

AP Calculus AB · Definition of the Derivative and Basic Rules · Test Answer Key

Common Core AP Calculus AB Unit 2 · mathovore.fr

This ap calculus derivative test answer key gives the full solution and the scoring guide for each of the 9 test questions on definition of the derivative and basic rules.

Award points as shown in each scoring note. Accept equivalent forms of a derivative and correct work shown in a different order. Half points are allowed only where the note says so.

AP Calculus AB Common Core: AP Calculus AB Unit 2

AP Calculus Derivative Test Answer Key: questions 1-5

Question 1: Limit definition (2 points)

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-3(x+h)-x^2+3x}{h} = \frac{2xh+h^2-3h}{h} = 2x+h-3. \text{ As } h \rightarrow 0, f'(x) = 2x-3.$$

Scoring: 1 point for a correct difference quotient simplified to remove h from the denominator; 1 point for the limit and answer.

Question 2: Basic rules (3 points)

- a. $20x^3 - x^{-1/2}$, which is $20x^3 - \frac{1}{\sqrt{x}}$
- b. $3e^x - 4\cos x + \frac{2}{x}$
- c. Write $6x^{-2}$: $-\frac{12}{x^3} - 3\sin x$

Scoring: 1 point per part for a fully correct derivative; no partial credit unless a single sign or coefficient error is made, in which case award 0.5.

Question 3: Product rule (2 points)

$$f'(x) = 3x^2e^x + x^3e^x = x^2e^x(x+3). \text{ Then } f'(1) = e(4) = 4e.$$

Scoring: 1 point for the product rule applied correctly; 1 point for the value at $x = 1$ equal to $4e$.

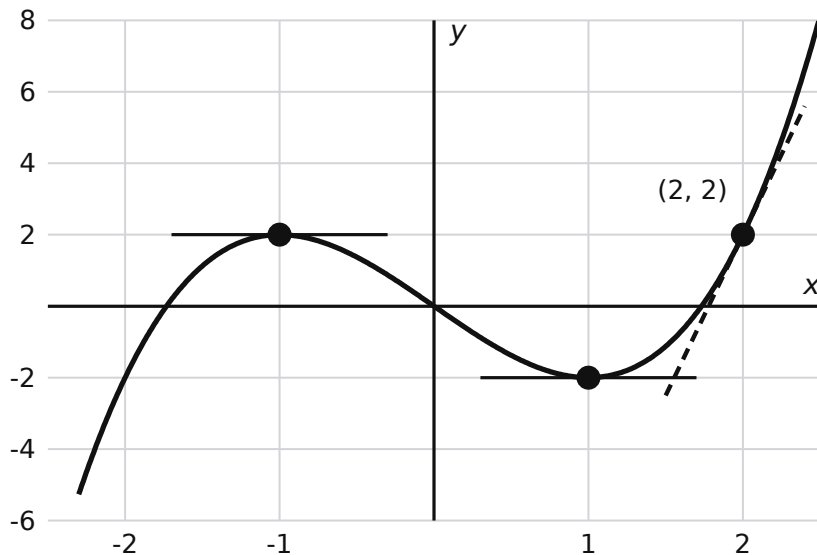
Question 4: Quotient rule (2 points)

$$g'(x) = \frac{2(x^2+1) - (2x+1)(2x)}{(x^2+1)^2} = \frac{-2x^2-2x+2}{(x^2+1)^2}. \text{ At } x=0: \frac{2}{1} = 2.$$

Scoring: 1 point for the correct quotient rule setup (numerator in the right order); 1 point for the value at $x = 0$ equal to 2.

Question 5: Tangent lines to a cubic (3 points)

- $f(2)=2$ and $f'(x)=3x^2-3$, so $f'(2)=9$. The line is $y=9x-16$.
- $3x^2-3=0$ gives $x=\pm 1$.
- $3x^2-3=6$ gives $x^2=3$, so $x=\pm\sqrt{3}$.



The tangent at $x = 2$ and the horizontal tangents.

Scoring: 1 point per part. Part (a) requires both the slope 9 and the point (2, 2) used correctly.

AP Calculus Derivative Test Answer Key: questions 6-9

Question 6: Differentiable piecewise function (2 points)

Slopes must match: $2x=4$ at $x=2$, so $a=4$. Values must match: $2^2+1=5=2a+b$, so $b=5-8=-3$.

Scoring: 1 point for continuity (values equal); 1 point for matching derivatives. Award the points for a correct method even if arithmetic slips once.

Question 7: Estimating rates from data (2 points)

- $\frac{152-190}{4-0} = -9.5$ °F per minute
- $T'(6) \approx \frac{132-152}{8-4} = -5$ °F per minute. At 6 minutes the coffee is cooling by about 5 degrees each minute.

Scoring: part (a) 1 point for the value with units; part (b) 1 point for the estimate and an interpretation in context.

Question 8: Derivative with tangent (2 points)

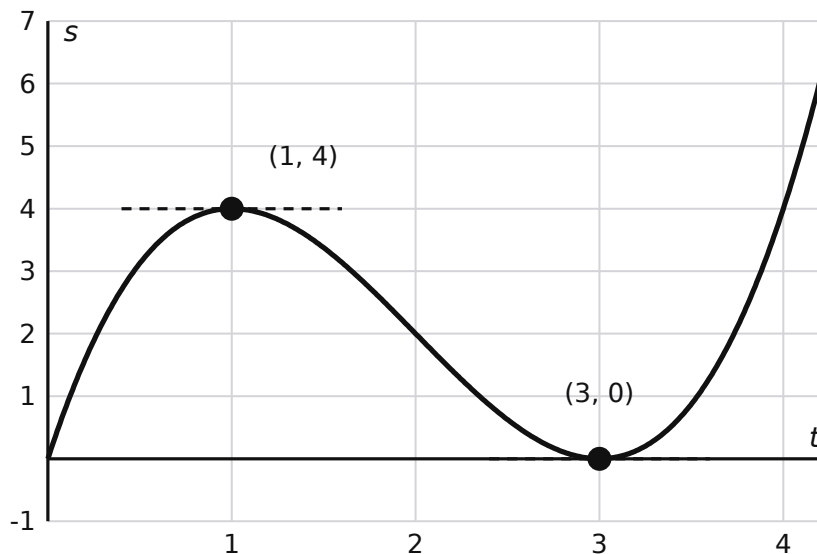
Product rule: $f'(x) = \tan x + x \sec^2 x$. At $\frac{\pi}{4}$: $\tan \frac{\pi}{4} = 1$ and $\sec^2 \frac{\pi}{4} = 2$, so the value is $1 + \frac{\pi}{2}$.

Scoring: 1 point for the derivative; 1 point for the exact value at $\pi/4$.

Question 9: Motion along a line (2 points)

a. $v(t) = 3t^2 - 12t + 9$

b. $3(t-1)(t-3) = 0$ gives $t = 1$ and $t = 3$. $s(1) = 4$ ft and $s(3) = 0$ ft.



Horizontal tangents at $t = 1$ and $t = 3$.

Scoring: part (a) 1 point; part (b) 1 point for both times (0.5 if one is missing) together with the positions.

Scoring summary

Total
20 points: add the points of every question of this ap calculus derivative test answer key.

How to use this ap calculus derivative test answer key

Use this ap calculus derivative test answer key to score the test fairly and to see which skills need more practice.

