

AP Calculus AB · Limits and Continuity · Worksheet Answer Key

Common Core AP Calculus AB Unit 1 · mathovore.fr

This ap calculus limits worksheet answer key solves all 20 problems of the limits and continuity worksheet step by step.

Every solution shows the algebra or the graph reading behind the answer. Compare each step with your own work, and redo any limit where your first substitution gave the form 0/0.

AP Calculus ABCommon Core: AP Calculus AB Unit 1

AP Calculus Limits Worksheet Answer Key: problems 1-5

Problem 1: Estimate a limit from a table

The outputs approach 0.25 from both sides, so the limit is about 0.25. To confirm, multiply by the conjugate: $\frac{(x+1)-4}{(x-3)(\sqrt{x+1}+2)} = \frac{1}{\sqrt{x+1}+2}$. At $x=3$ this is $\frac{1}{4}$. The limit is $\frac{1}{4}$.

Problem 2: Limits from a graph

- Coming from the left, the graph heads to height 2: **2**
- From the right it heads to 3: **3**
- The one-sided limits differ: **DNE**
- The filled point is at height 1: **1**
- Both sides head to 0, even though the point is elsewhere: **0**
- The filled point is at height 3: **3**

Problem 3: Direct substitution

- The function is continuous, so substitute: $3(4)-10+1 = \mathbf{3}$
- The denominator is 3, not 0: $\frac{-1-2}{3} = -1$
- $\sqrt{16+9} = \sqrt{25} = \mathbf{5}$

Problem 4: Factor and cancel

- $\frac{(x-5)(x+5)}{x-5} = x+5$, which tends to **10**
- $x^2+x-6 = (x+3)(x-2)$, so the quotient is $x-2$, which tends to **-5**
- $x^3-8 = (x-2)(x^2+2x+4)$; substitute $x=2$: $4+4+4 = \mathbf{12}$
- $\frac{(x-1)(x+1)}{(x-1)(x+2)} = \frac{x+1}{x+2}$, which tends to $\frac{2}{3}$

Problem 5: Conjugates and complex fractions

- a. Multiply by $\sqrt{x}+3$: $\frac{x-9}{(x-9)(\sqrt{x}+3)} = \frac{1}{\sqrt{x}+3}$, which tends to $\frac{1}{6}$
- b. Multiply by $\sqrt{x+4}+2$: the numerator becomes x , leaving $\frac{1}{\sqrt{x+4}+2}$, which tends to $\frac{1}{4}$
- c. The numerator becomes $(x+7)-9 = x-2$, leaving $\frac{1}{\sqrt{x+7}+3}$, which tends to $\frac{1}{6}$
- d. Combine: $\frac{1}{x+3} - \frac{1}{3} = \frac{-x}{3(x+3)}$. Dividing by x leaves $\frac{-1}{3(x+3)}$, which tends to $-\frac{1}{9}$

AP Calculus Limits Worksheet Answer Key: problems 6-10

Problem 6: Trigonometric limits

- a. $\frac{\sin 5x}{x} = 5 \cdot \frac{\sin 5x}{5x}$, which tends to **5**
- b. $\frac{\sin 3x}{\sin 7x} = \frac{3}{7} \cdot \frac{\sin 3x/3x}{\sin 7x/7x}$, which tends to $\frac{3}{7}$
- c. This is one of the given limits: **0**
- d. $\frac{\tan 2x}{x} = 2 \cdot \frac{\sin 2x}{2x} \cdot \frac{1}{\cos 2x}$, which tends to $2 \cdot 1 \cdot 1 = 2$

Problem 7: One-sided limits of a piecewise function

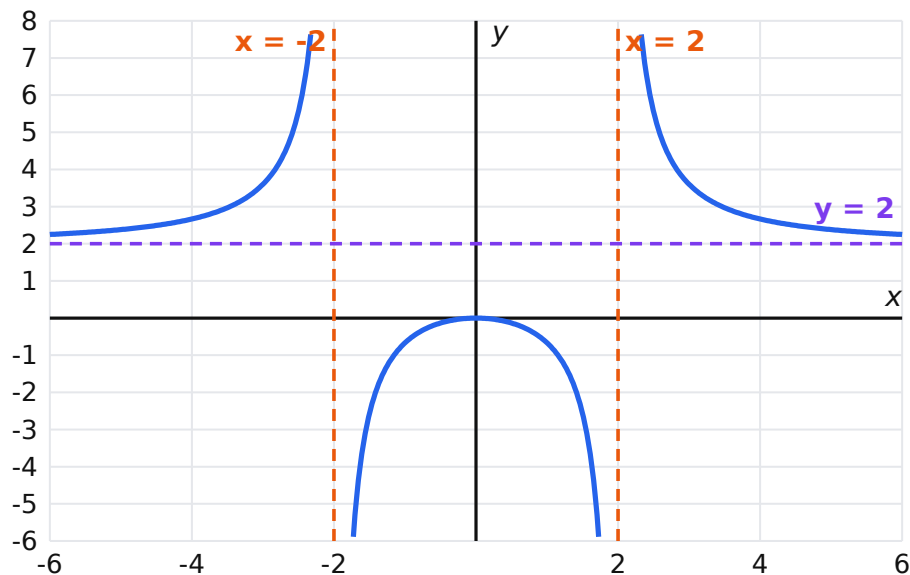
- a. Use the first rule: $2^2+1 = 5$
- b. Use the second rule: $8-2 = 6$
- c. $5 \neq 6$, so **DNE**
- d. $x=2$ uses the second rule: **6**

Problem 8: Limits at infinity

- a. Equal degrees: the ratio of leading coefficients is $\frac{3}{5}$
- b. The numerator has the smaller degree: **0**
- c. The numerator has the larger degree and positive values: ∞ (the limit does not exist as a number)
- d. For $x < 0$, $\sqrt{4x^2} = -2x$. Dividing by x gives $\frac{-2}{1}$, so the limit is -2

Problem 9: Vertical and horizontal asymptotes

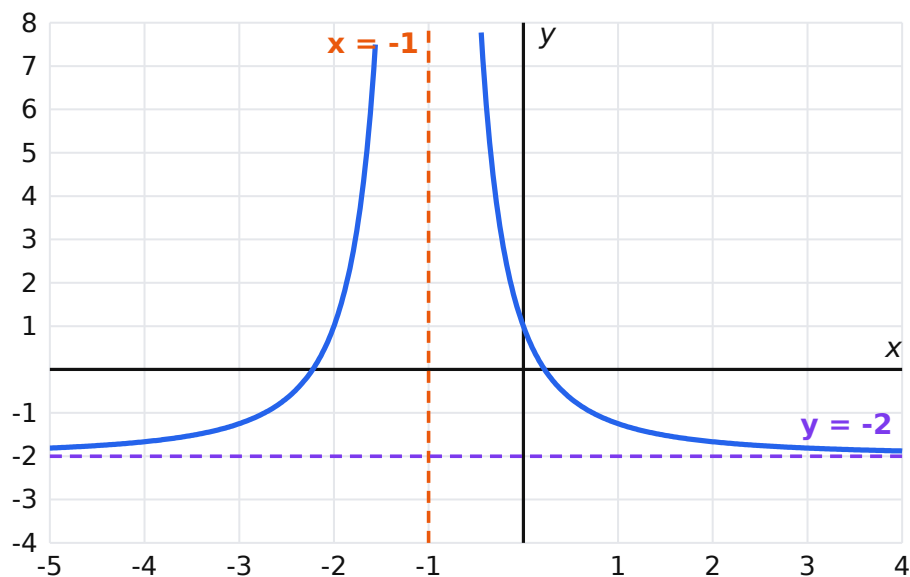
- a. The denominator $(x-2)(x+2)$ is zero at ± 2 and the numerator is not, so $x=2$ **and** $x=-2$
- b. Equal degrees: $\lim_{x \rightarrow \pm\infty} f(x) = \frac{2}{1}$, so $y=2$
- c. Near $x=2$ the numerator is about 8. For x slightly above 2, the denominator is a small positive number, giving ∞ . Slightly below 2, it is a small negative number, giving $-\infty$.



The graph with its asymptotes.

Problem 10: Infinite limits from a graph

- The curve climbs without bound: ∞
- Same on the right: ∞
- Both sides agree on ∞ , so the limit is ∞ (it is unbounded, so it does not exist as a number)
- The curve levels off near $y = -2$: -2
- Vertical asymptote $x = -1$; horizontal asymptote $y = -2$



The same graph with its asymptotes drawn.

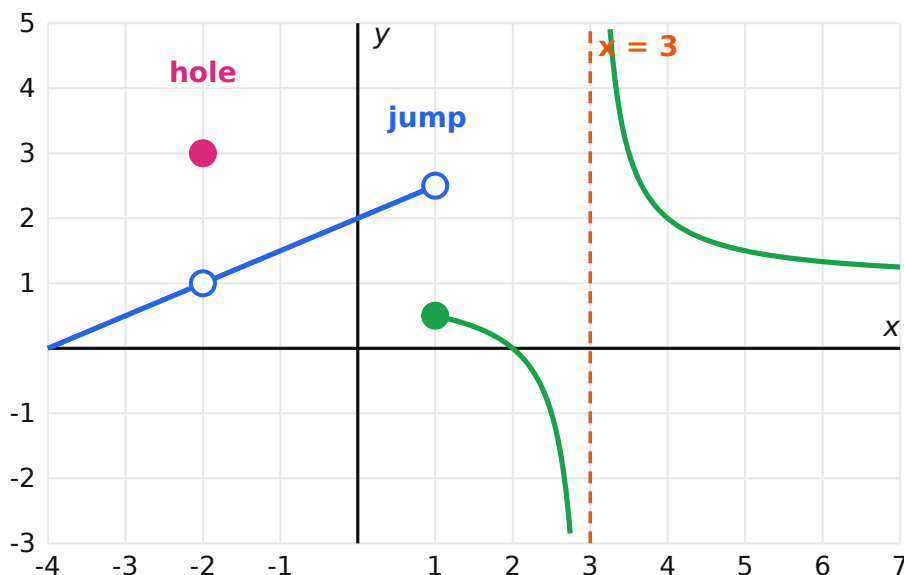
AP Calculus Limits Worksheet Answer Key: problems 11-15

Problem 11: Squeeze Theorem

- Both bounds tend to 5 as $x \rightarrow 0$, so **5**
- Since $-1 \leq \cos \frac{1}{x} \leq 1$, we have $-x^2 \leq x^2 \cos \frac{1}{x} \leq x^2$. Both bounds tend to 0, so **0**
- For $x > 0$, $-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$. Both bounds tend to 0, so **0**

Problem 12: Discontinuities from a graph

- $x = -2$, $x = 1$ and $x = 3$
- At $x = -2$ the limit is 1 but $f(-2) = 3$: **removable**. At $x = 1$ the one-sided limits are 2.5 and 0.5: **jump**. At $x = 3$ the curve is unbounded: **infinite**.
- Only the removable one: define $f(-2) = 1$.



The three discontinuities labeled.

Problem 13: Continuity at a point

Condition 1: $f(3) = 4$ is defined. Condition 2: $\frac{(x-3)(x+2)}{x-3} = x+2$, so $\lim_{x \rightarrow 3} f(x) = 5$ exists. Condition 3 fails: $5 \neq 4$. The discontinuity is **removable**. Setting $f(3) = 5$ makes f continuous.

Problem 14: Find k for continuity

Left side: $f(2) = 2k + 1$. Right limit: $4 - k$. Set them equal: $2k + 1 = 4 - k$, so $3k = 3$ and $k = 1$.
Check: $2(1) + 1 = 3$ and $4 - 1 = 3$. The common value is **3**.

Problem 15: Intermediate Value Theorem from a table

- Yes.** f is continuous and $f(0) = -3 < 0 < 5 = f(2)$, so the Intermediate Value Theorem gives a zero in $(0, 2)$.
- Yes.** Since $f(2) = 5$ and $f(4) = 2$, the value 4 lies between them, so $f(c) = 4$ for some c in $(2, 4)$, which is inside $(2, 6)$.

- c. **Not guaranteed.** The table values never reach 6, and the theorem only promises values between known outputs. (The function might exceed 6 between table entries, but nothing here forces it.)

AP Calculus Limits Worksheet Answer Key: problems 16-20

Problem 16: Show a root exists

g is a polynomial, so it is continuous on $[1,2]$. $g(1)=1+2-5=-2$ and $g(2)=8+4-5=7$.

Because 0 lies between -2 and 7 , the **Intermediate Value Theorem** guarantees a number c in $(1,2)$ with $g(c)=0$.

Problem 17: Parking garage fees

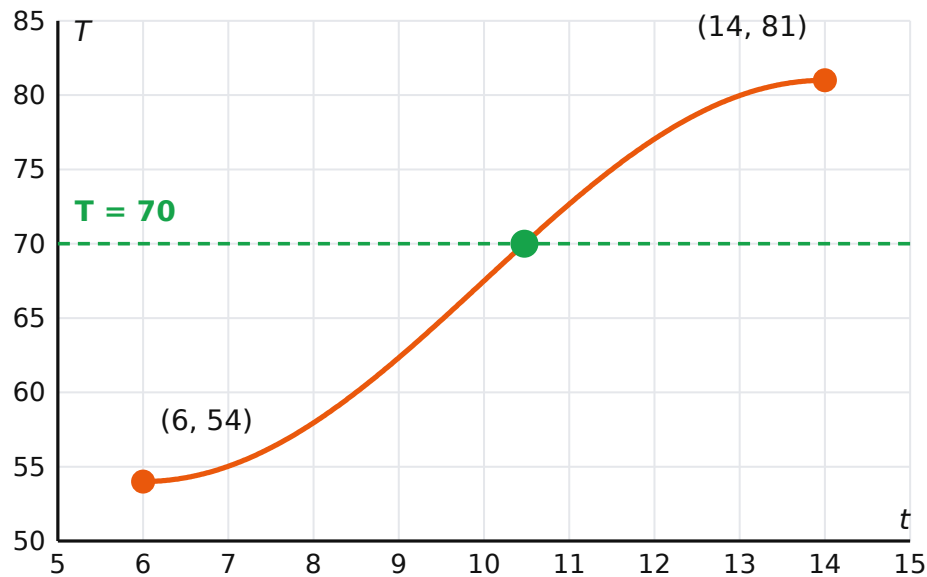
- From the left the cost is **\$5**; from the right it is **\$8**.
- The one-sided limits differ, so the limit **does not exist**, and C is **not continuous** at $t=2$ (a jump of \$3).
- $C(3)=$ **\$8** (the filled point), and the limit from the right is **\$11**.

Problem 18: Medicine in the bloodstream

$C(4)=\frac{120}{24}=5$ **mg/L**. For the limit, divide top and bottom by t : $\frac{30}{3+12/t} \rightarrow \frac{30}{3} = 10$ **mg/L**; the concentration levels off at 10 mg/L in the long run. Solve $\frac{30t}{3t+12}=9$: $30t=27t+108$, so $3t=108$ and $t=36$ **hours**.

Problem 19: Temperature must reach 70 degrees

- T is continuous on $[6,14]$, $T(6)=54$ and $T(14)=81$. Since $54 < 70 < 81$, the **Intermediate Value Theorem** gives a time c with $T(c)=70$.
- No.** 85 is not between 54 and 81, so the theorem says nothing about it. On this graph the maximum is 81.



The line $T = 70$ crosses the curve.

Problem 20: Shipping rates without a jump

At $w = 5$: $0.8(5) + 2 = \$6.00$. The other rule must also give \$6.00: $0.5(5) + k = 6$, so $k = 3.5$. For $w = 10$: $0.5(10) + 3.5 = \$8.50$.

How to use this ap calculus limits worksheet answer key

Read this ap calculus limits worksheet answer key after trying each problem yourself, and redo any problem you missed.

