

AP Statistics · Inference for Proportions · Lesson

Common Core AP Statistics Unit 3 (2026-27) · mathovore.fr

This ap statistics proportions lesson explains inference for proportions in one printable page, with definitions, a method and a worked example.

Inference for proportions uses a sample proportion to estimate or test a population proportion. A confidence interval gives plausible values for p , while a significance test measures how surprising the sample would be if a claimed value were true. Both require random data, independence, and enough successes and failures for a normal model.

AP StatisticsCommon Core: AP Statistics Unit 3 (2026-27)

AP Statistics Proportions Lesson: key facts

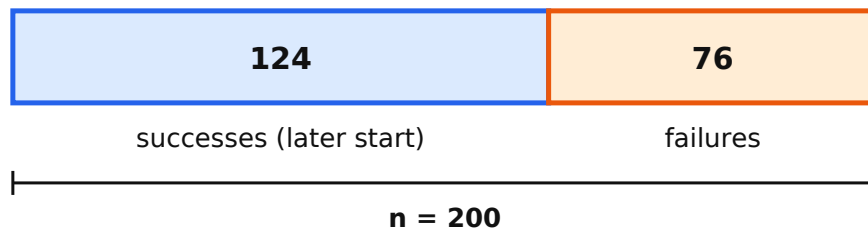
- Check three conditions: random, 10% (independence), and Large Counts (at least 10 successes and 10 failures).
- A one-proportion z interval is the sample proportion plus or minus z^* times the standard error; z^* is 1.645, 1.960 or 2.576 for 90%, 95% or 99%.
- A P-value is computed assuming H_0 is true; a small P-value is evidence against H_0 .
- A Type I error has probability α ; power equals 1 minus the probability of a Type II error.
- The sample size for a margin of error ME is $n = (z^*/ME)^2 p(1-p)$, rounded up.

What conditions must hold before doing inference for a proportion?

Inference about a population proportion p uses the sample proportion $\hat{p}$. Three conditions justify a normal model for $\hat{p}$.

- **Random:** the data come from a random sample or a randomized experiment.
- **10% condition:** when sampling without replacement, n is at most 10% of the population.
- **Large Counts:** for an interval, $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$. For a test, use $np_0 \geq 10$ and $n(1-p_0) \geq 10$.

Sample of 200 students



Large Counts: 124 and 76 are both at least 10.

How is a confidence interval for a proportion built?

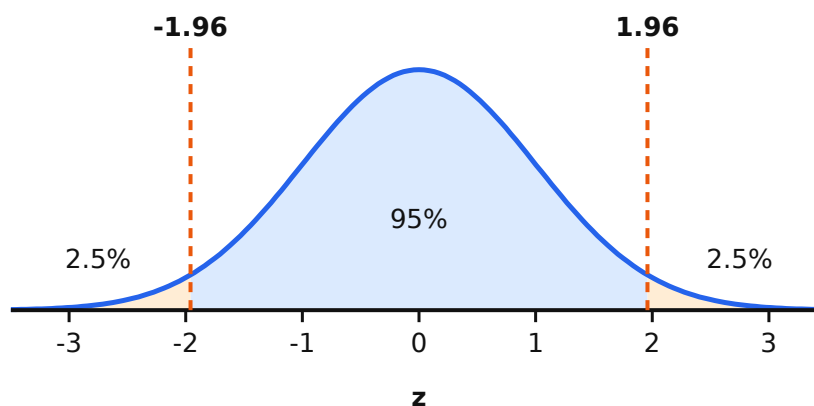
One-proportion z interval

statistic $\pm$ (critical value)(standard error)

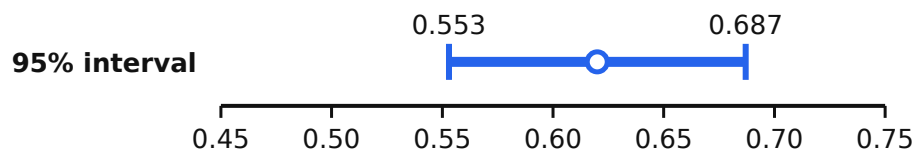
$$\hat{p} - z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \text{ to } \hat{p} + z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Critical values: $z^* = 1.645$ for 90%, 1.960 for 95%, 2.576 for 99%.

Example. A random sample of 200 students from a large district includes 124 who prefer a later start time, so $\hat{p} = 0.620$. The standard error is $\sqrt{\frac{0.62(0.38)}{200}} = 0.034$, and the margin of error is $1.960(0.034) = 0.067$. The 95% interval is 0.553 to 0.687. We are 95% confident that the interval captures the true proportion of all students in the district who prefer a later start.



A 95% interval uses $z^* = 1.960$ because the middle 95% of a normal curve lies within 1.960 standard deviations.



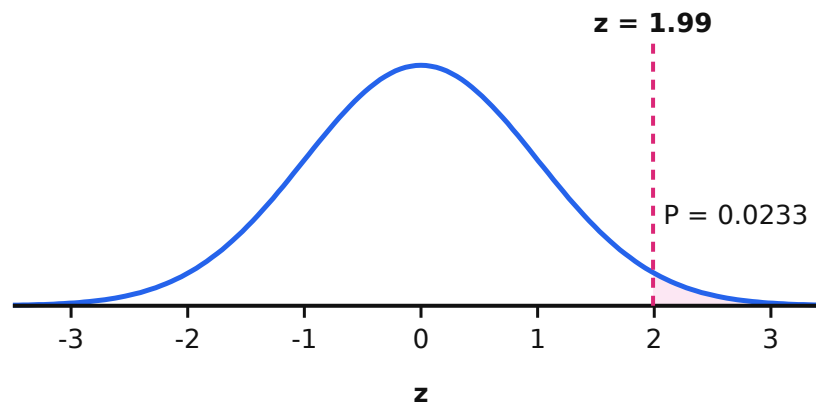
The interval 0.553 to 0.687 is centered at the sample proportion 0.620.

The confidence level describes the method: about 95% of intervals built this way capture $\mathcal{P}$. To compare two groups, the interval for $p_1 - p_2$ uses $\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$. If it contains 0, the data do not give convincing evidence of a difference.

How does a significance test for a proportion work?

1. State $H_0: p = p_0$ and H_a , and define $\mathcal{P}$.
2. Check the three conditions, using $\mathcal{P}_0$.
3. Compute $z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$ and the P-value.
4. Compare the P-value with α and conclude in context.

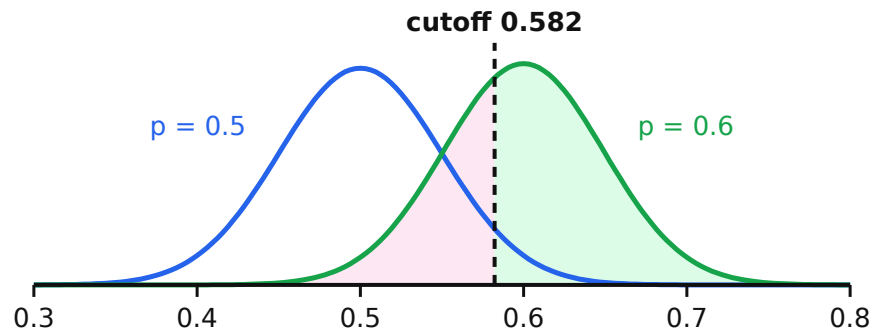
The **P-value** is the probability of a result at least as extreme as the one observed, assuming H_0 is true. With $H_0: p = 0.55$ and $H_a: p > 0.55$, the sample above gives $z = 1.99$ and $P = 0.0233$. Since $0.0233 < 0.05$, reject H_0 : there is convincing evidence that more than 55% prefer a later start. For two proportions, the test uses the pooled proportion $\hat{p}_c$ in the standard error.



The P-value is the shaded area to the right of $z = 1.99$.

What are Type I error, Type II error and power?

A **Type I error** rejects a true H_0 ; its probability is α . A **Type II error** fails to reject a false H_0 ; its probability is β . The **power** is $1 - \beta$, the chance of detecting a real difference. Power grows when n increases, when α increases, or when the true $\mathcal{P}$ is farther from $\mathcal{P}_0$.



■ alpha (Type I)
 ■ beta (Type II)
 ■ power

Blue: sampling distribution if $p = 0.5$. Green: if $p = 0.6$. The cutoff 0.582 gives $\alpha = 0.05$ and power about 0.64.

How large must the sample be for a given margin of error?

Solve $z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq ME$ for n : $n \geq \left(\frac{z^*}{ME}\right)^2 \hat{p}(1-\hat{p})$. With no prior estimate, use $\hat{p} = 0.5$, then round up.

Inference for Proportions: frequently asked questions

What is the difference between a confidence level and a confidence interval?

The confidence level, such as 95%, is the long-run success rate of the method. The confidence interval is the specific range of values computed from one sample.

Why does a test use p_0 but an interval use the sample proportion?

A test assumes the null value is true, so the standard error and Large Counts check use p_0 . An interval has no assumed value, so it estimates the standard error with the sample proportion.

What does a P-value of 0.03 mean?

If the null hypothesis were true, a sample result at least this extreme would happen about 3% of the time by chance alone. It is not the probability that the null hypothesis is true.

How can a confidence interval for a difference justify a claim?

If the interval for p_1 minus p_2 contains only positive values, there is convincing evidence that p_1 is larger. If it contains 0, the data do not show a difference.

How do you increase the power of a test?

Take a larger sample, raise the significance level, or study a situation where the true

proportion is farther from the null value. A larger sample does not raise the Type I error rate.

How to use this ap statistics proportions lesson

Use this ap statistics proportions lesson before the worksheet, then check what you learned with the quiz.

