

AP Calculus AB · Integration and Accumulation of Change · Test Answer Key

Common Core AP Calculus AB Unit 6 · mathovore.fr

This ap calculus integration test answer key gives the full solution and the scoring guide for each of the 8 test questions on integration and accumulation of change.

Worked solutions and scoring guides for the Unit 6 test. Each answer shows the setup an AP reader expects: the Riemann sum terms, the antiderivative, the limits of integration and a clear final statement with units. Award points exactly as the scoring note describes and total them out of 20.

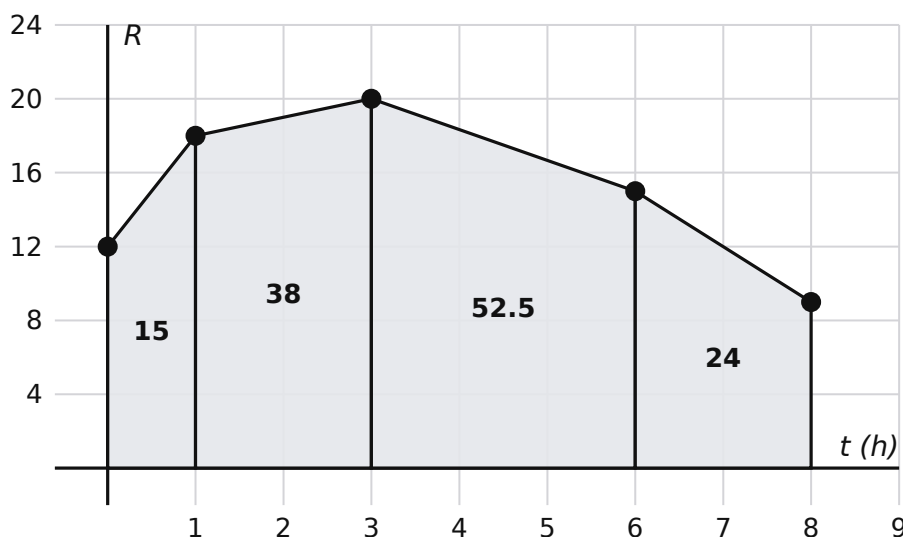
AP Calculus ABCommon Core: AP Calculus AB Unit 6

AP Calculus Integration Test Answer Key: questions 1-5

Question 1: Estimating from a table (3 points)

(a) The widths are 1,2,3,2 . Left sum: $12(1)+18(2)+20(3)+15(2)=12+36+60+30=138$.

(b) Trapezoidal: $\frac{12+18}{2}(1)+\frac{18+20}{2}(2)+\frac{20+15}{2}(3)+\frac{15+9}{2}(2)=15+38+52.5+24=129.5$.



The four trapezoid areas add to 129.5 cars.

(c) $\int_0^8 R(t) dt$ is the total number of cars that enter the garage between 6 A.M. and 2 P.M., measured in cars.

Scoring: (a) 1 point for the correct sum 138 (setup with the widths 1,2,3,2). (b) 1 point for 129.5. (c) 1 point for the meaning (total or net number of cars entering) with the time span and the unit "cars".

Question 2: Properties of integrals (2 points)

(a) $\int_0^7 f = \int_0^4 f + \int_4^7 f = 10 + (-3) = 7.$

(b) $3(7) - 6 + 2(7 - 0) = 21 - 6 + 14 = 29.$

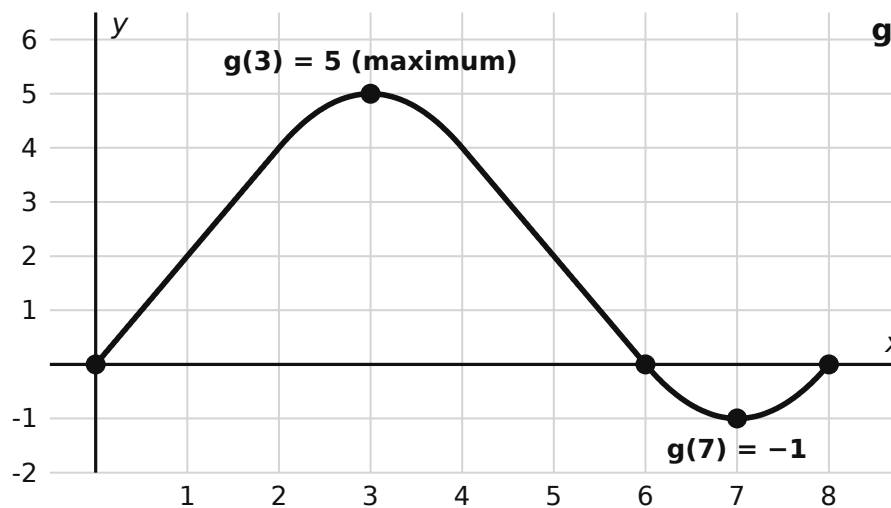
Scoring: (a) 1 point for 7. (b) 1 point for 29; the constant term must be integrated as $2 \cdot 7 = 14$.

Question 3: Accumulation from a graph (3 points)

(a) $\int_0^2 f = 2(2) = 4$; $\int_2^4 f = \frac{2+(-2)}{2}(2) = 0$; $\int_4^6 f = -2(2) = -4$. So $g(6) = 4 + 0 - 4 = 0$.

(b) $g'(5) = f(5) = -2.$

(c) $g' = f$, which is positive on $[0,3)$, negative on $(3,7)$ and positive on $(7,8]$. So g has candidates $g(0) = 0$, $g(3) = 4 + \frac{1}{2}(1)(2) = 5$, $g(7) = 0 + \left(-\frac{1}{2}(1)(2)\right) = -1$ and $g(8) = 0$. The absolute maximum is at $x = 3$.



The graph of $g(x) = \int f$ from 0 to x .

Scoring: (a) 1 point for 0 with signed areas shown. (b) 1 point for -2 . (c) 1 point for $x = 3$ with a justification (sign change of g' or comparison of candidate values).

Question 4: Fundamental Theorem evaluation (2 points)

An antiderivative is $2x^{3/2} - \frac{2}{x}$. Then $\left[2x^{3/2} - \frac{2}{x}\right]_1^4 = \left(16 - \frac{1}{2}\right) - (2 - 2) = \frac{31}{2}.$

Scoring: 1 point for the correct antiderivative; 1 point for evaluating the limits to get $\frac{31}{2}.$

Question 5: Substitution (2 points)

(a) Let $u = x^2 - 5$, so $du = 2x \, dx$ and $4x \, dx = 2 \, du$. Then $\int 2u^3 \, du = \frac{u^4}{2} + C = \frac{(x^2 - 5)^4}{2} + C$.

(b) Let $u = \ln x$, so $du = \frac{1}{x} dx$, with limits $u = 0$ to $u = 1$. Then $\int_0^1 u^2 \, du = \frac{1}{3}$.

Scoring: (a) 1 point for $\frac{(x^2 - 5)^4}{2} + C$; omit the point if $+C$ is missing. (b) 1 point for $\frac{1}{3}$.

AP Calculus Integration Test Answer Key: questions 6-8

Question 6: Water in a tank (3 points)

(a) $V(t) = 30 + \int_0^t (12 - 2s) \, ds = 30 + 12t - t^2$.

(b) $V'(t) = r(t) = 12 - 2t = 0$ at $t = 6$. r is positive before $t = 6$ and negative after, so V increases then decreases and the maximum is at $t = 6$: $V(6) = 30 + 72 - 36 = 66$ gallons.

(c) $V(8) = 30 + 96 - 64 = 62$ gallons. Since $r(8) = -4 < 0$, the amount of water is decreasing at $t = 8$.

Scoring: (a) 1 point for $30 + 12t - t^2$. (b) 1 point for $t = 6$ with the sign-change justification and 66 gallons. (c) 1 point for 62 gallons and the explanation using $r(8) < 0$.

Question 7: Division and completing the square (3 points)

(a) Divide: $x^2 - 1 = (x + 3)(x - 3) + 8$, so the integrand is $x - 3 + \frac{8}{x + 3}$. Then $\int \left(x - 3 + \frac{8}{x + 3} \right) dx = \frac{x^2}{2} - 3x + 8 \ln|x + 3| + C$.

(b) $x^2 + 6x + 10 = (x + 3)^2 + 1$, so $\int \frac{1}{(x + 3)^2 + 1} \, dx = \tan^{-1}(x + 3) + C$.

Scoring: (a) 1 point for the correct division; 1 point for the antiderivative with $+C$. (b) 1 point for $\tan^{-1}(x + 3) + C$.

Question 8: Differentiating an integral (2 points)

(a) By the Fundamental Theorem and the chain rule, $G'(x) = (x^2 + 3) \cdot 2x$.

(b) $G'(2) = (4 + 3)(4) = 28$.

Scoring: (a) 1 point for $2x(x^2 + 3)$; the factor $2x$ must appear. (b) 1 point for 28.

Scoring summary

Total

20 points: add the points of every question of this ap calculus integration test answer key.

How to use this ap calculus integration test answer key

Use this ap calculus integration test answer key to score the test fairly and to see which skills need more practice.

