

AP Calculus AB · Definition of the Derivative and Basic Rules · Worksheet Answer Key

Common Core AP Calculus AB Unit 2 · mathovore.fr

This ap calculus derivative worksheet answer key solves all 20 problems of the definition of the derivative and basic rules worksheet step by step.

Each solution shows the rule used and the algebra behind the answer. If your result differs, check whether you applied the product or quotient rule in the right order before simplifying.

AP Calculus AB Common Core: AP Calculus AB Unit 2

AP Calculus Derivative Worksheet Answer Key: problems 1-5

Problem 1: Average rate of change

$f(1) = 4$ and $f(4) = 16 + 12 = 28$. The average rate is $\frac{28-4}{4-1} = \frac{24}{3} = 8$. It is the slope of the secant line through $(1, 4)$ and $(4, 28)$.

Problem 2: Derivative at a point by definition

a. $\frac{(3+h)^2 - 9}{h} = \frac{6h + h^2}{h} = 6 + h \rightarrow 6$

b. $f(2) = 12$. $\frac{(2+h)^2 + 4(2+h) - 12}{h} = \frac{4h + h^2 + 4h}{h} = 8 + h \rightarrow 8$

Problem 3: Limit definition for a function

$f(x+h) - f(x) = 2(x^2 + 2xh + h^2) - 5x - 5h - 2x^2 + 5x = 4xh + 2h^2 - 5h$. Divide by h : $4x + 2h - 5$. Let $h \rightarrow 0$: $f'(x) = 4x - 5$. Then $f'(1) = 4 - 5 = -1$.

Problem 4: Definition with a reciprocal

$\frac{2}{x+h-1} - \frac{2}{x-1} = \frac{2(x-1) - 2(x+h-1)}{(x-1)(x+h-1)} = \frac{-2h}{(x-1)(x+h-1)}$. Divide by h : $\frac{-2}{(x-1)(x+h-1)}$. Let $h \rightarrow 0$:
 $g'(x) = -\frac{2}{(x-1)^2}$.

Problem 5: Power rule

a. $8x^7$

b. $6 \cdot 5x^4 = 30x^4$

c. Write $3x^{-2}$: $-6x^{-3} = -\frac{6}{x^3}$

d. Write $x^{1/3} \cdot \frac{1}{3}x^{-2/3}$

AP Calculus Derivative Worksheet Answer Key: problems 6-10

Problem 6: Sum and constant multiple rules

$f'(x) = 12x^2 - 14x + 5$. $f'(2) = 48 - 28 + 5 = 25$. Solve $12x^2 - 14x + 5 = 5$: $2x(6x - 7) = 0$, so $x = 0$ or $x = \frac{7}{6}$.

Problem 7: Derivatives of sin, cos, e^x and $\ln x$

a. $3\cos x + 2\sin x$

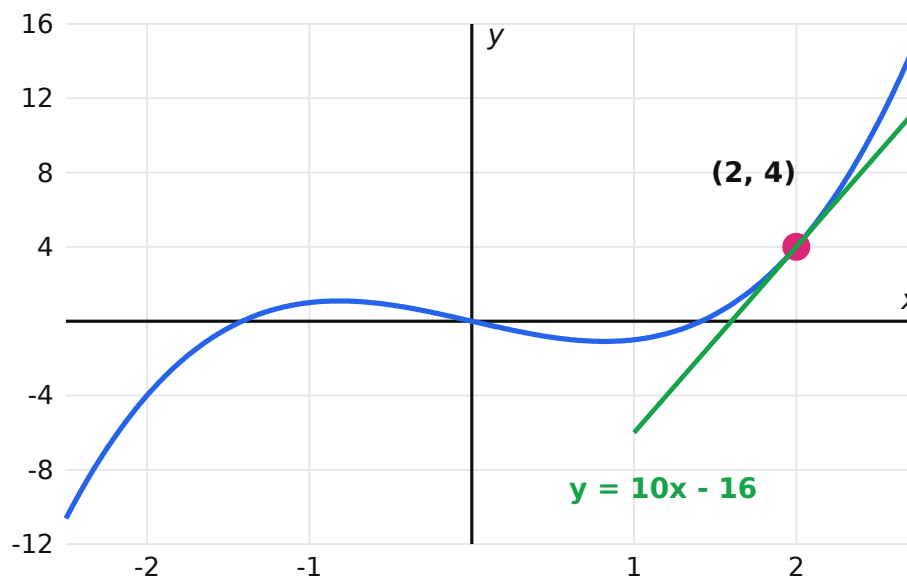
b. $5e^x + \frac{2}{x}$

c. $g'(x) = e^x + \cos x$, so $g'(0) = 1 + 1 = 2$

d. $h'(x) = \frac{4}{x} - 3x^2$, so $h'(1) = 4 - 3 = 1$

Problem 8: Tangent line

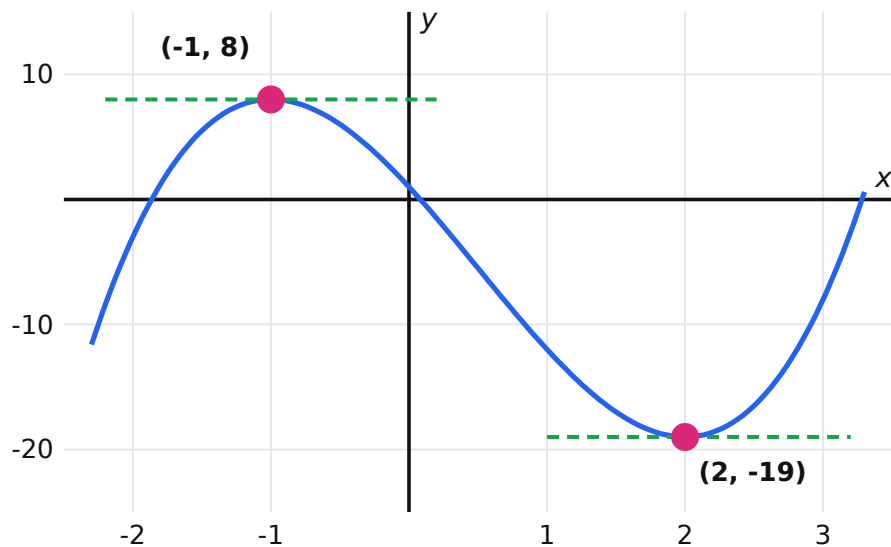
$f(2) = 8 - 4 = 4$. $f'(x) = 3x^2 - 2$, so $f'(2) = 10$. Point-slope form: $y - 4 = 10(x - 2)$, so $y = 10x - 16$.



The tangent line at $(2, 4)$.

Problem 9: Horizontal tangent lines

$f'(x) = 6x^2 - 6x - 12 = 6(x - 2)(x + 1) = 0$, so $x = 2$ and $x = -1$. $f(2) = 16 - 12 - 24 + 1 = -19$ and $f(-1) = -2 - 3 + 12 + 1 = 8$. The points are $(2, -19)$ and $(-1, 8)$.



Horizontal tangents at the two points.

Problem 10: Product rule

- $2x(x^3-4x)+(x^2+1)(3x^2-4)=2x^4-8x^2+3x^4-x^2-4=5x^4-9x^2-4$. Check: expanding first gives x^5-3x^3-4x , whose derivative is the same.
- $2x\sin x+x^2\cos x$
- $e^x\ln x+\frac{e^x}{x}$

AP Calculus Derivative Worksheet Answer Key: problems 11-15

Problem 11: Quotient rule

- $\frac{2x(x+1)-x^2 \cdot 1}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2}$
- $\frac{x\cos x - \sin x}{x^2}$
- $\frac{e^x x^2 - e^x \cdot 2x}{x^4} = \frac{e^x(x-2)}{x^3}$

Problem 12: Derivatives of tan, sec, csc and cot

- $f'(x) = \sec^2 x$. At $\frac{\pi}{4}$, $\sec \frac{\pi}{4} = \sqrt{2}$, so the value is $(\sqrt{2})^2 = 2$
- $3\sec x \tan x$
- $-2\csc^2 x - \csc x \cot x$

Problem 13: Estimate derivatives from a table

- $\frac{5.0-2.0}{2-1} = 3$
- $f'(2) \approx \frac{7.7-3.1}{2.5-1.5} = 4.6$
- $y-5=4.6(x-2)$, so $y=4.6x-4.2$

Problem 14: Is it differentiable?

- Left of 2, $f(x)=2-x$ with slope -1 ; right of 2, $f(x)=x-2$ with slope 1. The slopes differ, so **not differentiable** at $x=2$ (a corner).
- Continuity: $1^2=1$ and $2(1)-1=1$. Slopes: $2x=2$ from the left and 2 from the right. They match, so p is differentiable and $p'(1)=2$.

Problem 15: Height of a ball

- $h(1)=37$ and $h(2)=-64+96+5=37$, so the average velocity is **0 ft/s**.
- $h'(t)=-32t+48$, so $h'(1)=$ **16 ft/s** (moving upward).
- $h'(t)=0$ when $t=1.5$. $h(1.5)=-36+72+5=$ **41 ft** at **1.5 s**.

AP Calculus Derivative Worksheet Answer Key: problems 16-20

Problem 16: Marginal cost of bicycles

$C'(x)=0.04x+3$, so $C'(100)=$ **7**. When 100 helmets are made, cost rises at about \$7 per additional helmet. Exactly, $C(101)-C(100)=0.02(201)+3=$ **\$7.02**, very close to the derivative.

Problem 17: Learning curve

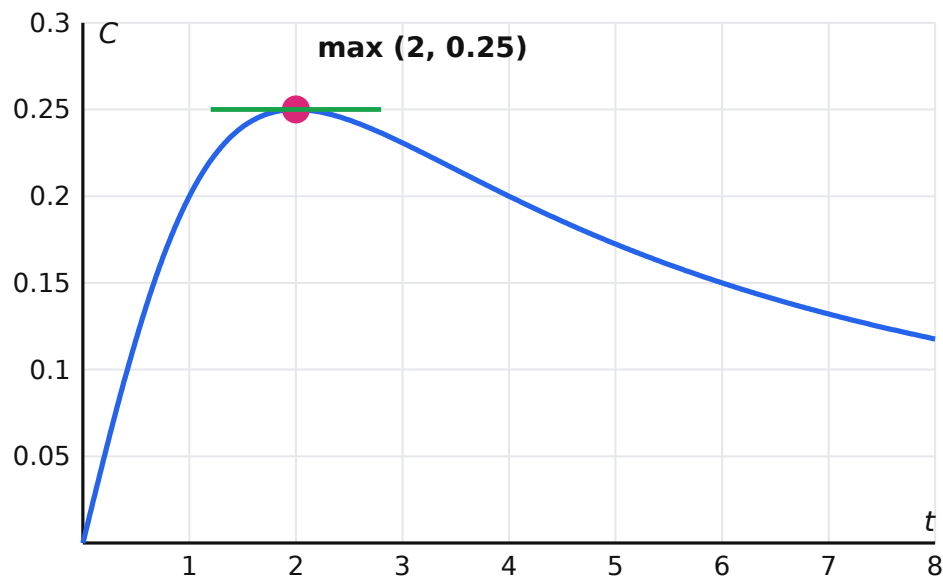
$S'(t)=\frac{15}{t}$. $S'(5)=$ **3** words per minute per hour and $S'(10)=$ **1.5**. Each extra hour of practice helps less as time goes on.

Problem 18: Revenue growth

$p(10)=25$, $q(10)=260$, $p'=0.5$, $q'=-4$. Product rule: $R'(10)=0.5(260)+25(-4)=130-100=$ **30**. Revenue is growing by about \$30 per day on day 10.

Problem 19: Concentration of a medicine

- $\frac{1(t^2+4)-t(2t)}{(t^2+4)^2} = \frac{4-t^2}{(t^2+4)^2}$
- $C'(1)=\frac{3}{25}=$ **0.12**. At 1 hour the concentration is rising by about 0.12 mg/L per hour.
- $C'(t)=0$ at $t=2$ (for $t>0$); $C'>0$ before and $C'<0$ after. $C(2)=\frac{2}{8}=$ **0.25 mg/L** at **2 hours**.



Horizontal tangent at the maximum.

Problem 20: Derivatives from a table of values

a. $h'(2) = f'g + fg' = (-1)(5) + (3)(4) = 7$

b. $k'(2) = \frac{f'g - fg'}{g^2} = \frac{-5 - 12}{25} = -\frac{17}{25}$

c. $m'(2) = 2(-1) - 3(4) = -14$

How to use this ap calculus derivative worksheet answer key

Read this ap calculus derivative worksheet answer key after trying each problem yourself, and redo any problem you missed.

