

AP Calculus AB · Integration and Accumulation of Change · Worksheet Answer Key

Common Core AP Calculus AB Unit 6 · mathovore.fr

This ap calculus integration worksheet answer key solves all 26 problems of the integration and accumulation of change worksheet step by step.

Full worked solutions for all 26 integration problems. Each answer shows the antiderivative or substitution, the evaluation of the limits where they apply, and the final result in bold. Compare each step with your own work, not only the final result.

AP Calculus ABCommon Core: AP Calculus AB Unit 6

AP Calculus Integration Worksheet Answer Key: problems 1-5

Problem 1: Power rule antiderivative

Apply the power rule term by term: $\int 6x^2 dx = 2x^3$, $\int -4x dx = -2x^2$, $\int 5 dx = 5x$.

Answer: $2x^3 - 2x^2 + 5x + C$. Check: the derivative is $6x^2 - 4x + 5$.

Problem 2: Fractional and negative powers

Rewrite as $\int (3x^{1/2} - 2x^{-2}) dx$. Then $\int 3x^{1/2} dx = 3 \cdot \frac{x^{3/2}}{3/2} = 2x^{3/2}$ and $\int -2x^{-2} dx = -2 \cdot \frac{x^{-1}}{-1} = \frac{2}{x}$.

Answer: $2x^{3/2} + \frac{2}{x} + C$.

Problem 3: Trigonometric and exponential

$\int 4\cos x dx = 4\sin x$, $\int -3e^x dx = -3e^x$, and $\int \frac{1}{x} dx = \ln|x|$.

Answer: $4\sin x - 3e^x + \ln|x| + C$.

Problem 4: Initial value problem

Integrate: $f(x) = 2x^3 - x^2 + C$. Use $f(1) = 4$: $2 - 1 + C = 4$, so $C = 3$.

Answer: $f(x) = 2x^3 - x^2 + 3$.

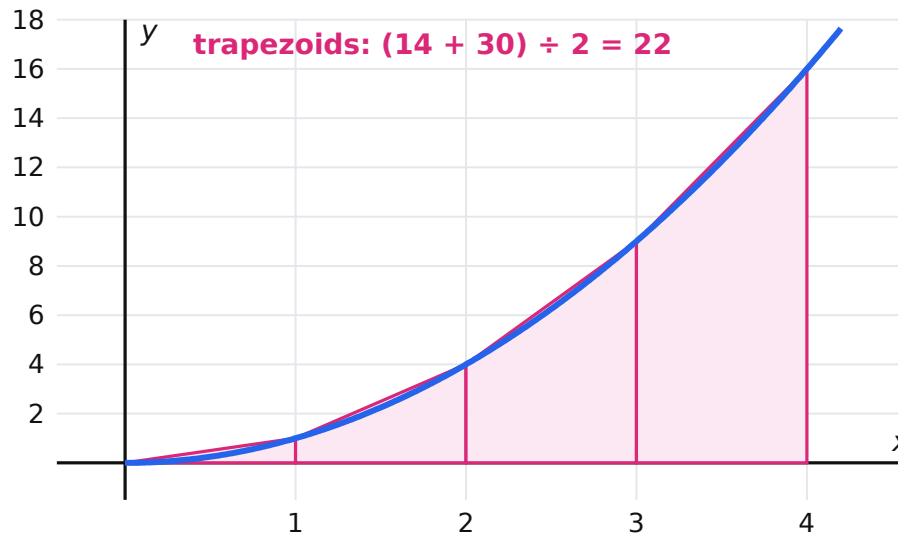
Problem 5: Four Riemann sums

$\Delta x = 1$. Left endpoints 0,1,2,3: $L = 0 + 1 + 4 + 9 = 14$. Right endpoints 1,2,3,4: $R = 1 + 4 + 9 + 16 = 30$.

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Midpoints 0.5,1.5,2.5,3.5 : $M = 0.25 + 2.25 + 6.25 + 12.25 = 21$. Trapezoidal: $T = \frac{14+30}{2} = 22$.



Trapezoids over x^2 give 22; the exact area is $64/3 \approx 21.33$.

Answer: $L = 14$, $R = 30$, $M = 21$, $T = 22$. The exact value is $\frac{64}{3} \approx 21.33$.

AP Calculus Integration Worksheet Answer Key: problems 6-10

Problem 6: Sums from a table

The subinterval widths are 2,3,4,1.

- Left: $3(2) + 5(3) + 8(4) + 6(1) = 6 + 15 + 32 + 6 = 59$ ft.
- Right: $5(2) + 8(3) + 6(4) + 4(1) = 10 + 24 + 24 + 4 = 62$ ft.
- Trapezoidal: $\frac{3+5}{2}(2) + \frac{5+8}{2}(3) + \frac{8+6}{2}(4) + \frac{6+4}{2}(1) = 8 + 19.5 + 28 + 5 = 60.5$ ft.

Answer: **59 ft, 62 ft and 60.5 ft.**

Problem 7: Overestimate or underestimate

Since f is decreasing, the left endpoints give the tallest rectangles and the right endpoints the shortest. Since f is concave up, chords lie above the curve and tangent lines lie below it.

- Left sum: **overestimate**.
- Right sum: **underestimate**.
- Midpoint sum: **underestimate** (concave up).
- Trapezoidal sum: **overestimate** (chords above the curve).

Left sum with $\Delta x = 1$: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12} \approx 2.083$, which exceeds the exact value $\ln 5 \approx 1.609$, as expected.

Problem 8: Properties of definite integrals

- a. $\int_3^5 f = \int_1^5 f - \int_1^3 f = 7 - 2 = 5.$
- b. $2(7) - 3(-3) = 14 + 9 = 23.$
- c. Reversing the limits: $-7.$
- d. $\int_1^5 f \, dx + \int_1^5 4 \, dx = 7 + 4(4) = 23.$

Answer: **5, 23, -7 , 23.**

Problem 9: Integrals from geometry

- a. The area of a semicircle with $r = 3$: $\frac{1}{2}\pi(3)^2 = \frac{9\pi}{2}.$
- b. A quarter circle: $\frac{9\pi}{4}.$
- c. Add the rectangle of height 2 and width 6: $\frac{9\pi}{2} + 12.$

Answer: $\frac{9\pi}{2}, \frac{9\pi}{4}, \frac{9\pi}{2} + 12.$

Problem 10: Evaluate with the Fundamental Theorem

An antiderivative is $x^4 - 3x^2$. Then $[x^4 - 3x^2]_1^3 = (81 - 27) - (1 - 3) = 54 + 2 = 56.$

Answer: **56.**

AP Calculus Integration Worksheet Answer Key: problems 11-15

Problem 11: Trigonometric definite integral

An antiderivative is $-3\cos x + 2\sin x$. At $\frac{\pi}{2}$: $0 + 2 = 2$. At 0: $-3 + 0 = -3$. The difference is $2 - (-3) = 5$.

Answer: **5.**

Problem 12: Logarithmic and exponential limits

- a. $[3\ln x]_1^{e^2} = 3(2) - 3(0) = 6.$
- b. $[4e^x]_0^{\ln 3} = 4(3) - 4(1) = 8.$

Answer: **6 and 8.**

Problem 13: Derivatives of accumulation functions

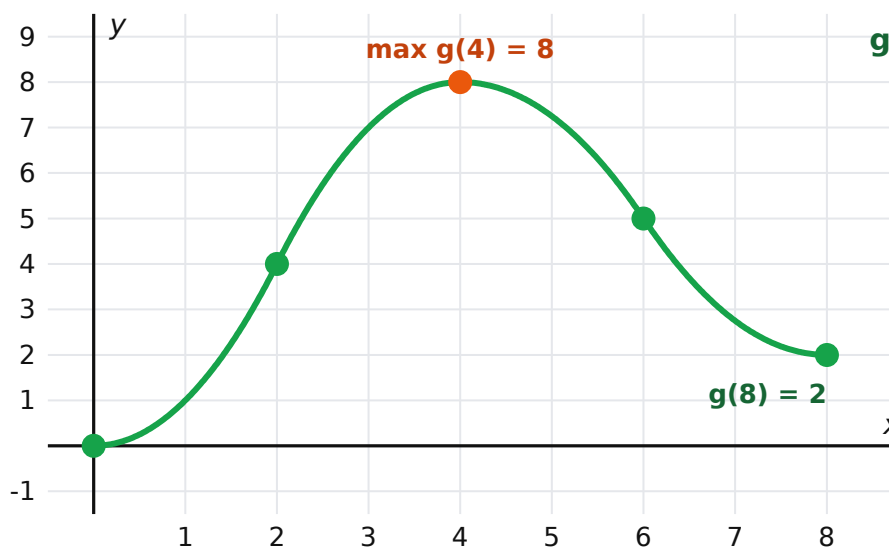
- a. By FTC Part 1: $\sqrt{x^3 + 1}.$
- b. Substitute the upper limit and multiply by its derivative: $\sin(x^2) \cdot 2x.$
- c. Swap the limits to introduce a negative sign: $-\ln(x^2 + 1).$

Answer: $\sqrt{x^3 + 1}; 2x\sin(x^2); -\ln(x^2 + 1).$

Problem 14: Accumulation from a graph

Use triangle areas (signed).

- a. $g(2) = \frac{1}{2}(2)(4) = 4$; $g(4) = \frac{1}{2}(4)(4) = 8$; $g(6) = 8 - \frac{1}{2}(2)(3) = 5$; $g(8) = 5 - \frac{1}{2}(2)(3) = 2$.
- b. $g'(6) = f(6) = -3$.
- c. $g' = f$ changes from positive to negative at $x = 4$, and $g(0) = 0$, $g(8) = 2$ are smaller, so the maximum is $g(4) = 8$.
- d. $g'' = f'$, and f is decreasing on $(2, 6)$, so g is concave down on $(2, 6)$.



The accumulation function g peaks at $x = 4$.

Answer: **4, 8, 5, 2; -3; maximum 8 at $x = 4$; concave down on $(2, 6)$.**

Problem 15: Substitution with a power

Let $u = 3x^2 + 1$, so $du = 6x \, dx$. Then $\int u^4 \, du = \frac{u^5}{5} + C$.

Answer: $\frac{(3x^2+1)^5}{5} + C$.

AP Calculus Integration Worksheet Answer Key: problems 16-20

Problem 16: Substitution with cosine

Let $u = x^3$, so $du = 3x^2 \, dx$ and $x^2 \, dx = \frac{1}{3} du$. Then $\frac{1}{3} \int \cos u \, du = \frac{1}{3} \sin u + C$.

Answer: $\frac{1}{3} \sin(x^3) + C$.

Problem 17: Substitution giving a logarithm

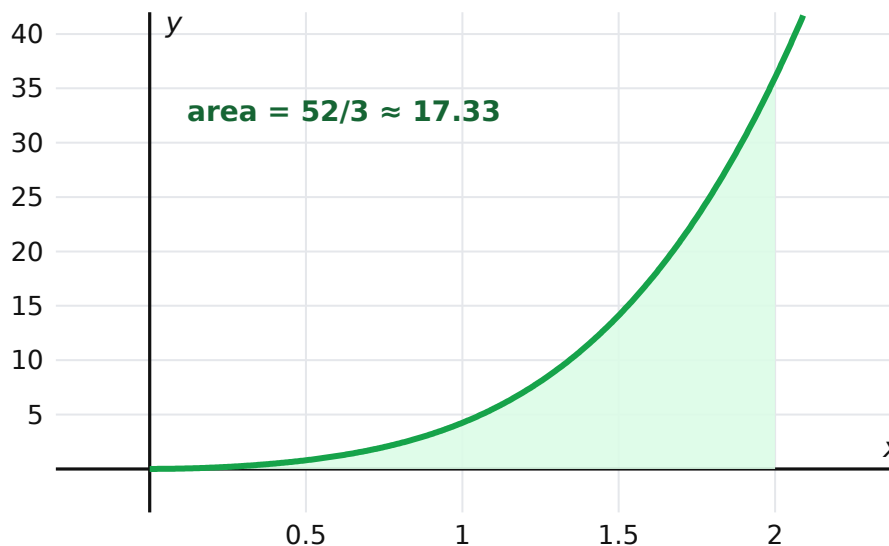
Let $u = x^2 + 4$, so $du = 2x \, dx$ and $x \, dx = \frac{1}{2} du$. Then $\frac{1}{2} \int \frac{1}{u} \, du = \frac{1}{2} \ln|u| + C$, and $x^2 + 4 > 0$.

Answer: $\frac{1}{2}\ln(x^2+4)+C$.

Problem 18: Definite substitution

Let $u = x^3 + 1$, so $du = 3x^2 dx$. When $x = 0$, $u = 1$; when $x = 2$, $u = 9$.

$$\int_1^9 u^{1/2} du = \left[\frac{2}{3} u^{3/2} \right]_1^9 = \frac{2}{3}(27 - 1) = \frac{52}{3}.$$



The area under $y = 3x^2\sqrt{x^3 + 1}$ on $[0, 2]$.

Answer: $\frac{52}{3}$.

Problem 19: Substitution with changed limits

Let $u = \cos x$, so $du = -\sin x dx$. When $x = 0$, $u = 1$; when $x = \frac{\pi}{2}$, $u = 0$.

$$\int_1^0 -u^2 du = \int_0^1 u^2 du = \left[\frac{u^3}{3} \right]_0^1 = \frac{1}{3}.$$

Answer: $\frac{1}{3}$.

Problem 20: Long division first

Divide: $x^2 + 3x + 5 = (x+1)(x+2) + 3$, so $\frac{x^2 + 3x + 5}{x+1} = x + 2 + \frac{3}{x+1}$.

Integrate: $\frac{x^2}{2} + 2x + 3\ln|x+1| + C$.

Answer: $\frac{x^2}{2} + 2x + 3\ln|x+1| + C$.

AP Calculus Integration Worksheet Answer Key: problems 21-25

Problem 21: Completing the square

Complete the square: $x^2 - 6x + 25 = (x - 3)^2 + 16$. Let $w = \frac{x-3}{4}$, so $dx = 4 dw$ and $(x-3)^2 + 16 = 16(w^2 + 1)$.

$$\int \frac{4 dw}{16(w^2 + 1)} = \frac{1}{4} \tan^{-1} w + C.$$

Answer: $\frac{1}{4} \tan^{-1} \left(\frac{x-3}{4} \right) + C$.

Problem 22: Water in a tank

Amount at $t = 8$ = initial amount + net change:

$$40 + \int_0^8 (20 - 2t) dt = 40 + [20t - t^2]_0^8 = 40 + (160 - 64) = 136.$$

Answer: **136 gallons**.

Problem 23: Customers entering a store

The total is $\int_0^{12} 100 \sin\left(\frac{\pi t}{12}\right) dt$. Let $u = \frac{\pi t}{12}$, so $dt = \frac{12}{\pi} du$, and the limits become 0 to π .

$$\frac{1200}{\pi} \int_0^{\pi} \sin u du = \frac{1200}{\pi} [-\cos u]_0^{\pi} = \frac{1200}{\pi} (1 + 1) = \frac{2400}{\pi} \approx 763.9.$$

Answer: **about 764 customers**.

Problem 24: Leaking water

$$\int_0^4 6e^{-0.5t} dt = [-12e^{-0.5t}]_0^4 = -12e^{-2} + 12 = 12 - 12e^{-2}.$$

Since $e^{-2} \approx 0.1353$, the amount is about $12 - 1.624 = 10.38$.

Answer: $12 - 12e^{-2} \approx 10.38$ **gallons**.

Problem 25: Saving for a bike

$$\text{Net change: } \int_0^{10} (40 + 6t) dt = [40t + 3t^2]_0^{10} = 400 + 300 = 700.$$

Total: $200 + 700 = 900$. Answer: **\$900**.

AP Calculus Integration Worksheet Answer Key: problems 26-26

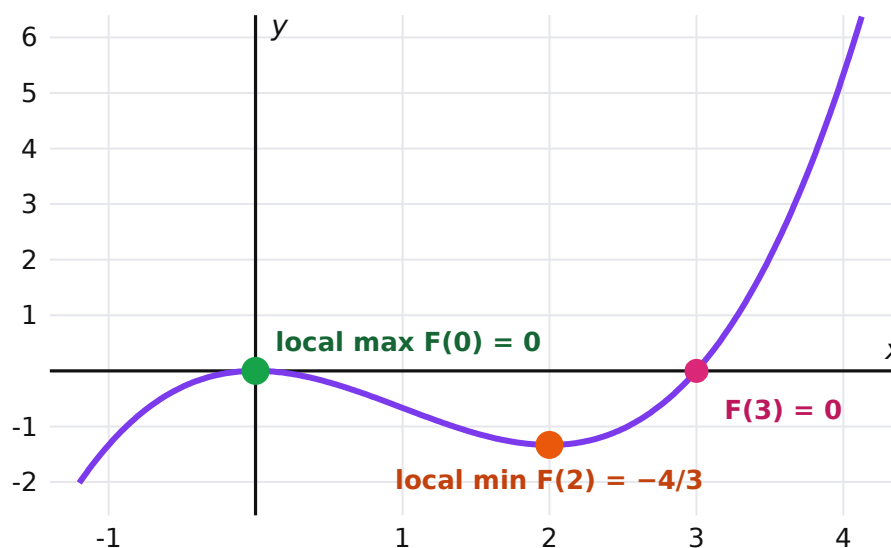
Problem 26: Analyzing an accumulation function

a. By FTC Part 1: $F'(x) = x^2 - 2x$.

b. $F(x) = \left[\frac{1}{3}x^3 - x^2 \right]_0^x = \frac{x^3}{3} - x^2$, so $F(3) = 9 - 9 = 0$.

c. $F'(x) = x(x - 2) = 0$ gives $x = 0$ and $x = 2$. $F''(x) = 2x - 2$: $F''(0) = -2 < 0$, a local

maximum with $F'(0)=0$; $F''(2)=2>0$, a local minimum with $F'(2)=\frac{8}{3}-4=-\frac{4}{3}$.



The graph of $F(x) = x^3/3 - x^2$.

Answer: x^2-2x ; $F'(3)=0$; **local maximum at $x=0$, local minimum at $x=2$.**

How to use this ap calculus integration worksheet answer key

Read this ap calculus integration worksheet answer key after trying each problem yourself, and redo any problem you missed.

