

AP Calculus AB · Limits and Continuity · Worksheet

Common Core AP Calculus AB Unit 1 · mathovore.fr

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This ap calculus limits worksheet gives 20 problems on limits and continuity, ordered from easy to hard, with a separate answer key.

These 20 problems cover AP Calculus AB Unit 1: reading limits from tables and graphs, algebraic techniques, limits at infinity, asymptotes, the Squeeze Theorem, continuity and the Intermediate Value Theorem.

They run from easy to hard and finish with applications such as parking fees, drug concentration and temperature. Show each step, and write DNE only when no limit exists.

AP Calculus ABCommon Core: AP Calculus AB Unit 1

AP Calculus Limits Worksheet: problems 1-5

Problem 1: Estimate a limit from a table

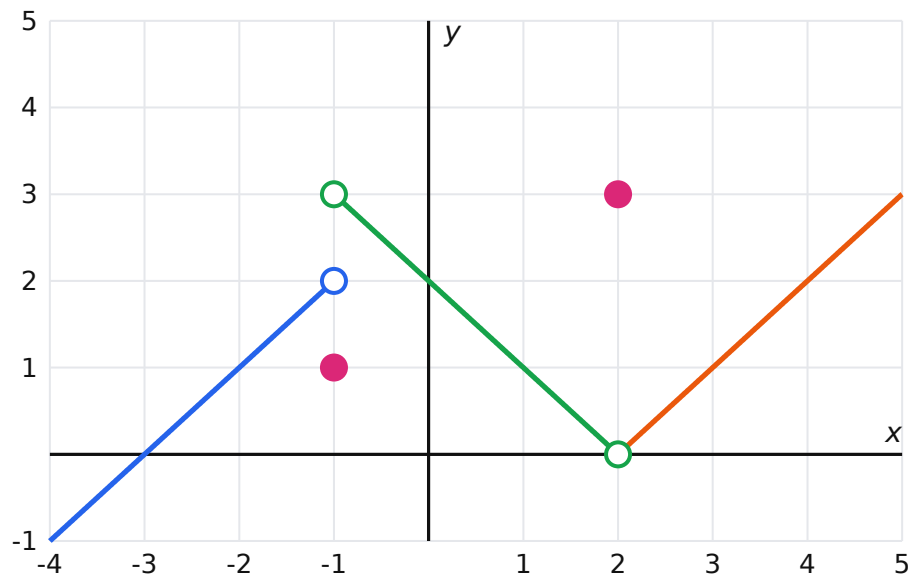
Let $f(x) = \frac{\sqrt{x+1}-2}{x-3}$. Values of f near $x=3$ are shown.

x	f(x)
2.9	0.25158
2.99	0.25016
2.999	0.25002
3.001	0.24998
3.01	0.24984
3.1	0.24846

Estimate $\lim_{x \rightarrow 3} f(x)$ from the table, then confirm it algebraically.

Problem 2: Limits from a graph

Use the graph of f to find each value, or write DNE if it does not exist.



Graph of f with breaks at $x = -1$ and $x = 2$.

- $\lim_{x \rightarrow -1^-} f(x)$
- $\lim_{x \rightarrow -1^+} f(x)$
- $\lim_{x \rightarrow -1} f(x)$
- $f(-1)$
- $\lim_{x \rightarrow 2} f(x)$
- $f(2)$

Problem 3: Direct substitution

Evaluate each limit.

- $\lim_{x \rightarrow 2} (3x^2 - 5x + 1)$
- $\lim_{x \rightarrow -1} \frac{x^3 + 2x}{x + 4}$
- $\lim_{x \rightarrow 4} \sqrt{x^2 + 9}$

Problem 4: Factor and cancel

Evaluate each limit.

- $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$
- $\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x + 3}$
- $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$
- $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 + x - 2}$

Problem 5: Conjugates and complex fractions

Evaluate each limit.

- $\lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x-9}$
- $\lim_{x \rightarrow 0} \frac{\sqrt{x+4}-2}{x}$
- $\lim_{x \rightarrow 2} \frac{\sqrt{x+7}-3}{x-2}$
- $\lim_{x \rightarrow 0} \frac{\frac{1}{x+3}-\frac{1}{3}}{x}$

AP Calculus Limits Worksheet: problems 6-10

Problem 6: Trigonometric limits

Use $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ and $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$ to evaluate each limit.

- $\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$
- $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 7x}$
- $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$
- $\lim_{x \rightarrow 0} \frac{\tan 2x}{x}$

Problem 7: One-sided limits of a piecewise function

Let $f(x) = x^2 + 1$ when $x < 2$ and $f(x) = 8 - x$ when $x \geq 2$. Find each value.

- $\lim_{x \rightarrow 2^-} f(x)$
- $\lim_{x \rightarrow 2^+} f(x)$
- $\lim_{x \rightarrow 2} f(x)$
- $f(2)$

Problem 8: Limits at infinity

Evaluate each limit.

- $\lim_{x \rightarrow \infty} \frac{3x^2 - x}{5x^2 + 2}$
- $\lim_{x \rightarrow \infty} \frac{4x + 1}{x^2 + 3}$
- $\lim_{x \rightarrow \infty} \frac{x^3}{2x^2 + 1}$
- $\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 + 1}}{x + 3}$

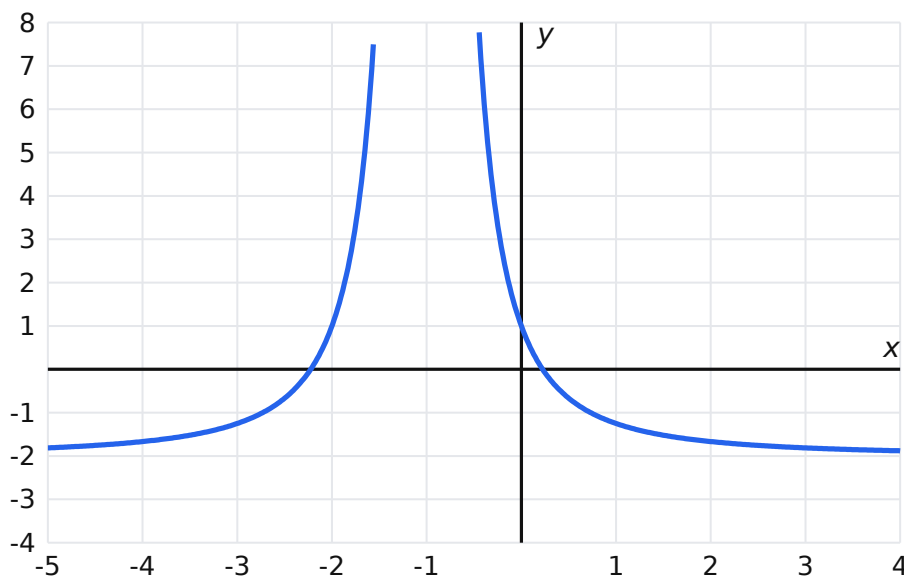
Problem 9: Vertical and horizontal asymptotes

Let $f(x) = \frac{2x^2}{x^2 - 4}$.

- Find every vertical asymptote.
- Find the horizontal asymptote.
- Find $\lim_{x \rightarrow 2^+} f(x)$ and $\lim_{x \rightarrow 2^-} f(x)$.

Problem 10: Infinite limits from a graph

Use the graph of h to find each limit.



The graph of h .

- a. $\lim_{x \rightarrow -1^-} h(x)$
- b. $\lim_{x \rightarrow -1^+} h(x)$
- c. $\lim_{x \rightarrow -1} h(x)$
- d. $\lim_{x \rightarrow \infty} h(x)$
- e. Write the equation of each asymptote.

AP Calculus Limits Worksheet: problems 11-15

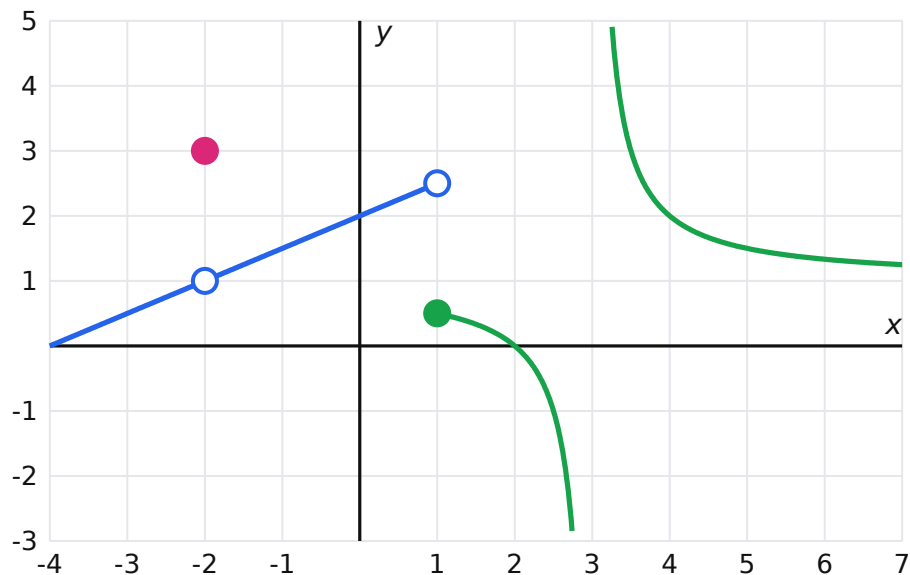
Problem 11: Squeeze Theorem

Use the Squeeze Theorem to evaluate each limit.

- a. g satisfies $5 - x^2 \leq g(x) \leq 5 + x^2$ for all x . Find $\lim_{x \rightarrow 0} g(x)$.
- b. $\lim_{x \rightarrow 0} x^2 \cos \frac{1}{x}$
- c. $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$

Problem 12: Discontinuities from a graph

Use the graph of f .



Graph of f .

- List every x -value where f is not continuous.
- Classify each as removable, jump or infinite.
- Which discontinuity can be removed by changing the value of f at one point, and what value should be used?

Problem 13: Continuity at a point

Let $f(x) = \frac{x^2 - x - 6}{x - 3}$ for $x \neq 3$ and $f(3) = 4$. Check the three conditions for continuity at $x = 3$, classify the discontinuity, and redefine $f(3)$ to remove it.

Problem 14: Find k for continuity

Let $f(x) = kx + 1$ when $x \leq 2$ and $f(x) = x^2 - k$ when $x > 2$. Find the value of k that makes f continuous at $x = 2$, then give the common value.

Problem 15: Intermediate Value Theorem from a table

A function f is continuous on $[0, 6]$ with the values below.

x	0	2	4	6
$f(x)$	-3	5	2	-1

- Must $f(x) = 0$ for some x in $(0, 2)$? Justify.
- Must $f(x) = 4$ for some x in $(2, 6)$? Justify.
- Must $f(x) = 6$ for some x in $(0, 6)$? Explain.

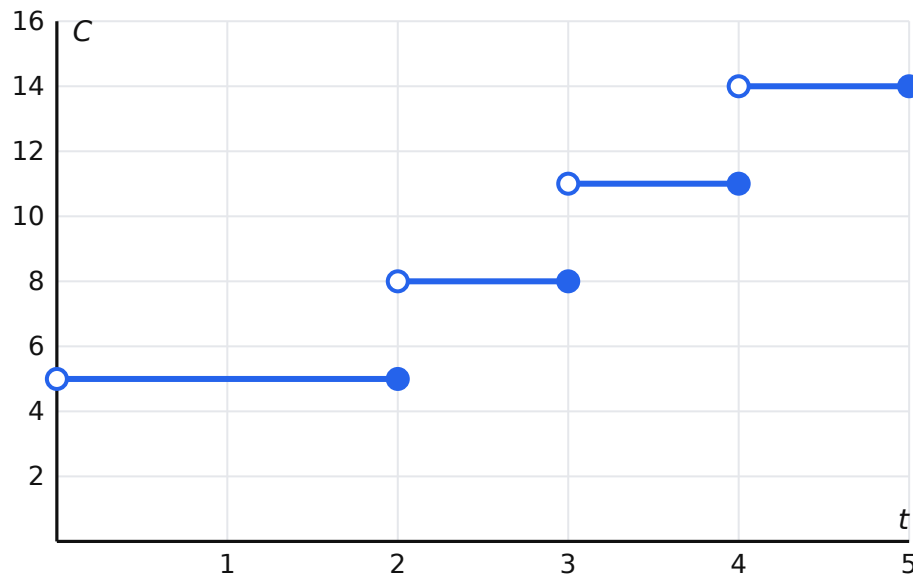
AP Calculus Limits Worksheet: problems 16-20

Problem 16: Show a root exists

Show that $g(x) = x^3 + 2x - 5$ has a zero in the interval $(1, 2)$. State the theorem and check its hypothesis.

Problem 17: Parking garage fees

A garage charges \$5 for up to 2 hours, then an extra \$3 for each additional hour or part of an hour (up to 5 hours). The graph shows the cost $C(t)$ in dollars after t hours.



Cost of parking $C(t)$ in dollars.

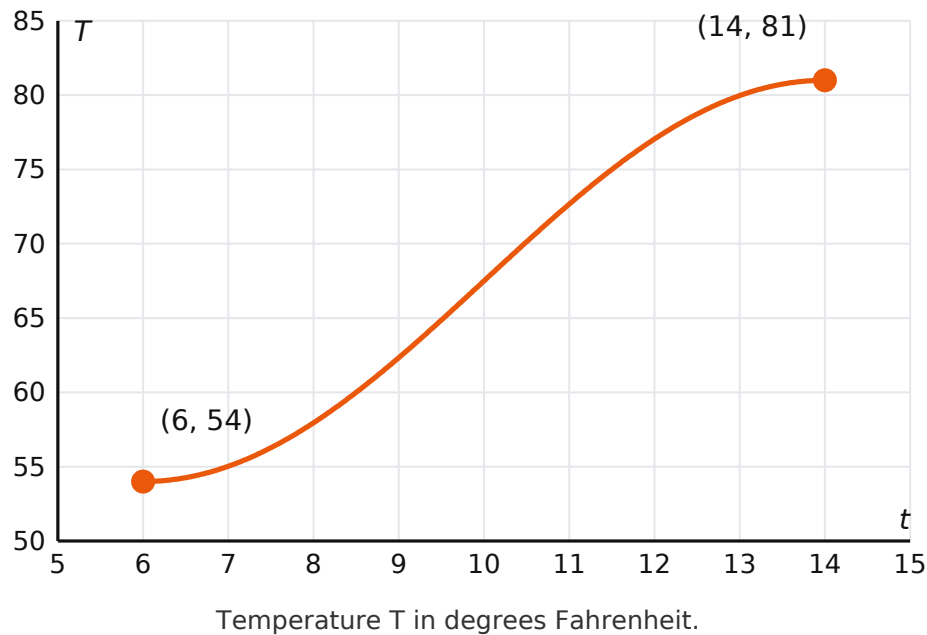
- $\lim_{t \rightarrow 2^-} C(t)$ and $\lim_{t \rightarrow 2^+} C(t)$
- Does $\lim_{t \rightarrow 2} C(t)$ exist? Is C continuous at $t = 2$?
- Find $C(3)$ and $\lim_{t \rightarrow 3^+} C(t)$.

Problem 18: Medicine in the bloodstream

The concentration of a drug, in mg/L, is $C(t) = \frac{30t}{3t+12}$ at t hours after an injection. Find $C(4)$, find $\lim_{t \rightarrow \infty} C(t)$ and interpret it, and find when the concentration reaches 9 mg/L.

Problem 19: Temperature must reach 70 degrees

Starting at 6 a.m. the temperature rises continuously from 54°F to 81°F at 2 p.m. (hour 14), as shown.



- Explain why the temperature was exactly 70°F at some time between 6 a.m. and 2 p.m.
- Can you be sure the temperature reached 85°F ? Explain.

Problem 20: Shipping rates without a jump

A company charges $C(w) = 0.8w + 2$ dollars for a package of w pounds when $0 \leq w \leq 5$, and $C(w) = 0.5w + k$ dollars when $w > 5$. Find k so the price has no jump at $w = 5$, then find the cost of a 10-pound package.

How to use this ap calculus limits worksheet

Print this ap calculus limits worksheet as a PDF, solve every problem on paper, then compare with the answer key.

