

AP Calculus AB · Differential Equations · Quiz

Common Core AP Calculus AB Unit 7 · mathovore.fr

Name: _____ Date: ____/____/____ Score: _____

Ten multiple-choice questions check slope fields, verifying solutions, separation of variables and exponential models. Choose one answer for each question, then read the explanation to see the typical mistake.

AP Calculus ABCommon Core: AP Calculus AB Unit 7

Questions

Question 1

Which function is a solution of $\frac{dy}{dx} = 3y$ with $y(0) = 5$?

- A. $y = 5e^{3x}$
- B. $y = 3e^{5x}$
- C. $y = 15x$
- D. $y = 5e^x + 3$

Question 2

Find the slope given by $\frac{dy}{dx} = xy - 1$ at the point $(2, 3)$.

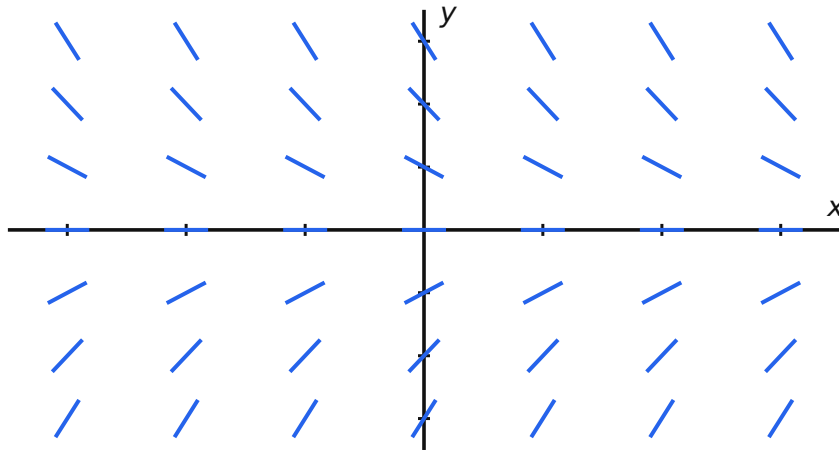
- A. 5
- B. 6
- C. 4
- D. -1

Question 3

What is the first step in separating $\frac{dy}{dx} = \frac{x}{y^2}$?

- A. $dy = \frac{x}{y^2} dx$
- B. $y^2 dy = x dx$
- C. $\frac{dy}{y^2} = x dx$
- D. $y dy = x^2 dx$

Question 4



A slope field.

Which differential equation produces this slope field?

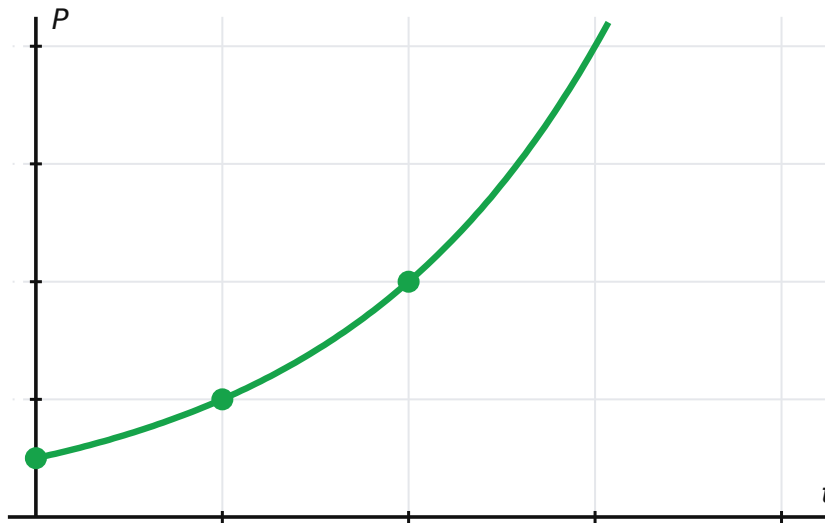
- A. $\frac{dy}{dx} = y$
- B. $\frac{dy}{dx} = x$
- C. $\frac{dy}{dx} = -y$
- D. $\frac{dy}{dx} = -x$

Question 5

If $\frac{dy}{dx} = -2y$ and $y(0) = 4$, what is $y(\ln 2)$?

- A. 2
- B. 16
- C. $\frac{1}{4}$
- D. 1

Question 6



The population doubles every 2 hours.

A population satisfies $\frac{dP}{dt} = kP$ and doubles every 2 hours from 100. What is $P(7)$, rounded to the nearest whole number?

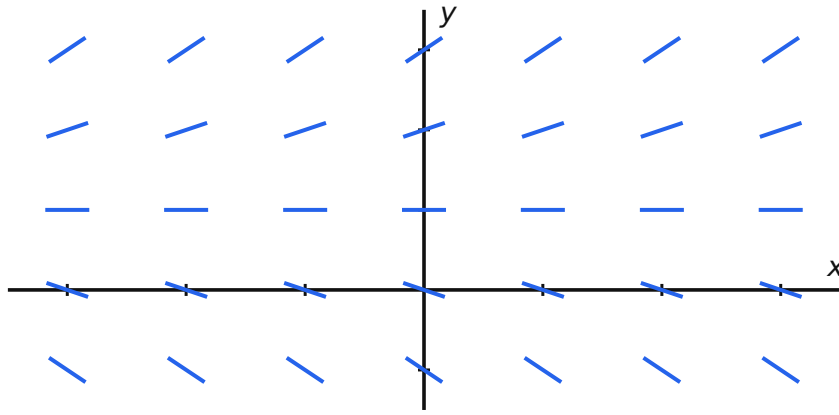
- A. 700
- B. 1,131
- C. 800
- D. 1,600

Question 7

A substance has a half-life of 10 years and obeys $\frac{dy}{dt} = ky$. What is k ?

- A. $\frac{\ln 2}{10}$
- B. $-\frac{10}{\ln 2}$
- C. $-\frac{1}{20}$
- D. $-\frac{\ln 2}{10}$

Question 8



A slope field.

Which constant function is a solution of the differential equation that has this slope field?

- A. $y = 0$
- B. $y = 2$
- C. $y = 1$
- D. $y = -1$

Question 9

Which function is a solution of $\frac{dy}{dx} = 2xy$ with $y(0) = 3$?

- A. $y = e^{x^2} + 2$
- B. $y = 3e^{2x}$
- C. $y = 3e^{x^2}$
- D. $y = x^2 + 3$

Question 10

The temperature of a cooling object satisfies $\frac{dT}{dt} = -0.1(T - 70)$ with $T(0) = 190$. What is $\lim_{t \rightarrow \infty} T(t)$?

- A. 0
- B. 190
- C. -70
- D. 70

Answer key

1. **A** - Differentiating $5e^{3x}$ gives $15e^{3x} = 3y$, and $y(0) = 5$. The choice $3e^{5x}$ swaps the constants.
2. **A** - Substitute: $2 \cdot 3 - 1 = 5$. Choosing 6 forgets the -1 .
3. **B** - Multiply both sides by $y^2 dx$ so each variable sits on its own side. Dividing by y^2 instead moves it the wrong way.
4. **C** - Rows are parallel, so the slope depends on y only. The slope is negative for $y > 0$, which means $-y$, not y .
5. **D** - $y = 4e^{-2x}$, so $y(\ln 2) = 4e^{-2\ln 2} = 4 \cdot \frac{1}{4} = 1$. Using $e^{-\ln 2}$ only once gives 2.
6. **B** - $P = 100 \cdot 2^{t/2}$, so $P(7) = 100 \cdot 2^{3.5} \approx 1131$. A linear guess gives 700.
7. **D** - Half-life means $e^{10k} = \frac{1}{2}$, so $k = -\frac{\ln 2}{10}$. A positive k would be growth.
8. **C** - A constant solution needs slope 0 everywhere along it. The horizontal segments lie on the line $y = 1$.
9. **C** - For $y = 3e^{x^2}$, $y' = 6xe^{x^2} = 2xy$ and $y(0) = 3$. The function $y = x^2 + 3$ has $y' = 2x$ only.
10. **D** - $T = 70 + 120e^{-0.1t}$, and the exponential term tends to 0. The object approaches room temperature, not 0.

Remember:

- Verify a solution by differentiating and substituting.
- A slope field shows $\frac{dy}{dx}$ at grid points; solutions follow the segments.
- Separable: $g(y) dy = h(x) dx$; integrate both sides, add C once, then use the initial condition.
- $\frac{dy}{dt} = ky$ has solution $y = y_0 e^{kt}$.