

AP Calculus AB · Derivatives in Context: Motion and Related Rates · Test Answer Key

Common Core AP Calculus AB Unit 4 · mathovore.fr

This ap calculus contextual test answer key gives the full solution and the scoring guide for each of the 8 test questions on derivatives in context: motion and related rates.

Worked solutions and a scoring guide for the Unit 4 test. Points are awarded for each correct step, so partial credit applies. The total is 20 points.

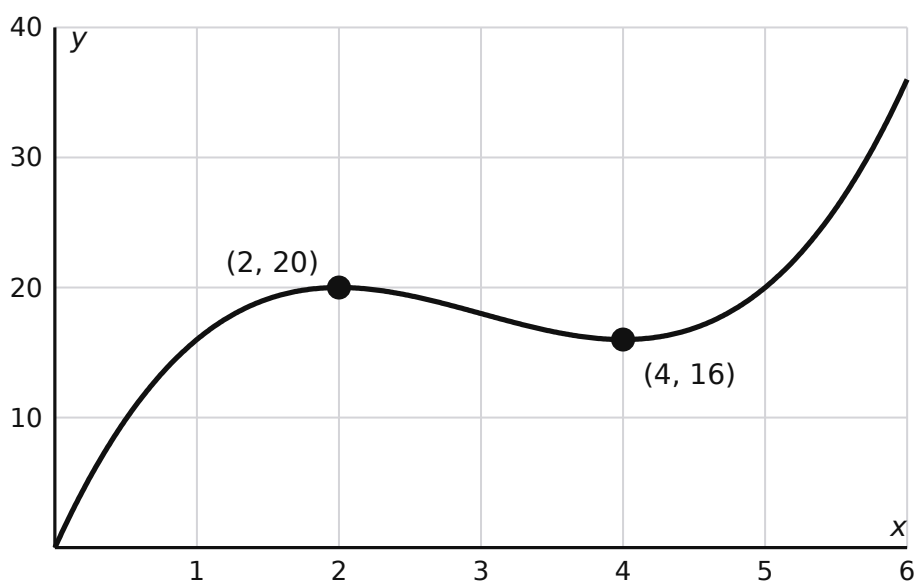
AP Calculus ABCommon Core: AP Calculus AB Unit 4

AP Calculus Contextual Test Answer Key: questions 1-5

Question 1: Particle motion (3 points)

- a. $v(t) = 3t^2 - 18t + 24$, $a(t) = 6t - 18$
- b. $v = 3(t-2)(t-4) = 0$: **$t = 2$ and $t = 4$.**
- c. $v(5) = 75 - 90 + 24 = 9 > 0$ and $a(5) = 12 > 0$. Same sign: **speed is increasing.**

Scoring: (a) 1 point for both functions. (b) 1 point for both times. (c) 1 point for evaluating both signs and the correct conclusion.



$s(t)$ in feet for $0 \leq t \leq 6$ seconds. Turning points where $v = 0$.

Question 2: Interpreting a derivative (2 points)

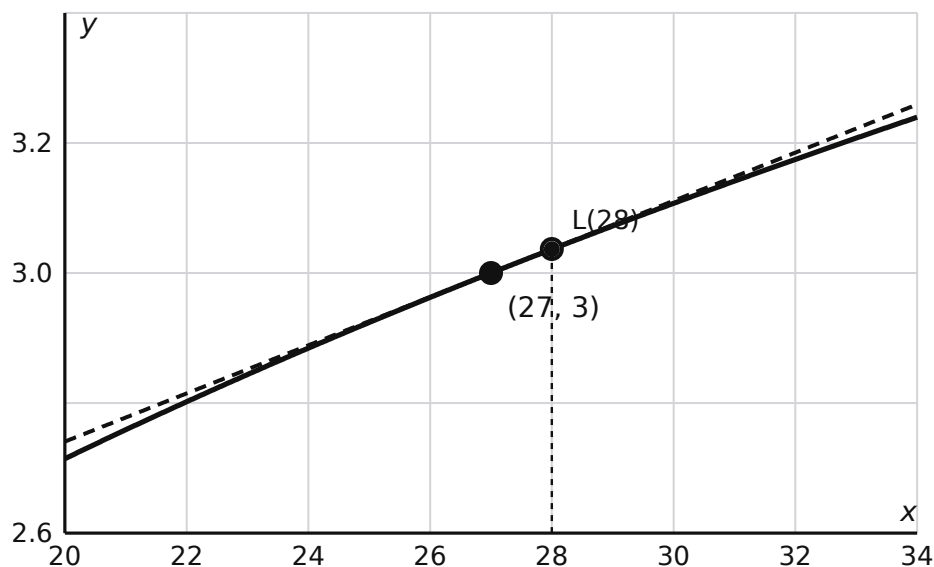
- a. **At 10 minutes the soup is cooling at about 1.8 °F per minute.**
- b. $H(12) \approx 140 + (-1.8)(2) = 136.4$. **About 136.4 °F.**

Scoring: (a) 1 point for °F per minute and the meaning of the sign. (b) 1 point for the linearization value.

Question 3: Local linearization (3 points)

- a. $f'(x) = \frac{1}{3}x^{-2/3}$, $f'(27) = \frac{1}{27}$: $L(x) = 3 + \frac{1}{27}(x - 27)$.
- b. $L(28) = 3 + \frac{1}{27} \approx 3.037$. $\sqrt[3]{28} \approx 3.037$
- c. $f''(x) = -\frac{2}{9}x^{-5/3} < 0$ so f is concave down and the tangent line lies above the graph: **overestimate.**

Scoring: 1 point per part. In (c) the reason must refer to concavity or the sign of the second derivative.



Cube root of x and its tangent line at $x = 27$. $L(28)$ lies just above $f(28)$.

Question 4: Melting ice cube (2 points)

$$\frac{dV}{dt} = 3s^2 \frac{ds}{dt} = 3(25)(-0.2) = -15.$$

$$-15 \text{ in}^3/\text{min}$$

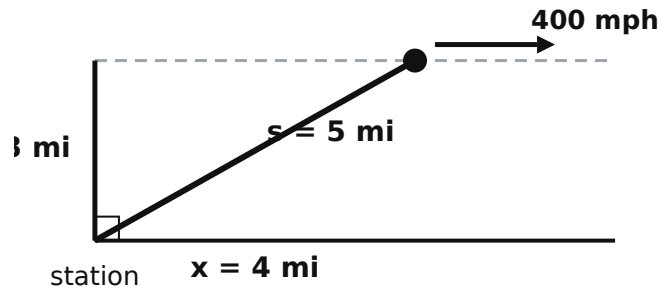
Scoring: 1 point for differentiating with the chain rule factor. 1 point for the value with the negative sign and units.

Question 5: Plane and radar station (3 points)

a. $s^2 = x^2 + 9$

b. $2s \frac{ds}{dt} = 2x \frac{dx}{dt}$. At $x = 4$, $s = \sqrt{25} = 5$. $\frac{ds}{dt} = \frac{4}{5}(400) = 320$. **320 mph**

Scoring: (a) 1 point. (b) 1 point for the differentiated equation, 1 point for $s = 5$ and the final value.



The plane flies at constant altitude 3 mi. At $x = 4$ mi the line of sight is $s = 5$ mi.

AP Calculus Contextual Test Answer Key: questions 6-8

Question 6: Limits with L'Hôpital (3 points)

a. Form $\frac{0}{0}$: $\frac{4\sin(4x)}{2x}$, still $\frac{0}{0}$; then $\frac{16\cos(4x)}{2} \rightarrow 8$. **8**

b. Form $\frac{\infty}{\infty}$: $\frac{1/x}{1/(2\sqrt{x})} = \frac{2}{\sqrt{x}} \rightarrow 0$. **0**

Scoring: (a) 1 point for naming the form and applying the rule once, 1 point for the value. (b) 1 point.

Question 7: Marginal revenue (2 points)

a. $R'(x) = 50 - 0.2x$, so $R'(100) = 30$.

b. **Selling the 101st scarf adds about \$30 in revenue.**

Scoring: 1 point for each part; the interpretation must include dollars per scarf.

Question 8: Speeding up and slowing down (2 points)

$v = (t-2)(t-4)$ and $a(t) = 2t-6$. Signs: $(0,2)$: $v > 0, a < 0$; $(2,3)$: $v < 0, a < 0$; $(3,4)$: $v < 0, a > 0$; $(4,\infty)$: $v > 0, a > 0$.

Speed increases on $(2, 3)$ and $(4, \infty)$.

Scoring: 1 point for the sign analysis of v and a , 1 point for both intervals.

Scoring summary

Total

20 points: add the points of every question of this ap calculus contextual test answer key.

How to use this ap calculus contextual test answer key

Use this ap calculus contextual test answer key to score the test fairly and to see which skills need more practice.

