

AP Calculus AB · Differential Equations · Test Answer Key

Common Core AP Calculus AB Unit 7 · mathovore.fr

Full solutions and a scoring guide for the differential equations test. Points are awarded the way an AP reader would: separate setup, antiderivative, constant and final answer each earn their own point.

AP Calculus AB Common Core: AP Calculus AB Unit 7

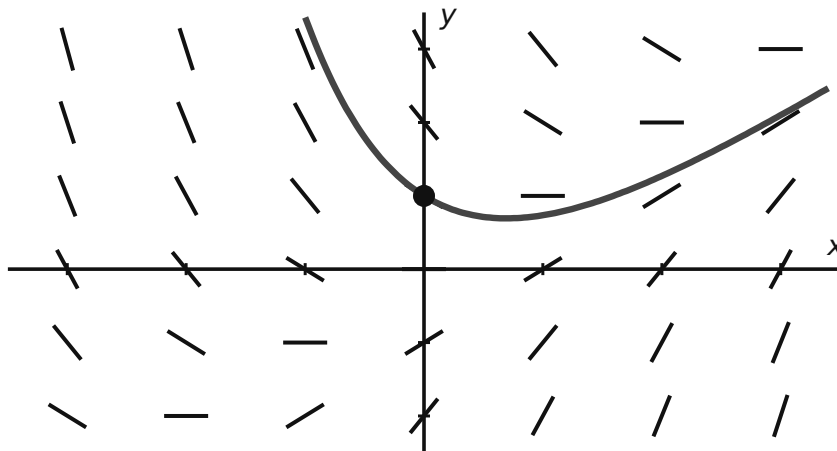
Question 1: Verify a solution (2 points)

- a. $\frac{dy}{dx} = -6e^{-3x}$. Also $-3(y-1) = -3(2e^{-3x}) = -6e^{-3x}$. The sides are equal.
b. $y(0) = 2e^0 + 1 = 3$.

(b) $y(0) = 3$

Scoring: 1 point for differentiating and substituting to show equality; 1 point for $y(0) = 3$.

Question 2: Slope field reading (3 points)



The solution through $(0,1)$ stays above its tangent line.

- a. At $(2,1)$: $2-1=1$. At $(-1,2)$: $-1-2=-3$.
b. $f'(0) = 0-1 = -1$. Tangent line: $y = 1-x$. $f(0.5) \approx 0.5$.
c. $\frac{d^2y}{dx^2} = 1 - \frac{dy}{dx} = 1 - (x-y) = 1-x+y$. At $(0,1)$ this is $2 > 0$, so the graph is concave up and the tangent line lies below it: the estimate is **too small**.

Scoring: 1 point for both slopes (1 and -3); 1 point for the tangent line and 0.5; 1 point for $\frac{d^2y}{dx^2}=1-x+y$ with the correct conclusion.

Question 3: Separable equation (3 points)

$$y^2 dy = 2x dx, \text{ so } \frac{y^3}{3} = x^2 + C.$$

$$y(0) = 2: \frac{8}{3} = C. \text{ Then } y^3 = 3x^2 + 8.$$

$$y = (3x^2 + 8)^{1/3}$$

Scoring: 1 point for separating the variables; 1 point for both antiderivatives (with C); 1 point for $C = \frac{8}{3}$ and the final function.

Question 4: Bacteria model (3 points)

- a. $P(t) = 400e^{kt}$.
- b. $400e^{3k} = 1600$, so $e^{3k} = 4$ and $k = \frac{\ln 4}{3}$ (about 0.4621).
- c. $P(5) = 400 \cdot 4^{5/3} \approx 400(10.0794) \approx 4032$.

(c) about 4,032 bacteria

Scoring: 1 point for $P = 400e^{kt}$; 1 point for $k = \frac{\ln 4}{3}$; 1 point for 4032.

Question 5: Write a differential equation (2 points)

- a. $\frac{dT}{dt} = k(T - 72)$.
- b. The pie is hotter than the room, so $T - 72 > 0$, but its temperature falls, so $\frac{dT}{dt} < 0$.
Therefore $k < 0$.

$$\frac{dT}{dt} = k(T - 72) \text{ with } k < 0$$

Scoring: 1 point for the equation; 1 point for $k < 0$ with the reason.

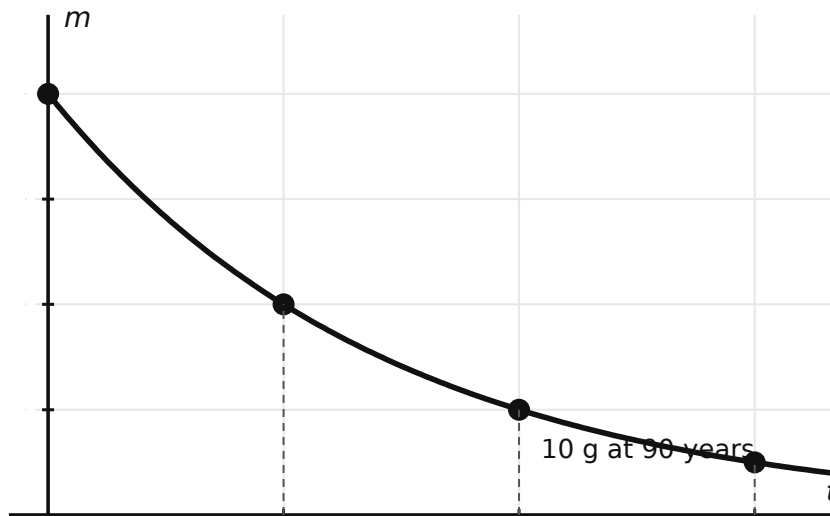
Question 6: Solution and domain (3 points)

- a. $y^{-2} dy = x dx$, so $-\frac{1}{y} = \frac{x^2}{2} + C$. With $f(0) = 1$, $C = -1$. Then $\frac{1}{y} = 1 - \frac{x^2}{2}$ and $f(x) = \frac{2}{2 - x^2}$.
- b. The denominator is zero when $x = \pm\sqrt{2}$, so the interval is $-\sqrt{2} < x < \sqrt{2}$.

(a) $f(x) = \frac{2}{2 - x^2}$; (b) $(-\sqrt{2}, \sqrt{2})$

Scoring: part (a) 1 point for separation and antiderivatives, 1 point for C and f ; part (b) 1 point.

Question 7: Radioactive decay (2 points)



Mass of the sample over time.

Half-life 30 years: 80 g becomes 40 g, 20 g, then 10 g after three half-lives. Equivalently $80 \cdot 2^{-t/30} = 10$ gives $2^{-t/30} = \frac{1}{8}$, so $t = 90$.

90 years

Scoring: 1 point for a correct model such as $m = 80 \cdot 2^{-t/30}$ (or $k = -\frac{\ln 2}{30}$); 1 point for $t = 90$.

Question 8: Identify an equation (2 points)

Segments are parallel along each horizontal row, so the slope depends only on y . The slope is positive above the x -axis and negative below it, so the equation is $\frac{dy}{dx} = \frac{y}{2}$ (not $-\frac{y}{2}$, which has the opposite signs).

The solution through $(0,1)$ is $f(x) = e^{x/2}$, so $\lim_{x \rightarrow \infty} f(x) = \infty$.

$\frac{dy}{dx} = \frac{y}{2}$; the limit is ∞ .

Scoring: 1 point for the equation with a valid reason; 1 point for the limit.

Total
20 points