

AP Calculus AB · Derivatives in Context: Motion and Related Rates · Worksheet Answer Key

Common Core AP Calculus AB Unit 4 · mathovore.fr

This ap calculus contextual worksheet answer key solves all 25 problems of the derivatives in context: motion and related rates worksheet step by step.

Each solution shows the model, the derivative, the substitution and the final answer with units. For related rates, check that you substituted only after differentiating.

AP Calculus ABCommon Core: AP Calculus AB Unit 4

AP Calculus Contextual Worksheet Answer Key: problems 1-5

Problem 1: Interpret a rate with units

T' has units of $^{\circ}\text{F}$ per minute. The negative sign means the temperature is dropping.

Five minutes after pouring, the cocoa is cooling at about 3.2°F per minute.

Problem 2: Velocity and acceleration functions

$$v(t) = s'(t) = 2t - 6 \text{ and } a(t) = v'(t) = 2.$$

$$v(t) = 2t - 6 \text{ ft/s, } a(t) = 2 \text{ ft/s}^2$$

Problem 3: When is the cart at rest?

$$v(4) = 2(4) - 6 = 2 \text{ ft/s.}$$

$$v(t) = 0 \text{ gives } 2t - 6 = 0, \text{ so } t = 3. \text{ Position: } s(3) = 9 - 18 + 5 = -4.$$

$$v(4) = 2 \text{ ft/s; at rest at } t = 3 \text{ s, } s = -4 \text{ ft}$$

Problem 4: Where the particle turns around

$$v(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3), \text{ so } v = 0 \text{ at } t = 1 \text{ and } t = 3.$$

$$s(1) = 1 - 6 + 9 = 4 \text{ and } s(3) = 27 - 54 + 27 = 0.$$

At rest at $t = 1$ (position 4) and $t = 3$ (position 0).

Problem 5: Direction of motion

$v > 0$ on $(0,1)$ and $(3,5)$. $v < 0$ on $(1,3)$ (test $v(2) = -3$).

Right on $(0, 1)$ and $(3, 5)$; left on $(1, 3)$.

AP Calculus Contextual Worksheet Answer Key: problems 6-10

Problem 6: Speeding up or slowing down

$a(t) = 6t - 12$, so $a < 0$ for $t < 2$ and $a > 0$ for $t > 2$.

Signs: on $(0,1)$ $v > 0, a < 0$ (slowing); on $(1,2)$ $v < 0, a < 0$ (speeding up); on $(2,3)$ $v < 0, a > 0$ (slowing); on $(3,5)$ $v > 0, a > 0$ (speeding up).

Speed increases on $(1, 2)$ and $(3, 5)$.

Problem 7: Total distance traveled

$v(t) = 3t^2 - 18t + 24 = 3(t-2)(t-4)$, so the particle turns at $t = 2$ and $t = 4$. Positions: $s(0) = 0$, $s(2) = 20$, $s(4) = 16$, $s(6) = 216 - 324 + 144 = 36$.

Distance: $|20 - 0| + |16 - 20| + |36 - 16| = 44$.

44 feet

Problem 8: Linearization of a cube

$f(2) = 8$, $f'(x) = 3x^2$, $f'(2) = 12$. So $L(x) = 8 + 12(x - 2)$.

$L(2.1) = 8 + 12(0.1) = 9.2$. (The exact value is 9.261.)

$f(2.1) \approx 9.2$

Problem 9: Estimate a square root

$f(49) = 7$, $f'(x) = \frac{1}{2\sqrt{x}}$, $f'(49) = \frac{1}{14}$. $L(x) = 7 + \frac{1}{14}(x - 49)$.

$L(50) = 7 + \frac{1}{14} \approx 7.0714$.

$\sqrt{50} \approx 7.071$. Because $f''(x) = -\frac{1}{4}x^{-3/2} < 0$, the graph is concave down, the tangent line is above it, and the estimate is **too large** (the exact value is 7.0711).

Problem 10: Estimate a logarithm

$f(1) = 0$, $f'(1) = 1$. $L(x) = x - 1$.

$$L(1.05) = 0.05.$$

$\ln(1.05) \approx 0.05$ (exact value 0.04879; the estimate is a little large since $\ln x$ is concave down).

AP Calculus Contextual Worksheet Answer Key: problems 11-15

Problem 11: Estimate a cube root

$$f(8) = 2, f'(x) = \frac{1}{3}x^{-2/3}, f'(8) = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}.$$

$$L(8.3) = 2 + \frac{1}{12}(0.3) = 2.025.$$

$$\sqrt[3]{8.3} \approx 2.025$$

Problem 12: L'Hôpital with sine

The form is $\frac{0}{0}$. Differentiate top and bottom: $\frac{3\cos(3x)}{1} \rightarrow 3$.

3

Problem 13: L'Hôpital with a quartic

At $x = 1$ the form is $\frac{0}{0}$. Then $\frac{4x^3}{3x^2} \rightarrow \frac{4}{3}$.

$$\frac{4}{3}$$

Problem 14: Apply L'Hôpital twice

Form $\frac{0}{0}$. First: $\frac{2e^{2x}-2}{2x}$, again $\frac{0}{0}$. Second: $\frac{4e^{2x}}{2} \rightarrow 2$.

2

Problem 15: Logarithm over a root

Form $\frac{\infty}{\infty}$. Then $\frac{1/x}{1/(2\sqrt{x})} = \frac{2}{\sqrt{x}} \rightarrow 0$.

The limit exists: 0

AP Calculus Contextual Worksheet Answer Key: problems 16-20

Problem 16: Polynomial over exponential

Form $\frac{\infty}{\infty}$. First: $\frac{3x^2}{2e^{2x}}$, second: $\frac{6x}{4e^{2x}}$ (both still $\frac{\infty}{\infty}$). Third: $\frac{6}{8e^{2x}} \rightarrow 0$.

0

Problem 17: Cosine and a square

The form is $\frac{0}{0}$. Then $\frac{3\sin 3x}{2x}$ is again $\frac{0}{0}$, and $\frac{9\cos 3x}{2} \rightarrow \frac{9}{2}$.

$$\frac{9}{2}$$

Problem 18: Expanding ripple

$A = \pi r^2$, so $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$. Substitute $r = 10$ and $\frac{dr}{dt} = 2$: $2\pi(10)(2) = 40\pi$.

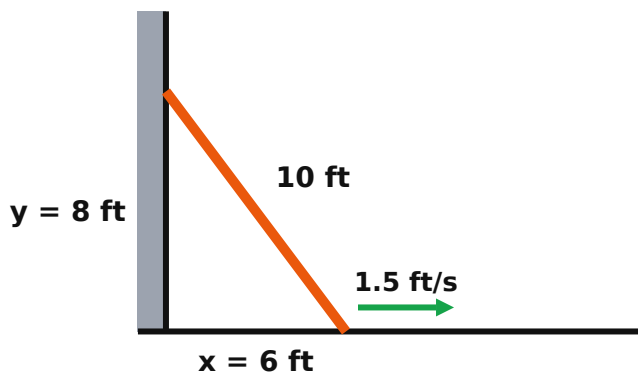
$$40\pi \approx 125.7 \text{ ft}^2/\text{s}$$

Problem 19: Sliding ladder

$x^2 + y^2 = 100$. At $x = 6$: $y = \sqrt{100 - 36} = 8$.

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$ gives $\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt} = -\frac{6}{8}(1.5) = -1.125$.

The top slides down at 1.125 ft/s.



With $x = 6$ and $y = 8$: $dy/dt = -1.125$ ft/s.

Problem 20: Inflating a ball

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. So $30 = 4\pi(9) \frac{dr}{dt}$ and $\frac{dr}{dt} = \frac{30}{36\pi} = \frac{5}{6\pi}$.

$$\frac{5}{6\pi} \approx 0.265 \text{ in/s}$$

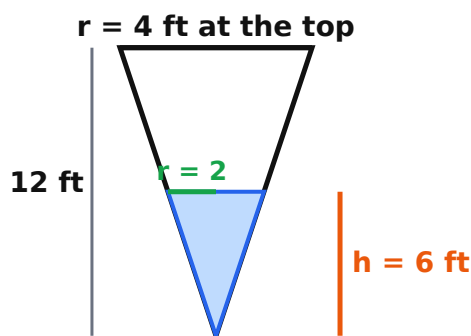
AP Calculus Contextual Worksheet Answer Key: problems 21-25

Problem 21: Filling a conical tank

Similar triangles: $\frac{r}{h} = \frac{4}{12}$, so $r = \frac{h}{3}$. Then $V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h = \frac{\pi}{27}h^3$.

$\frac{dV}{dt} = \frac{\pi}{9}h^2 \frac{dh}{dt}$. At $h = 6$: $5 = \frac{\pi}{9}(36) \frac{dh}{dt} = 4\pi \frac{dh}{dt}$.

$$\frac{dh}{dt} = \frac{5}{4\pi} \approx 0.398 \text{ ft/min}$$



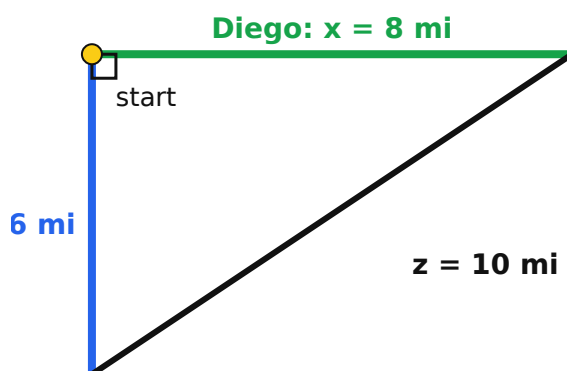
Similar triangles give $r = h/3$, so at $h = 6$ ft the surface radius is 2 ft.

Problem 22: Two walkers

After 2 h: $x = 8$, $y = 6$, $z = \sqrt{64+36} = 10$.

$z^2 = x^2 + y^2$ gives $z \frac{dz}{dt} = x \frac{dx}{dt} + y \frac{dy}{dt}$, so $10 \frac{dz}{dt} = 8(4) + 6(3) = 50$.

$$\frac{dz}{dt} = 5 \text{ mph}$$



After 2 hours: legs 8 mi and 6 mi, hypotenuse 10 mi.

Problem 23: Shadow of a walker

Let x be the distance from the post and s the shadow length. Similar triangles: $\frac{s}{6} = \frac{x+s}{15}$, so $15s = 6x + 6s$ and $9s = 6x$, $s = \frac{2}{3}x$.

$\frac{ds}{dt} = \frac{2}{3}(4) = \frac{8}{3}$. The tip is at $x+s = \frac{5}{3}x$, so its speed is $\frac{5}{3}(4) = \frac{20}{3}$.

Tip: $20/3 \approx 6.67$ ft/s. Shadow length: $8/3 \approx 2.67$ ft/s.

Problem 24: Marginal cost

$C'(x) = 0.04x + 4$, so $C'(100) = 8$. Interpretation: after 100 bracelets, the next one costs about \$8.

$C(101) - C(100) = 0.02(10201 - 10000) + 4 = 4.02 + 4 = 8.02$, so the actual cost is \$8.02.

$C'(100)$ = \$8 per bracelet; the exact cost of the 101st is \$8.02.

Problem 25: Pool volume

$V'(t) = 40 - 2t$. Set $40 - 2t = -10$: $t = 25$.

$V(25) = 600 + 1000 - 625 = 975$.

At $t = 25$ minutes, the pool holds 975 gallons and is draining at 10 gal/min.

How to use this ap calculus contextual worksheet answer key

Read this ap calculus contextual worksheet answer key after trying each problem yourself, and redo any problem you missed.

