

AP Calculus AB · Differential Equations · Lesson

Common Core AP Calculus AB Unit 7 · mathovore.fr

A differential equation relates an unknown function to its derivative. In AP Calculus AB you verify solutions, read slope fields, and solve separable equations such as $\frac{dy}{dx} = ky$, whose solution $y = y_0 e^{kt}$ models exponential growth and decay. An initial condition picks one particular solution.

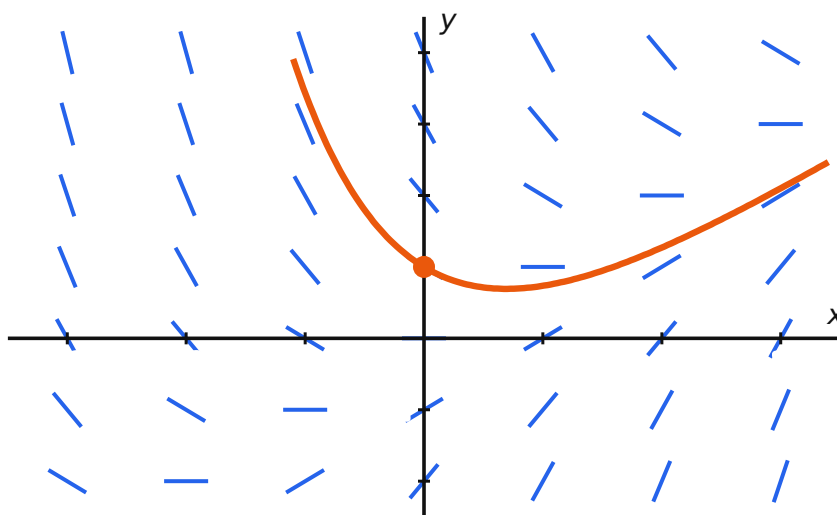
AP Calculus ABCommon Core: AP Calculus AB Unit 7

Key facts:

- Verify a solution by differentiating and substituting.
- A slope field shows $\frac{dy}{dx}$ at grid points; solutions follow the segments.
- Separable: $g(y) dy = h(x) dx$; integrate both sides, add C once, then use the initial condition.
- $\frac{dy}{dt} = ky$ has solution $y = y_0 e^{kt}$.

What is a differential equation?

A **differential equation** is an equation that contains a derivative of an unknown function. A **solution** is a function that makes the equation true on an interval. For example, $y = 3e^{2x}$ solves $\frac{dy}{dx} = 2y$ because $y' = 6e^{2x} = 2y$. To *verify* a solution, differentiate it and substitute.



Slope field of $\frac{dy}{dx} = x - y$ with the solution through $(0,1)$ in orange.

Key vocabulary

General solution: a family of functions with an arbitrary constant C .

Initial condition: a point (a,b) on the graph, written $y(a)=b$. It picks one particular solution.

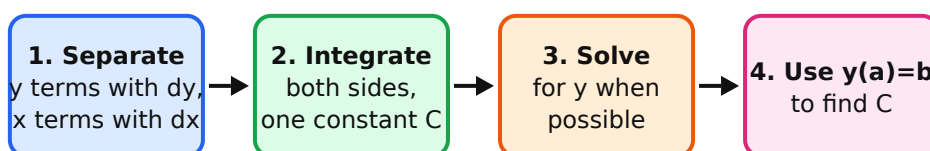
Slope field: short segments drawn at grid points (x,y) with slope $\frac{dy}{dx}$ from the equation.

How do slope fields help?

At every grid point, substitute x and y into $\frac{dy}{dx}$ and draw a short segment with that slope. A solution curve follows the segments. In the field above, $\frac{dy}{dx} = x - y$ is zero on the line $y = x$, so segments there are horizontal. If the slope depends only on x , the segments in each vertical column are parallel. If it depends only on y , the segments in each horizontal row are parallel.

How does separation of variables work?

An equation is **separable** when it can be written as $g(y) dy = h(x) dx$.



The four steps of separation of variables.

Worked example. Solve $\frac{dy}{dx} = 2xy$ with $y(0) = 3$.

1. Separate: $\frac{1}{y} dy = 2x dx$.
2. Integrate: $\ln|y| = x^2 + C$.
3. Solve: $y = Ae^{x^2}$, where $A = \pm e^C$.
4. Use $y(0) = 3$: $3 = Ae^0$, so $A = 3$ and $y = 3e^{x^2}$.

Always include the constant on one side *before* you use the initial condition. Forgetting it is a classic way to lose points on the AP exam.

What are exponential growth and decay?

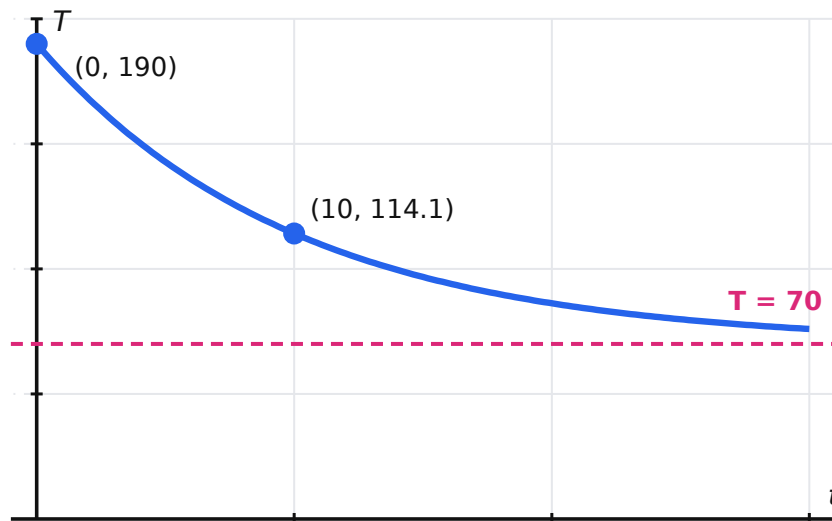
When a quantity changes at a rate proportional to its size, $\frac{dy}{dt} = ky$, the solution is $y = y_0 e^{kt}$. Growth means $k > 0$; decay means $k < 0$.



Solutions of $\frac{dy}{dt} = ky$: growth when $k > 0$, decay when $k < 0$.

Model	Equation	Solution
Exponential change	$\frac{dy}{dt} = ky$	$y = y_0 e^{kt}$
Doubling time T	$k = \frac{\ln 2}{T}$	$y = y_0 \cdot 2^{t/T}$
Newton cooling	$\frac{dT}{dt} = k(T - T_s)$	$T = T_s + (T_0 - T_s)e^{kt}$

A cooling object follows the third row: its temperature moves toward the surrounding temperature T_s , never past it.



Newton cooling: $T = 70 + 120e^{-0.1t}$ approaches the room temperature 70.

Differential Equations: frequently asked questions

What is a differential equation in AP Calculus AB?

It is an equation relating a function to one or more of its derivatives, such as $\frac{dy}{dx} = x - y$. A solution is a function that satisfies the equation, and an initial condition selects one particular solution.

How do you verify that a function is a solution?

Differentiate the function, then substitute both the function and its derivative into the equation. If both sides simplify to the same expression, the function is a solution. Also check any initial condition.

What does a slope field show?

A slope field shows the slope $\frac{dy}{dx}$ at many points. Solution curves follow the segments, so you can sketch a solution through a given point without solving the equation.

When can you use separation of variables?

You can use it when all the y factors can be moved to one side with dy and all the x factors to the other side with dx . Then you integrate both sides and add one constant C .

Why is the solution of $dy/dt = ky$ exponential?

Separating gives $\frac{1}{y} dy = k dt$, so $\ln|y| = kt + C$ and $y = y_0 e^{kt}$. The rate of change is proportional to the amount, so the amount grows or decays by a constant factor in equal time intervals.