

AP Calculus AB · Integration and Accumulation of Change · Worksheet

Common Core AP Calculus AB Unit 6 · mathovore.fr

Name: _____ Date: ____/____/____ Score: _____

This ap calculus integration worksheet gives 26 problems on integration and accumulation of change, ordered from easy to hard, with a separate answer key.

These 26 problems practice AP Calculus AB Unit 6: antiderivatives, Riemann sums, definite integral properties, the Fundamental Theorem of Calculus, accumulation functions, and u-substitution, long division and completing the square.

Problems move from basic antiderivatives to applied accumulation questions. Show every antiderivative and every substitution before you evaluate limits.

AP Calculus ABCommon Core: AP Calculus AB Unit 6

AP Calculus Integration Worksheet: problems 1-5

Problem 1: Power rule antiderivative

Find $\int (6x^2 - 4x + 5) dx$.

Problem 2: Fractional and negative powers

Find $\int \left(3\sqrt{x} - \frac{2}{x^2} \right) dx$.

Problem 3: Trigonometric and exponential

Find $\int \left(4\cos x - 3e^x + \frac{1}{x} \right) dx$ for $x > 0$.

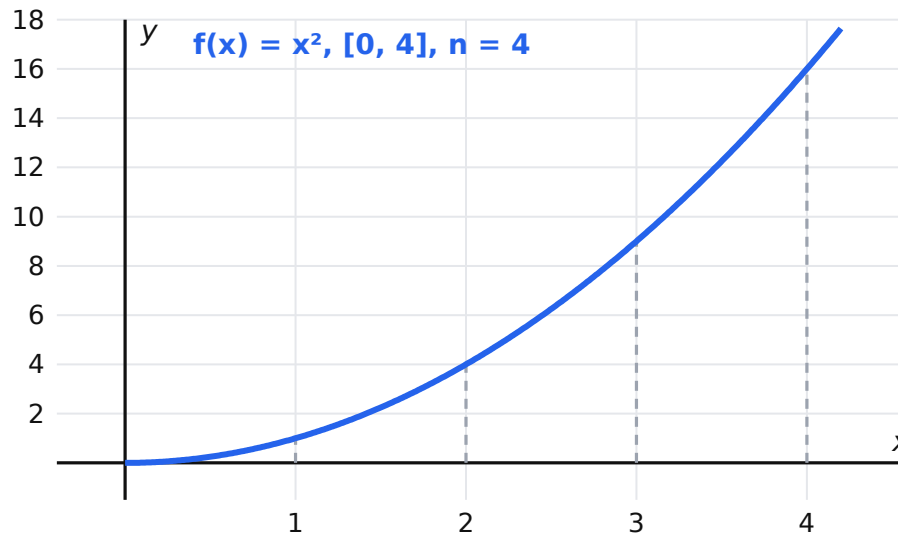
Problem 4: Initial value problem

The function f satisfies $f'(x) = 6x^2 - 2x$ and $f(1) = 4$. Find $f(x)$.

Problem 5: Four Riemann sums

Let $f(x) = x^2$ on $[0, 4]$ with $n = 4$ equal subintervals. Find the left, right, midpoint and

trapezoidal sums.



The interval $[0, 4]$ split into four equal subintervals.

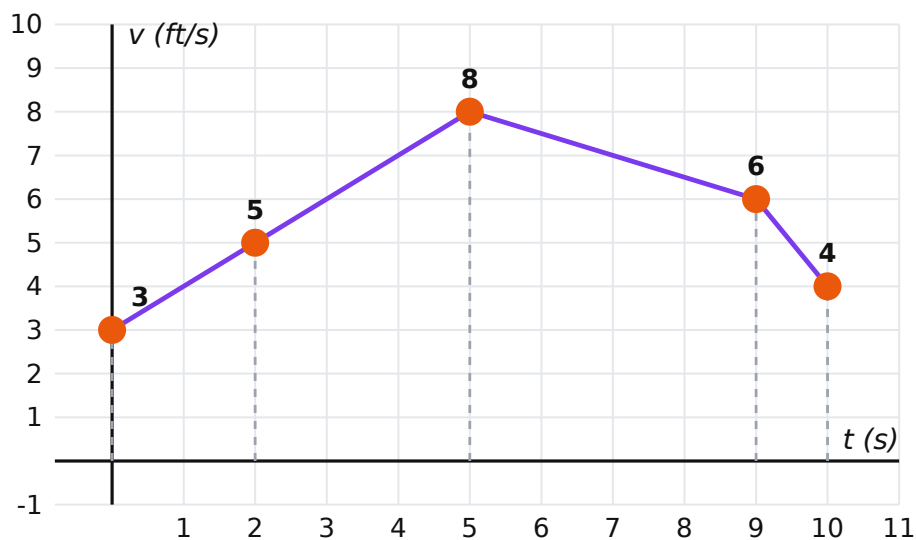
AP Calculus Integration Worksheet: problems 6–10

Problem 6: Sums from a table

Maya jogs and records her speed $v(t)$ in feet per second.

t (s)	0	2	5	9	10
$v(t)$	3	5	8	6	4

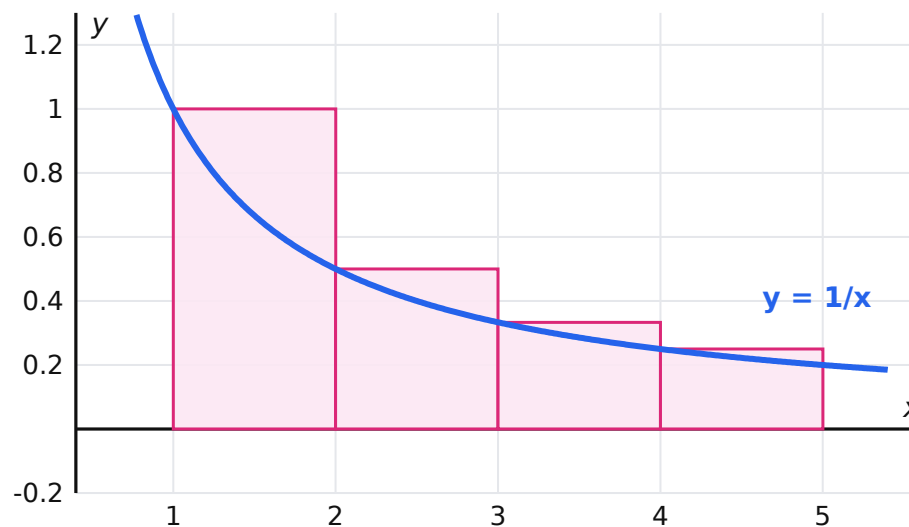
- Use a left Riemann sum with the four subintervals to estimate the distance she jogs from $t = 0$ to $t = 10$.
- Use a right Riemann sum.
- Use a trapezoidal sum.



Problem 7: Overestimate or underestimate

For $f(x) = \frac{1}{x}$ on $[1, 5]$, which is decreasing and concave up, decide whether each estimate of $\int_1^5 \frac{1}{x} dx$ is an overestimate or an underestimate, and then compute the left sum with $n = 4$.

- Left sum
- Right sum
- Midpoint sum
- Trapezoidal sum



Left-endpoint rectangles for $y = 1/x$ on $[1, 5]$.

Problem 8: Properties of definite integrals

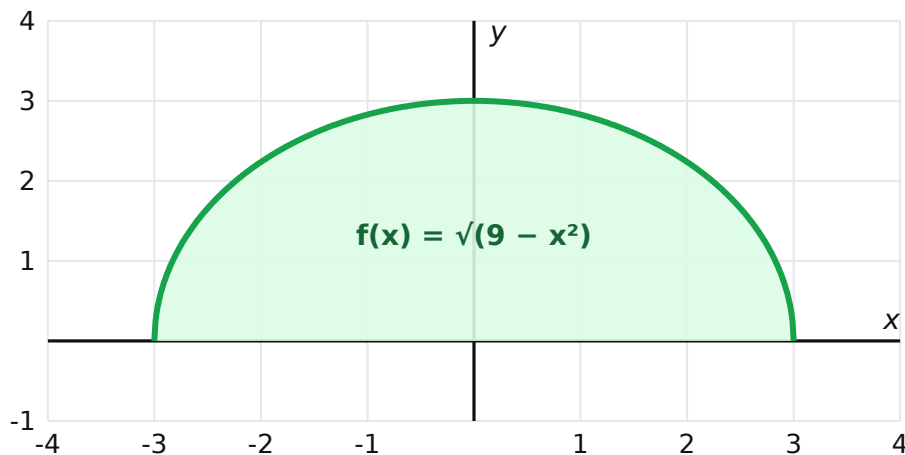
Suppose $\int_1^5 f(x) dx = 7$, $\int_1^3 f(x) dx = 2$ and $\int_1^5 g(x) dx = -3$. Find each value.

- $\int_3^5 f(x) dx$
- $\int_1^5 (2f(x) - 3g(x)) dx$
- $\int_5^1 f(x) dx$
- $\int_1^5 (f(x) + 4) dx$

Problem 9: Integrals from geometry

The graph of $f(x) = \sqrt{9 - x^2}$ is a semicircle of radius 3. Use geometry to find each value.

- $\int_{-3}^3 f(x) dx$
- $\int_0^3 f(x) dx$
- $\int_{-3}^3 (f(x) + 2) dx$



The graph of f is a semicircle of radius 3.

Problem 10: Evaluate with the Fundamental Theorem

Evaluate $\int_1^3 (4x^3 - 6x) dx$.

AP Calculus Integration Worksheet: problems 11-15

Problem 11: Trigonometric definite integral

Evaluate $\int_0^{\pi/2} (3\sin x + 2\cos x) dx$.

Problem 12: Logarithmic and exponential limits

Evaluate each integral.

a. $\int_{\frac{1}{e^2}}^{\frac{1}{e}} dx$

b. $\int_0^{\ln 3} 4e^x dx$

Problem 13: Derivatives of accumulation functions

Find each derivative.

a. $\frac{d}{dx} \int_2^x \sqrt{t^3 + 1} dt$

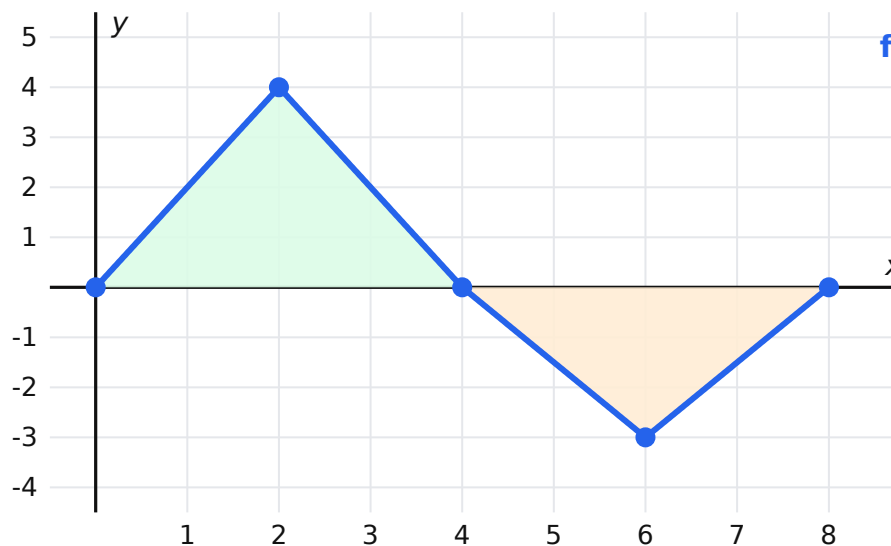
b. $\frac{d}{dx} \int_1^{x^2} \sin t \, dt$

c. $\frac{d}{dx} \int_x^5 \ln(t^2+1) \, dt$

Problem 14: Accumulation from a graph

The graph of f consists of line segments through $(0,0)$, $(2,4)$, $(4,0)$, $(6,-3)$ and $(8,0)$. Let $g(x) = \int_0^x f(t) \, dt$.

- Find $g(2)$, $g(4)$, $g(6)$ and $g(8)$.
- Find $g'(6)$.
- On $[0,8]$, at what x does g have its maximum value, and what is it?
- On what interval is the graph of g concave down?



The graph of f on $[0, 8]$.

Problem 15: Substitution with a power

Find $\int 6x(3x^2+1)^4 \, dx$.

AP Calculus Integration Worksheet: problems 16-20

Problem 16: Substitution with cosine

Find $\int x^2 \cos(x^3) \, dx$.

Problem 17: Substitution giving a logarithm

Find $\int \frac{x}{x^2+4} dx$.

Problem 18: Definite substitution

Evaluate $\int_0^2 3x^2\sqrt{x^3+1} dx$.

Problem 19: Substitution with changed limits

Evaluate $\int_0^{\pi/2} \sin x \cos^2 x dx$.

Problem 20: Long division first

Find $\int \frac{x^2+3x+5}{x+1} dx$.

AP Calculus Integration Worksheet: problems 21-25

Problem 21: Completing the square

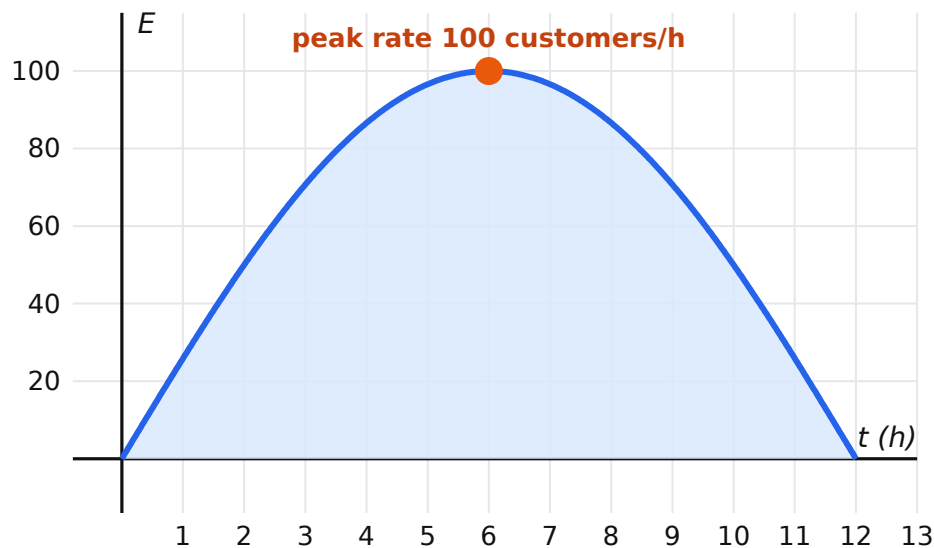
Find $\int \frac{1}{x^2-6x+25} dx$.

Problem 22: Water in a tank

A tank holds 40 gallons at $t=0$ minutes. Water flows in at the rate $r(t)=20-2t$ gallons per minute for $0 \leq t \leq 8$. How many gallons are in the tank at $t=8$?

Problem 23: Customers entering a store

Customers enter a store at the rate $E(t)=100\sin\left(\frac{\pi t}{12}\right)$ customers per hour for $0 \leq t \leq 12$, where t is hours after opening. Find the total number of customers who enter, to the nearest whole number.



The entry rate $E(t)$ over a 12-hour day.

Problem 24: Leaking water

Water leaks from a pipe at the rate $L(t) = 6e^{-0.5t}$ gallons per hour. Find the exact amount that leaks during the first 4 hours, then round to the nearest hundredth.

Problem 25: Saving for a bike

Ethan has \$200 saved and deposits money at the rate $D(t) = 40 + 6t$ dollars per week for 10 weeks. Find his total savings after 10 weeks.

AP Calculus Integration Worksheet: problems 26-26

Problem 26: Analyzing an accumulation function

Let $F(x) = \int_0^x (t^2 - 2t) dt$.

- Find $F'(x)$.
- Find a formula for $F(x)$ and evaluate $F(3)$.
- Find the critical numbers of F and classify each using F'' .

How to use this ap calculus integration worksheet

Print this ap calculus integration worksheet as a PDF, solve every problem on paper, then compare with the answer key.

