

AP Calculus AB · Integration and Accumulation of Change · Lesson

Common Core AP Calculus AB Unit 6 · mathovore.fr

This ap calculus integration lesson explains integration and accumulation of change in one printable page, with definitions, a method and a worked example.

Integration accumulates change. A definite integral is the limit of Riemann sums and gives signed area; the Fundamental Theorem of Calculus links it to antiderivatives so that $\int_a^b f \, dx = F(b) - F(a)$, and techniques such as u-substitution make those antiderivatives possible to find in practice.

AP Calculus ABCommon Core: AP Calculus AB Unit 6

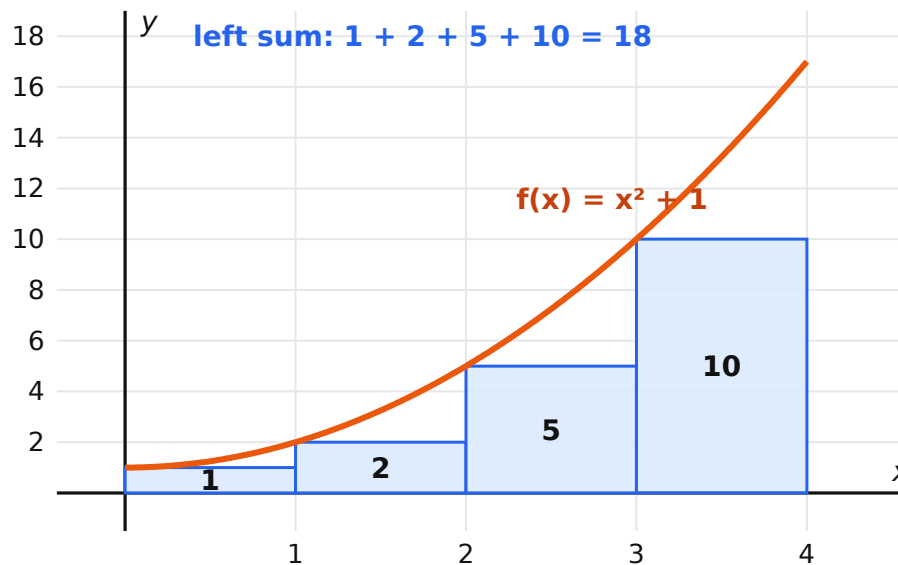
AP Calculus Integration Lesson: key facts

- AP Calculus AB Unit 6: Riemann sums, definite integrals, the Fundamental Theorem of Calculus, and antiderivative techniques.
- Left, right, midpoint and trapezoidal sums estimate $\int_a^b f \, dx$; for $f(x) = x^2 + 1$ on $[0, 4]$ with $n = 4$ they give 18, 34, 25 and 26.
- FTC Part 1: $\frac{d}{dx} \int_a^x f(t) \, dt = f(x)$. FTC Part 2: $\int_a^b f \, dx = F(b) - F(a)$.
- In u-substitution, change the limits of integration along with the variable.

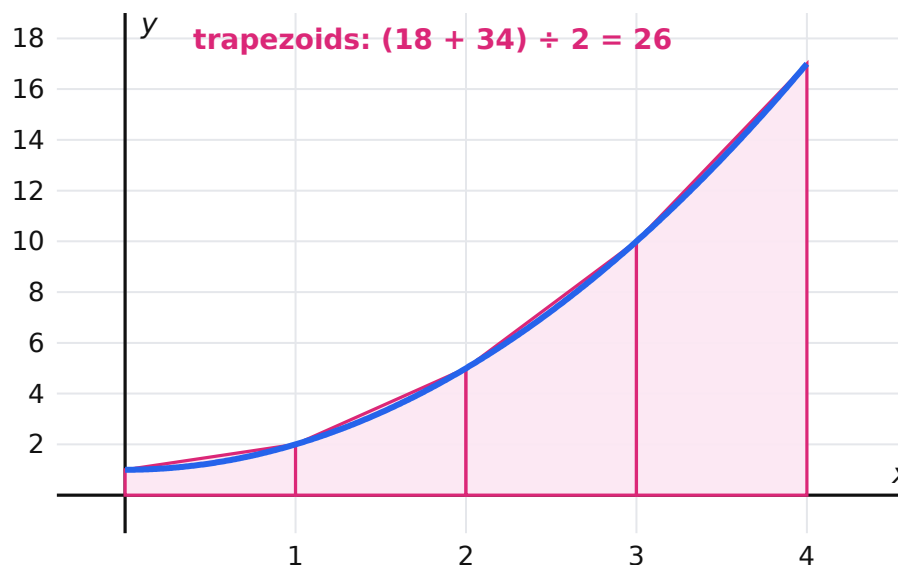
What is a Riemann sum?

A **Riemann sum** estimates the area between a curve and the x -axis by adding the areas of thin rectangles. Split $[a, b]$ into n subintervals of width Δx . A **left sum** uses the height at each left endpoint, a **right sum** uses the right endpoints, and a **midpoint sum** uses the middle of each subinterval. The **trapezoidal sum** averages the left and right sums when the widths are equal. For $f(x) = x^2 + 1$ on $[0, 4]$ with $n = 4$ and $\Delta x = 1$:

Sum	Heights used	Value
Left	1, 2, 5, 10	18
Right	2, 5, 10, 17	34
Midpoint	1.25, 3.25, 7.25, 13.25	25
Trapezoidal	average of left and right	26



A left Riemann sum with $n = 4$ uses the heights 1, 2, 5, 10.



The trapezoidal sum averages the left sum 18 and the right sum 34.

Because f is increasing here, the left sum underestimates and the right sum overestimates the true area.

What is a definite integral?

Definite integral

$\int_a^b f(x) dx$ is the limit of Riemann sums as $n \rightarrow \infty$. It gives the **signed area** between the graph and the x -axis: area above the axis counts as positive and area below counts as negative.

Useful properties: $\int_a^a f \, dx = 0$; $\int_b^a f \, dx = -\int_a^b f \, dx$; $\int_a^b k f \, dx = k \int_a^b f \, dx$;
 $\int_a^b (f+g) \, dx = \int_a^b f \, dx + \int_a^b g \, dx$; and $\int_a^c f \, dx = \int_a^b f \, dx + \int_b^c f \, dx$.

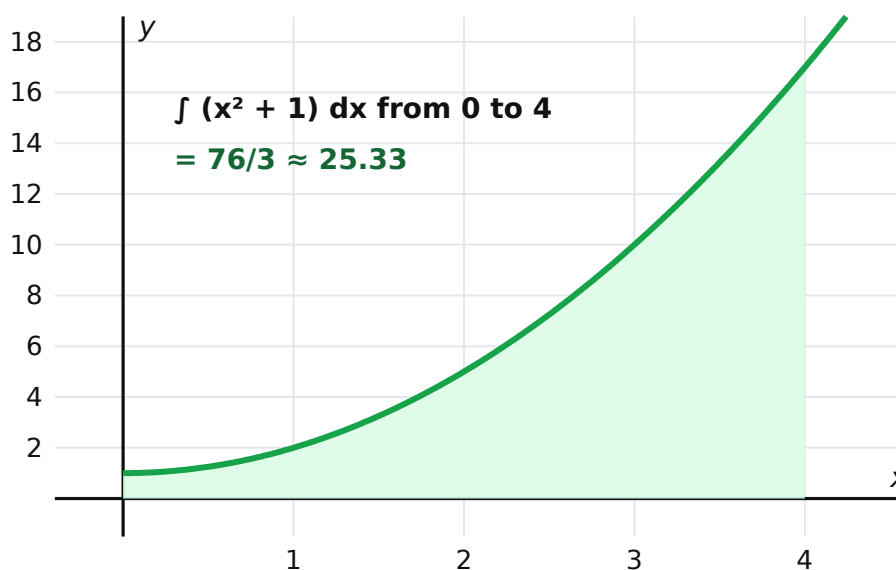
How does the Fundamental Theorem of Calculus connect integrals and antiderivatives?

Fundamental Theorem of Calculus

Part 1: if $F(x) = \int_a^x f(t) \, dt$, then $F'(x) = f(x)$.

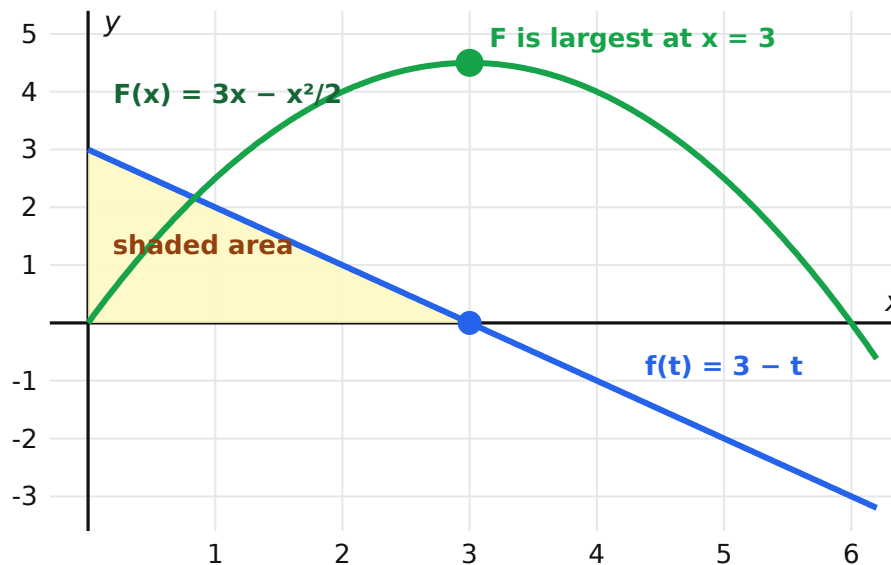
Part 2: if $G' = f$, then $\int_a^b f(x) \, dx = G(b) - G(a)$.

Part 2 gives the exact value: $\int_0^4 (x^2+1) \, dx = \left[\frac{x^3}{3} + x \right]_0^4 = \frac{64}{3} + 4 = \frac{76}{3} \approx 25.33$, which sits between the estimates 18 and 34.



The exact area, $76/3$, lies between the Riemann estimates 18 and 34.

Part 1 describes **accumulation**. For $F(x) = \int_0^x (3-t) \, dt = 3x - \frac{x^2}{2}$, the function F rises while $f(t) = 3-t$ is positive and reaches its maximum $F(3) = 4.5$ where f crosses zero.

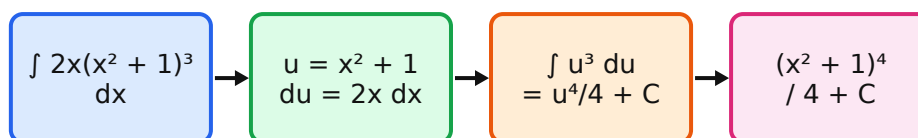


The accumulation function F rises while f is positive and peaks where f crosses zero.

How does u-substitution work?

1. Pick u as an inner function whose derivative (up to a constant) also appears.
2. Compute du and rewrite the whole integral in terms of u .
3. Integrate, then replace u with the original expression.

Example: $\int 2x(x^2+1)^3 dx$ with $u = x^2+1$ becomes $\int u^3 du = \frac{u^4}{4} + C = \frac{(x^2+1)^4}{4} + C$. For a definite integral, change the bounds too: $\int_0^1 2x(x^2+1)^3 dx = \int_1^2 u^3 du = \frac{16-1}{4} = \frac{15}{4}$. When the integrand is a fraction whose top has degree at least that of the bottom, divide first; when the bottom is an irreducible quadratic, complete the square.



Four steps: choose u , rewrite, integrate, substitute back.

Integration and Accumulation of Change: frequently asked questions

Which Riemann sum overestimates and which underestimates?

For an increasing function the left sum underestimates and the right sum overestimates; for a decreasing function the roles reverse. If the graph is concave up, the trapezoidal sum overestimates and the midpoint sum underestimates.

Why do we add C to an indefinite integral but not to a definite one?

An antiderivative is a whole family of functions that differ by a constant. In a definite integral the constants cancel in $G(b) - G(a)$, so no C is needed.

What does the integral of a rate of change represent?

It is the net change in the accumulated quantity. If $r(t)$ is a rate in gallons per minute, $\int_a^b r(t) dt$ is the net number of gallons gained between times a and b .

How do I differentiate an integral with a variable upper limit?

Use Part 1 and the chain rule: $\frac{d}{dx} \int_a^{g(x)} f(t) dt = f(g(x)) \cdot g'(x)$. Swapping the limits adds a negative sign.

How can I tell whether to use u-substitution?

Look for a function and its derivative (up to a constant factor) inside the same integrand, such as x with x^2+1 . If no such pair appears, try long division, completing the square, or basic rules first.

How to use this ap calculus integration lesson

Use this ap calculus integration lesson before the worksheet, then check what you learned with the quiz.

