

AP Calculus AB · Differential Equations · Test

Common Core AP Calculus AB Unit 7 · mathovore.fr

Name: _____ Date: ____/____/____ Score: _____

This free-response style test covers verifying solutions, slope fields, separation of variables and exponential models. It has 8 questions worth 20 points. Show all work; a correct answer with no supporting work earns limited credit.

Test information

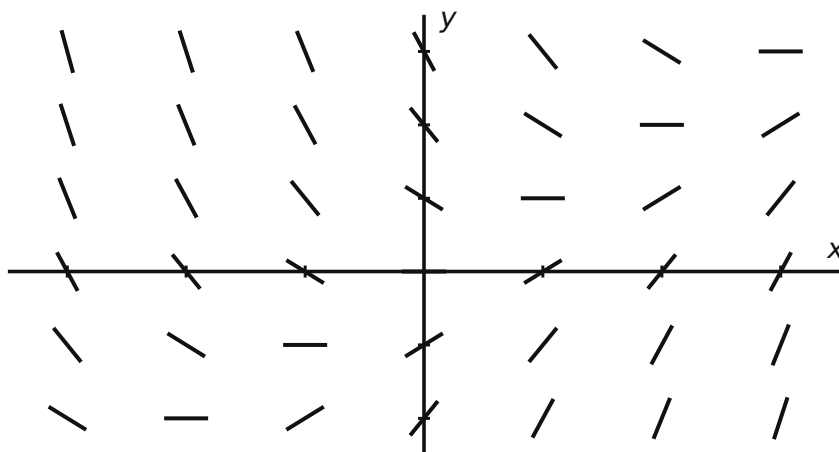
- **Grade:** AP Calculus AB
- **Chapter:** Differential Equations
- **Common Core:** AP Calculus AB Unit 7
- **Time:** 45 minutes
- **Total:** 20 points (Question 1: 2 pts; Question 2: 3 pts; Question 3: 3 pts; Question 4: 3 pts; Question 5: 2 pts; Question 6: 3 pts; Question 7: 2 pts; Question 8: 2 pts)

Question 1: Verify a solution (2 points)

Consider $\frac{dy}{dx} = -3(y-1)$.

- Show that $y = 2e^{-3x} + 1$ is a solution. (1 point)
- Find $y(0)$ for this solution. (1 point)

Question 2: Slope field reading (3 points)



Slope field of $\frac{dy}{dx} = x - y$.

The slope field shown is for $\frac{dy}{dx} = x - y$. Let $y = f(x)$ be the solution with $f(0) = 1$.

- Find the slope of the field at $(2,1)$ and at $(-1,2)$. (1 point)
- Write the tangent line to the graph of f at $x=0$ and use it to estimate $f(0.5)$. (1 point)
- Find $\frac{d^2y}{dx^2}$ in terms of x and y , and decide whether your estimate in part (b) is too large or too small. (1 point)

Question 3: Separable equation (3 points)

Solve $\frac{dy}{dx} = \frac{2x}{y^2}$ with $y(0) = 2$. Write y as a function of x .

Question 4: Bacteria model (3 points)

A bacteria culture grows so that $\frac{dP}{dt} = kP$, where t is in hours. There are 400 bacteria at $t = 0$ and 1,600 bacteria at $t = 3$.

- Write $P(t)$ in terms of k . (1 point)
- Find the exact value of k . (1 point)
- Find $P(5)$ to the nearest whole bacterium. (1 point)

Question 5: Write a differential equation (2 points)

The temperature T of a pie, in $^{\circ}\text{F}$, changes at a rate proportional to the difference between T and the room temperature of 72°F . The pie comes out of the oven at $T(0) = 350$.

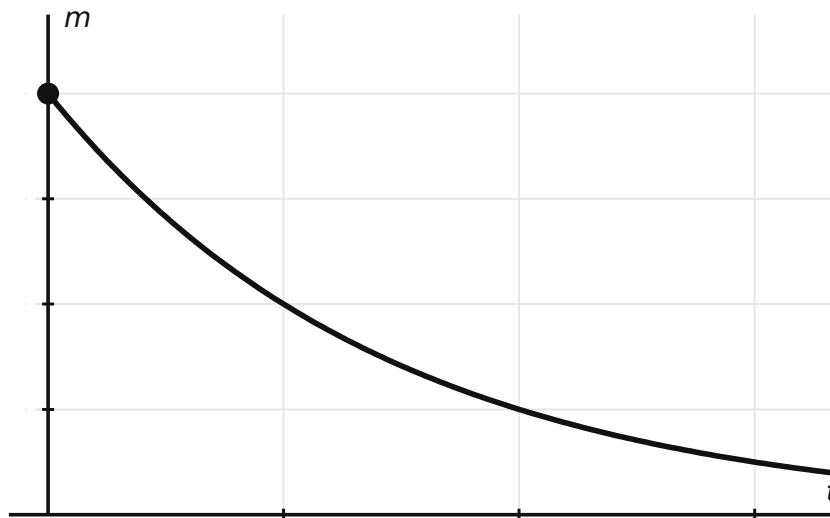
- Write a differential equation for T . (1 point)
- State the sign of the constant of proportionality and justify it. (1 point)

Question 6: Solution and domain (3 points)

Let $y = f(x)$ be the solution of $\frac{dy}{dx} = xy^2$ with $f(0) = 1$.

- Find $f(x)$. (2 points)
- State the largest open interval containing $x = 0$ on which f is defined. (1 point)

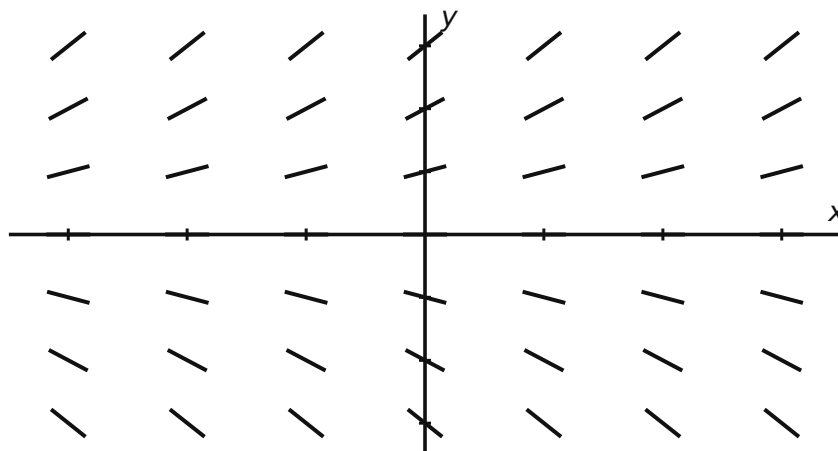
Question 7: Radioactive decay (2 points)



Mass of the sample over time.

An 80 g sample of an isotope decays so that $\frac{dm}{dt} = km$, with a half-life of 30 years. Find the number of years until 10 g remain.

Question 8: Identify an equation (2 points)



A slope field.

A slope field is shown. Which of these could be its differential equation: $\frac{dy}{dx} = x$, $\frac{dy}{dx} = \frac{y}{2}$ or $\frac{dy}{dx} = -\frac{y}{2}$? Justify, then state $\lim_{x \rightarrow \infty} f(x)$ for the solution with $f(0) = 1$.