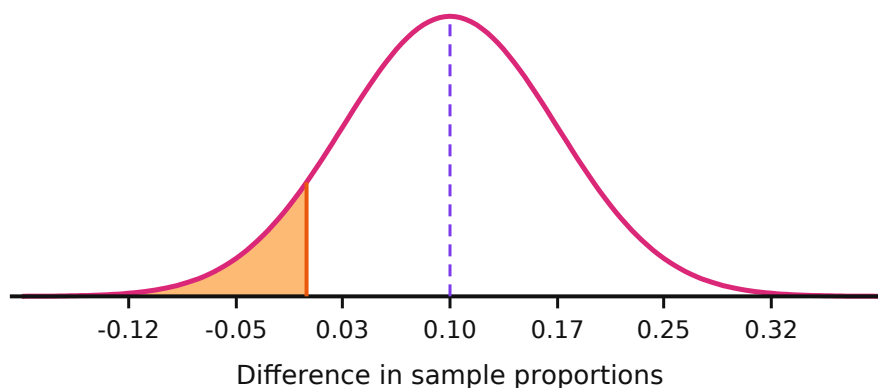
A statistic is **unbiased** when the center of its sampling distribution equals the parameter. **Variability** is the spread of that distribution. Random sampling keeps the statistic unbiased, and a larger sample reduces variability. A larger population does not: only n matters.

How do you handle a difference of two statistics?

For independent random samples, the sampling distribution of $\hat{p}_1 - \hat{p}_2$ has mean $p_1 - p_2$ and standard deviation $\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$. For $\bar{x}_1 - \bar{x}_2$, the mean is $\mu_1 - \mu_2$ and the standard deviation is $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$. Variances add, standard deviations do not.

With $p_1 = 0.55$, $n_1 = 80$, $p_2 = 0.45$, $n_2 = 100$: the mean is 0.10, the standard deviation is ≈ 0.0746 , and $P(\hat{p}_1 - \hat{p}_2 < 0) \approx 0.0901$.

$$P(\text{difference} < 0) = 0.0901$$



A negative difference is unlikely when the true difference is 0.10.

Sampling Distributions: frequently asked questions

What is the difference between a sample distribution and a sampling distribution?

A sample distribution shows the individual values in one sample. A sampling distribution shows the values of a statistic, such as the sample mean, across all possible samples of the same size. Inference depends on the second one.

Why does the standard deviation of the sample mean have a square root of n ?

Averaging reduces variability, but not proportionally. The spread of the mean is the population standard deviation divided by the square root of n , so cutting the spread in half takes four times as many observations.

When can I use the Central Limit Theorem?

Use it when the sample is random, the sample size is less than 10% of the population, and n is at least 30. If the population is strongly skewed or has outliers, a larger n may be needed; if the population is normal, any n works.

Does a larger population require a larger sample?

No. The variability of a statistic depends on the sample size, not on the population size, as long as the population is at least 10 times the sample. A sample of 1,000 is about as precise for a country as for a large state.

Why do the variances add when finding the standard deviation of a difference?

Each sample statistic varies independently, and the variability of the difference combines both sources. Variances of independent quantities add, so the standard deviation is the square root of the sum of the two variances.

How to use this ap statistics sampling distributions lesson

Use this ap statistics sampling distributions lesson before the worksheet, then check what you learned with the quiz.

