

AP Calculus AB · Definition of the Derivative and Basic Rules · Lesson

Common Core AP Calculus AB Unit 2 · mathovore.fr

This ap calculus derivative rules lesson explains definition of the derivative and basic rules in one printable page, with definitions, a method and a worked example.

The derivative of a function at a point is its instantaneous rate of change: the limit of the difference quotient as h approaches 0, and the slope of the tangent line. The power, sum, product and quotient rules, plus the derivatives of sine, cosine, e^x and $\ln x$, let you differentiate without computing limits.

AP Calculus AB Common Core: AP Calculus AB Unit 2

AP Calculus Derivative Rules Lesson: key facts

- AP Calculus AB Unit 2: $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ is the slope of the tangent line at $x = a$.
- Average rate of change on $[a, b]$ is $\frac{f(b) - f(a)}{b - a}$, the slope of a secant line.
- Power rule: $\frac{d}{dx} x^n = nx^{n-1}$. Product rule: $(fg)' = f'g + fg'$. Quotient rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$.
- A function is not differentiable at a corner, cusp, vertical tangent or discontinuity; differentiable implies continuous.
- $\frac{d}{dx} \sin x = \cos x$, $\frac{d}{dx} \cos x = -\sin x$, $\frac{d}{dx} e^x = e^x$, $\frac{d}{dx} \ln x = \frac{1}{x}$.

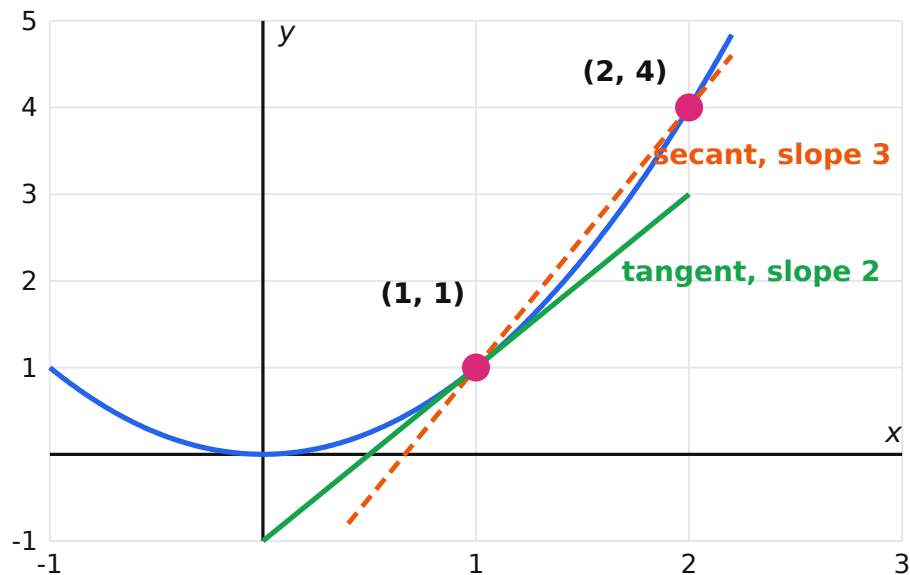
What is the derivative?

The **average rate of change** of f on $[a, b]$ is the slope of the secant line, $\frac{f(b) - f(a)}{b - a}$. Letting the second point slide toward the first gives the **instantaneous rate of change**, the slope of the tangent line:

Limit definition

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

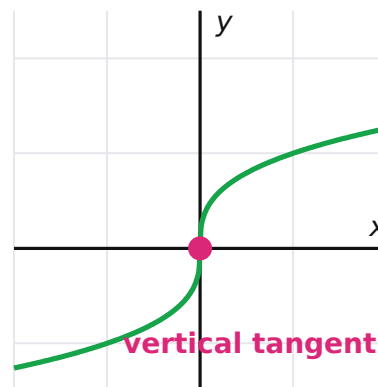
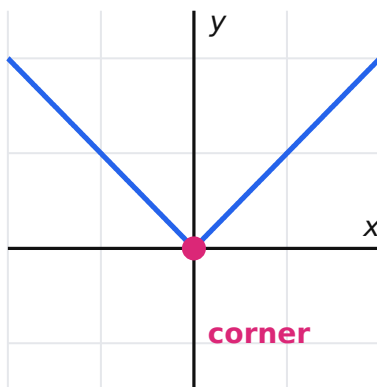
Also written $\frac{dy}{dx}$. For $f(x) = x^2$ at $a = 1$: $\frac{(1+h)^2 - 1}{h} = 2 + h \rightarrow 2$.



The secant slope 3 is an average rate of change; the tangent slope 2 is the instantaneous rate at $x = 1$.

When is a function not differentiable?

A derivative needs a single tangent slope, so f fails to be differentiable at a corner, a cusp, a vertical tangent, or any discontinuity. Differentiable always implies continuous, but not the other way around: $|x|$ is continuous at 0 with no derivative there.



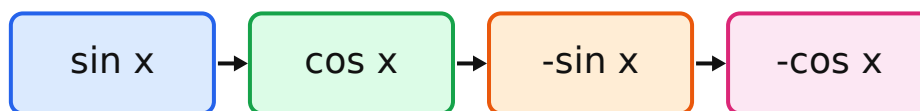
Left: $y = |x|$ has a corner at 0. Right: $y = x^{1/3}$ has a vertical tangent at 0. Neither has a derivative there.

What are the basic derivative rules?

Function	Derivative
c	0
x^n	nx^{n-1}
$\sin x, \cos x$	$\cos x, -\sin x$
$e^x, \ln x$	$e^x, \frac{1}{x}$

Function	Derivative
$\tan x, \cot x$	$\sec^2 x, -\csc^2 x$
$\sec x, \csc x$	$\sec x \tan x, -\csc x \cot x$

Constants factor out and sums split apart: $(cf \pm g)' = cf' \pm g'$.



Each arrow means "take the derivative." After four steps, sine returns to itself.

How do the product and quotient rules work?

Two rules

Product: $(fg)' = f'g + fg'$

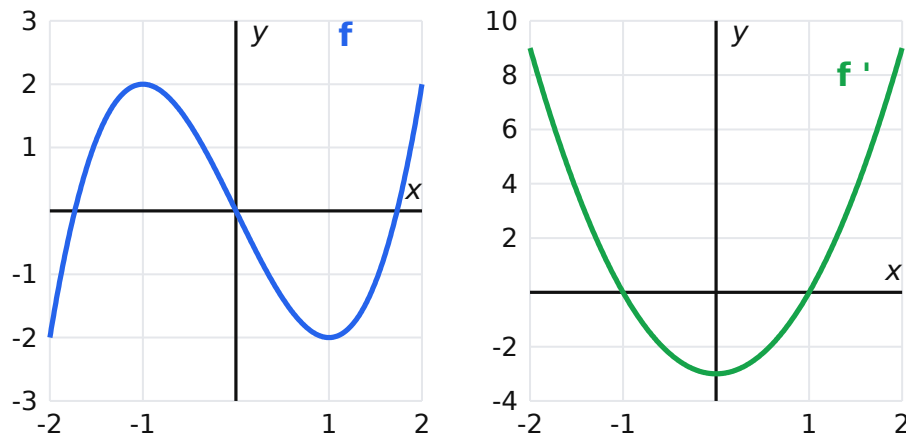
Quotient: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

Worked example: find the tangent line to $f(x) = x^2e^x$ at $x = 1$.

1. Product rule: $f'(x) = 2xe^x + x^2e^x$.
2. Evaluate: $f(1) = e$ and $f'(1) = 2e + e = 3e$.
3. Point-slope form: $y - e = 3e(x - 1)$, so $y = 3ex - 2e$.

How do you read a derivative from a graph or table?

Where f has a horizontal tangent, f' is zero. From a table, estimate $f'(a)$ with the slope between the nearest points on each side. The figure shows $f(x) = x^3 - 3x$ beside $f'(x) = 3x^2 - 3$.



Where f has a horizontal tangent ($x = -1$ and $x = 1$), the graph of the derivative crosses zero.

Definition of the Derivative and Basic Rules: frequently asked questions

What is the difference between average and instantaneous rate of change?

Average rate of change is the slope of a secant line over an interval, $\frac{f(b)-f(a)}{b-a}$. Instantaneous rate of change is the slope of the tangent line at one point, found as the limit of those slopes as the interval shrinks.

How do you use the limit definition of the derivative?

Write $\frac{f(x+h)-f(x)}{h}$, expand and simplify until h cancels from the numerator, then let $h \rightarrow 0$. For $f(x) = x^2$ the quotient simplifies to $2x+h$, so $f'(x) = 2x$.

When does a derivative not exist?

A derivative does not exist at a corner, cusp, vertical tangent, or discontinuity. At these points the left and right tangent slopes disagree or are infinite, so no single tangent slope exists.

What are the derivatives of $\sin x$, $\cos x$, e^x and $\ln x$?

The derivative of $\sin x$ is $\cos x$, of $\cos x$ is $-\sin x$, of e^x is e^x , and of $\ln x$ is $\frac{1}{x}$ for $x > 0$.

When should you use the product rule instead of the power rule?

Use the product rule when two non-constant factors multiply, such as $x^2 \sin x$. The power rule alone handles a single power of x . Multiplying out first is also fine for polynomials like $(x^2+1)(x^3-4x)$.

How to use this ap calculus derivative rules lesson

Use this ap calculus derivative rules lesson before the worksheet, then check what you learned with the quiz.

