

AP Calculus AB · Limits and Continuity · Lesson

Common Core AP Calculus AB Unit 1 · mathovore.fr

This ap calculus limits lesson explains limits and continuity in one printable page, with definitions, a method and a worked example.

A limit describes the value a function approaches as its input gets close to a number, whether or not the function is defined there. A function is continuous at a point when its value and its limit agree. AP Calculus AB Unit 1 covers one-sided limits, asymptotes, the Squeeze Theorem, discontinuities and the Intermediate Value Theorem.

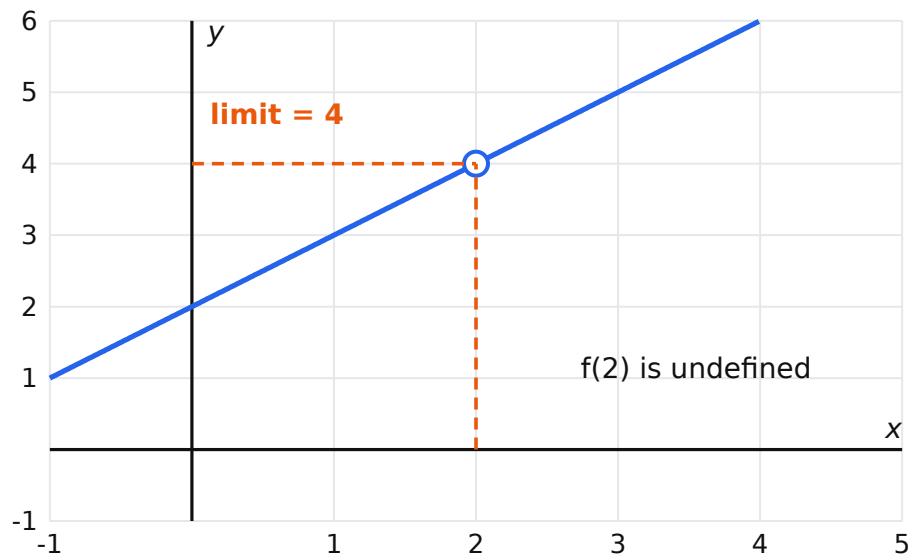
AP Calculus ABCommon Core: AP Calculus AB Unit 1

AP Calculus Limits Lesson: key facts

- AP Calculus AB Unit 1: $\lim_{x \rightarrow a} f(x) = L$ means $f(x)$ gets arbitrarily close to L as x gets close to a , from both sides.
- The two-sided limit exists only when $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$.
- Continuity at $x = a$ needs three things: $f(a)$ is defined, $\lim_{x \rightarrow a} f(x)$ exists, and the two are equal.
- Discontinuities come in three kinds: removable (hole), jump, and infinite (vertical asymptote).
- The Intermediate Value Theorem: if f is continuous on $[a, b]$ and k lies between $f(a)$ and $f(b)$, then $f(c) = k$ for some c in (a, b) .

What is a limit?

The **limit** of $f(x)$ as x approaches a is the one number L that $f(x)$ stays close to whenever x is close to a (but $x \neq a$). We write $\lim_{x \rightarrow a} f(x) = L$. The value $f(a)$ does not matter. For example, $f(x) = \frac{x^2 - 4}{x - 2}$ is undefined at $x = 2$, yet for every other x it equals $x + 2$, so $\lim_{x \rightarrow 2} f(x) = 4$.

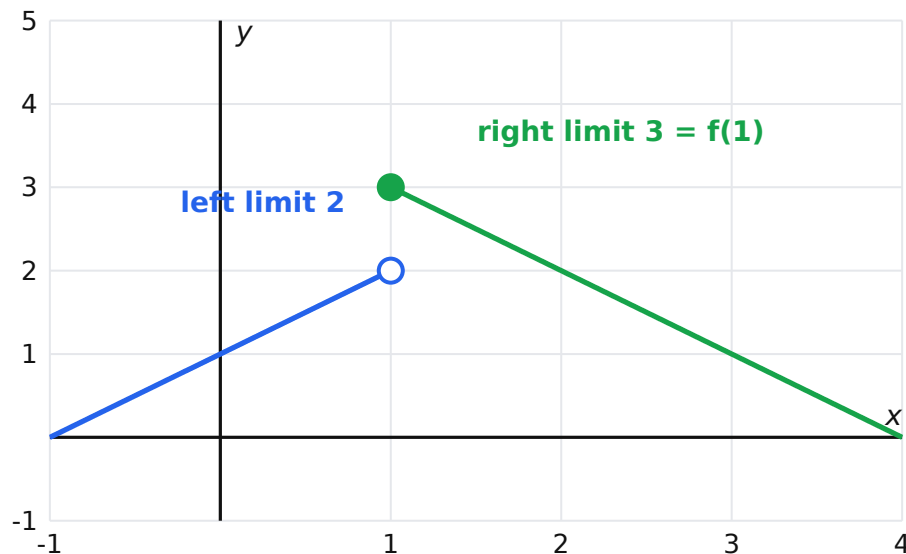


The graph of $f(x) = \frac{x^2 - 4}{x - 2}$ is the line $y = x + 2$ with a hole at $(2, 4)$.

One-sided limits

$\lim_{x \rightarrow a^-} f(x)$ uses only $x < a$; $\lim_{x \rightarrow a^+} f(x)$ uses only $x > a$.

The two-sided limit exists exactly when both one-sided limits exist and are equal.



A jump at $x = 1$: the one-sided limits are 2 and 3, so the two-sided limit does not exist.

How do you evaluate a limit algebraically?

Follow this order and stop as soon as you get a number.

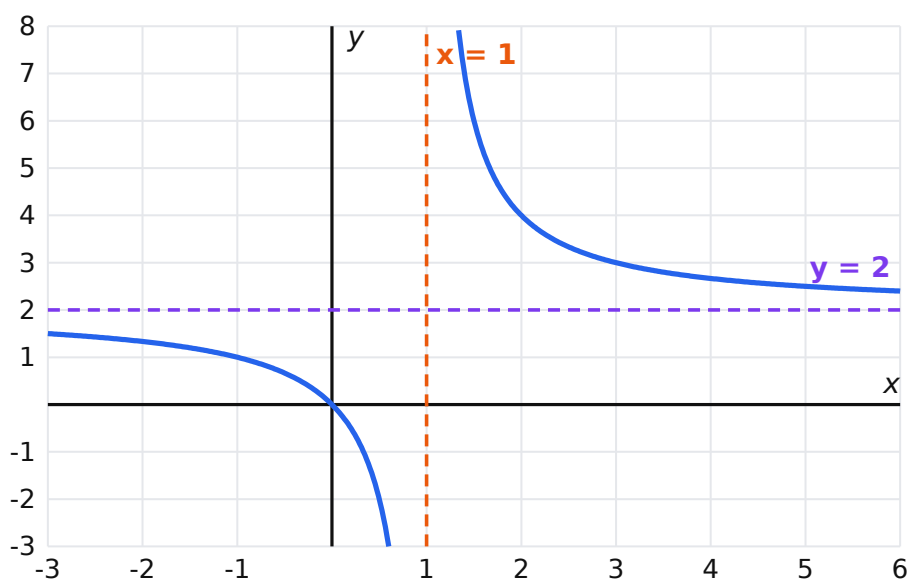
1. Substitute $x = a$. A real number answer is the limit when the function is continuous there.

2. If you get $\frac{0}{0}$, factor and cancel, or multiply by a conjugate.
3. Substitute again.
4. If you get a nonzero number over 0, check each side: the limit is ∞ , $-\infty$, or does not exist.

Worked example: $\lim_{x \rightarrow 3} \frac{x^2-9}{x-3}$ gives $\frac{0}{0}$. Factor: $\frac{(x-3)(x+3)}{x-3} = x+3$. Substitute: $3+3=6$.

What are limits at infinity and asymptotes?

If $\lim_{x \rightarrow \infty} f(x) = L$, the line $y = L$ is a horizontal asymptote. For a rational function, compare the highest powers. For $f(x) = \frac{2x}{x-1}$, divide top and bottom by x to get $\lim_{x \rightarrow \infty} f(x) = 2$. The denominator is 0 at $x = 1$, and $\lim_{x \rightarrow 1^+} f(x) = \infty$, so $x = 1$ is a vertical asymptote.



$f(x) = 2x/(x - 1)$ has the vertical asymptote $x = 1$ and the horizontal asymptote $y = 2$.

What does continuous mean?

Continuity at $x = a$

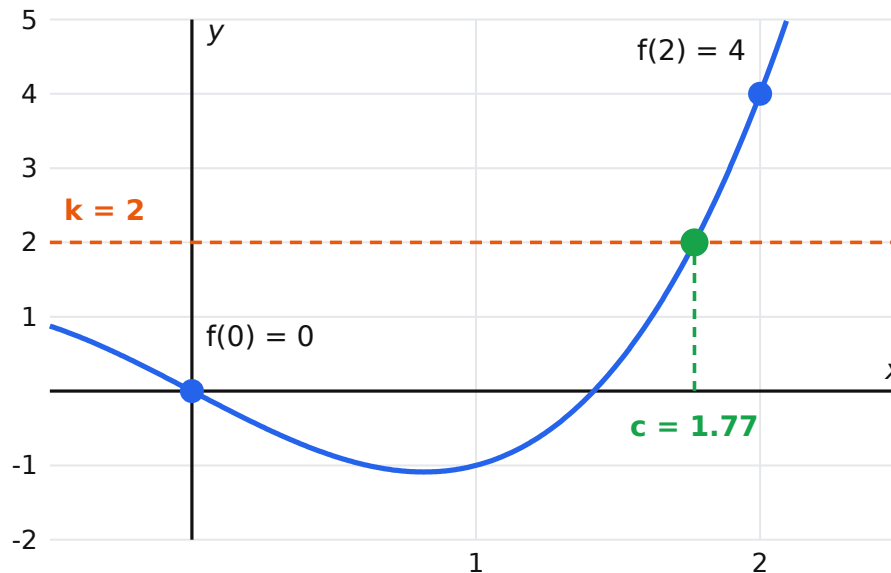
1. $f(a)$ is defined.
2. $\lim_{x \rightarrow a} f(x)$ exists.
3. $\lim_{x \rightarrow a} f(x) = f(a)$.

A **removable** discontinuity is a hole (the limit exists but condition 3 fails). A **jump** has unequal one-sided limits. An **infinite** discontinuity is a vertical asymptote.

What do the Squeeze Theorem and the Intermediate Value Theorem say?

If $g(x) \leq f(x) \leq h(x)$ near a and $\lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} h(x) = L$, then $\lim_{x \rightarrow a} f(x) = L$. Because $-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$, the limit at 0 is 0.

The Intermediate Value Theorem needs continuity on a closed interval $[a, b]$. It then guarantees every value between $f(a)$ and $f(b)$ is reached.



$f(0) = 0$ and $f(2) = 4$, so the continuous curve must cross $y = 2$ at some c between 0 and 2.

Limits and Continuity: frequently asked questions

What is a limit in calculus?

A limit is the value a function approaches as the input gets close to a number. It describes behavior near the point, not at it, so $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$ even though the function is undefined at $x = 2$.

When does a limit not exist?

A limit fails to exist when the left and right limits differ (a jump), when the function grows without bound (a vertical asymptote), or when it oscillates forever, as $\sin \frac{1}{x}$ does near 0.

How do you find a limit at infinity of a rational function?

Compare degrees. Equal degrees give the ratio of leading coefficients, a smaller numerator degree gives 0, and a larger numerator degree gives ∞ or $-\infty$. For example $\lim_{x \rightarrow \infty} \frac{3x^2 - x}{5x^2 + 2} = \frac{3}{5}$.

What are the three types of discontinuity?

A removable discontinuity is a hole where the limit exists. A jump discontinuity has different one-sided limits. An infinite discontinuity is a vertical asymptote where the function is

unbounded.

What does the Intermediate Value Theorem guarantee?

It guarantees that a function continuous on $[a,b]$ takes every value between $f(a)$ and $f(b)$ at least once. It does not say where, how many times, or anything about values outside that range.

How to use this ap calculus limits lesson

Use this ap calculus limits lesson before the worksheet, then check what you learned with the quiz.

