

AP Calculus AB · Chain Rule, Implicit Differentiation and Inverse Functions · Worksheet Answer Key

Common Core AP Calculus AB Unit 3 · mathovore.fr

This ap calculus chain worksheet answer key solves all 24 problems of the chain rule, implicit differentiation and inverse functions worksheet step by step.

Every solution below shows the identification of the outer and inner function or the implicit step, then the arithmetic. Compare the structure of your work, not only the final answer.

AP Calculus AB Common Core: AP Calculus AB Unit 3

AP Calculus Chain Worksheet Answer Key: problems 1-5

Problem 1: Power of a linear function

Outer function u^5 , inner function $u = 3x + 1$. Then $5(3x + 1)^4 \cdot 3$.

$$15(3x + 1)^4$$

Problem 2: Square root of a linear function

Write $y = (4x - 7)^{1/2}$. Then $y' = \frac{1}{2}(4x - 7)^{-1/2} \cdot 4$.

$$y' = \frac{2}{\sqrt{4x - 7}}$$

Problem 3: Sine of a multiple

The derivative of the outer function is $\cos(5x)$ and the inner derivative is 5.

$$5\cos(5x)$$

Problem 4: Exponential with a power

Outer e^u stays e^{x^2} ; inner derivative $2x$.

$$y' = 2x e^{x^2}$$

Problem 5: Log of a quadratic

Outer derivative $\frac{1}{x^2 + 1}$, inner derivative $2x$.

$$\frac{2x}{x^2+1}$$

AP Calculus Chain Worksheet Answer Key: problems 6-10

Problem 6: Cube of cosine

Think $y = (\cos x)^3$. Then $y' = 3(\cos x)^2 \cdot (-\sin x)$.

$$-3\cos^2 x \sin x$$

Problem 7: Product with a chain

Product rule: $f'(x) = 1 \cdot (x^2+3)^4 + x \cdot 4(x^2+3)^3 \cdot 2x$.

Factor out $(x^2+3)^3$: $(x^2+3)^3[(x^2+3)+8x^2]$.

$$f'(x) = (x^2+3)^3(9x^2+3)$$

Problem 8: Composite from given values

$$h(1) = f(g(1)) = f(2) = 7.$$

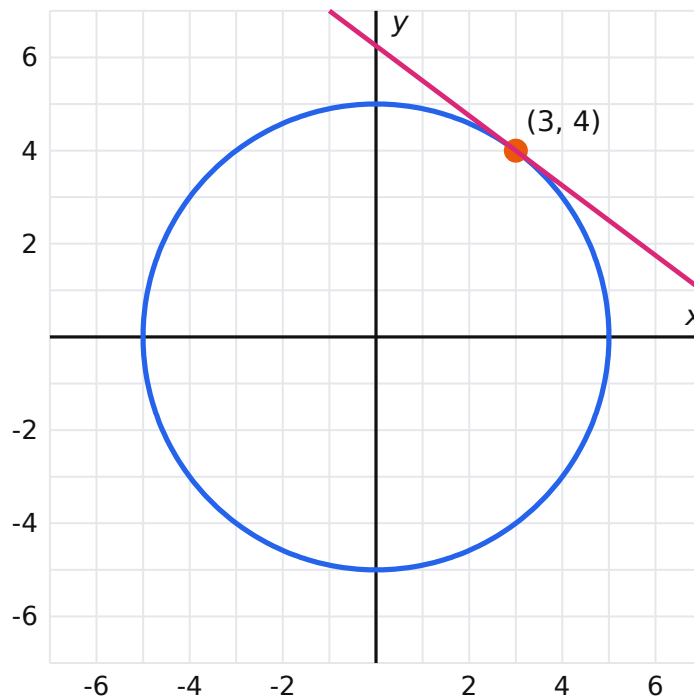
$$h'(1) = f'(g(1)) \cdot g'(1) = f'(2) \cdot 5 = (-4)(5).$$

$$h(1) = 7, \quad h'(1) = -20$$

Problem 9: Slope on a circle

a. $2x + 2y \frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{x}{y}$.

b. At $(3,4)$ the slope is $-\frac{3}{4}$. Then $y-4 = -\frac{3}{4}(x-3)$, so $y = -\frac{3}{4}x + \frac{25}{4}$.



The tangent line at (3, 4) drawn on the circle.

Problem 10: Implicit slope with a mixed term

Check: $1+2+4=7$. Differentiate: $2x+y+x\frac{dy}{dx}+2y\frac{dy}{dx}=0$.

Solve: $\frac{dy}{dx} = -\frac{2x+y}{x+2y}$. At (1,2): $-\frac{4}{5}$.

$$-\frac{4}{5}$$

AP Calculus Chain Worksheet Answer Key: problems 11-15

Problem 11: Tangent line to a cubic curve

Check: $1+8=9$. Differentiate: $3x^2+3y^2\frac{dy}{dx}=0$, so $\frac{dy}{dx} = -\frac{x^2}{y^2}$. At (1,2) the slope is $-\frac{1}{4}$.

$$y-2 = -\frac{1}{4}(x-1), \text{ or } y = -\frac{1}{4}x + \frac{9}{4}.$$

Problem 12: Product of x and y terms

Check: $2+8=10$. Differentiate with the product rule: $2xy+x^2\frac{dy}{dx}+3y^2\frac{dy}{dx}=0$.

$$\frac{dy}{dx} = -\frac{2xy}{x^2+3y^2}. \text{ At (1,2): } -\frac{4}{1+12}.$$

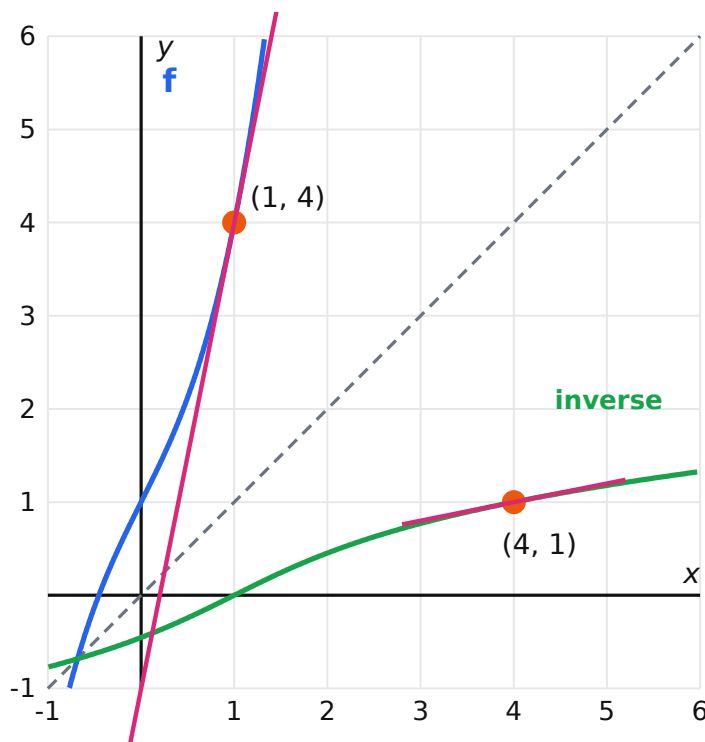
$$-\frac{4}{13}$$

Problem 13: Slope of an inverse function

Find α with $f(\alpha) = 4$: $f(1) = 1 + 2 + 1 = 4$, so $\alpha = 1$.

$f'(x) = 3x^2 + 2$, so $f'(1) = 5$.

$g'(4) = \frac{1}{f'(1)}$ gives $g'(4) = \frac{1}{5}$.



$f(x) = x^3 + 2x + 1$ and its inverse are mirror images across $y = x$. The tangent slopes at $(1, 4)$ and $(4, 1)$ are 5 and $1/5$.

Problem 14: Inverse of an exponential sum

Solve $f(\alpha) = 1$: $f(0) = 1 + 0 = 1$, so $\alpha = 0$.

$f'(x) = e^x + 1$, so $f'(0) = 2$.

$(f^{-1})'(1) = \frac{1}{2}$

Problem 15: Arctangent of a multiple

$$\frac{1}{1+(3x)^2} \cdot 3.$$

$$\frac{3}{1+9x^2}$$

AP Calculus Chain Worksheet Answer Key: problems 16-20

Problem 16: Arcsine of a power

$$y' = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x.$$

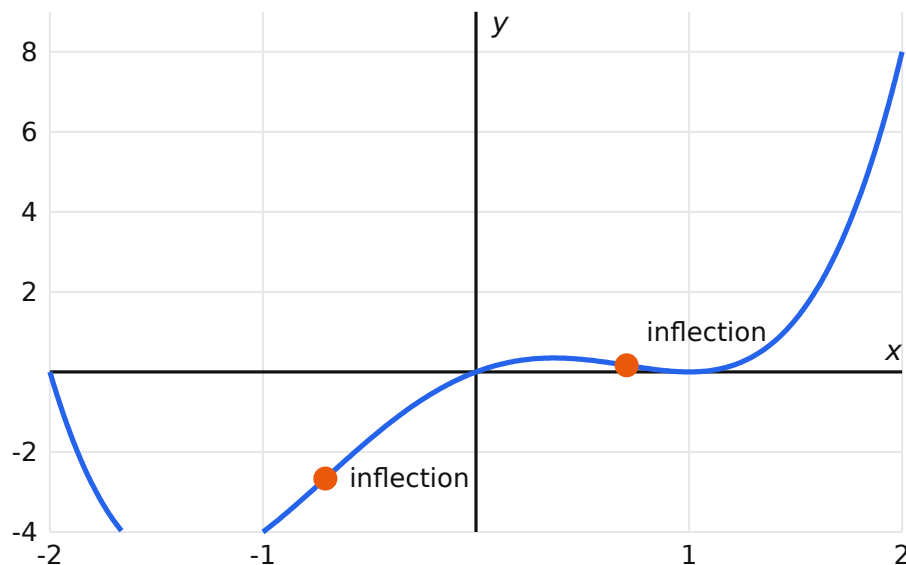
$$y' = \frac{2x}{\sqrt{1-x^4}}$$

Problem 17: Second derivative of a polynomial

$$f'(x) = 4x^3 - 6x + 2, \text{ so } f''(x) = 12x^2 - 6.$$

$$f''(2) = 12(4) - 6 = 42.$$

$12x^2 - 6 = 0$ gives $x^2 = \frac{1}{2}$, so $x = \pm \frac{1}{\sqrt{2}} \approx \pm 0.707$. The second derivative changes sign at both values.



$f(x) = x^4 - 3x^2 + 2x$. Concavity changes at $x = \pm 1/\sqrt{2} \approx \pm 0.707$.

Problem 18: Second derivative with a product

$$y' = e^{2x} + 2xe^{2x} = e^{2x}(1+2x).$$

$$y'' = 2e^{2x}(1+2x) + e^{2x} \cdot 2 = e^{2x}(4+4x).$$

$$y'' = 4(1+x)e^{2x}$$

Problem 19: Second derivative on a circle

$$\frac{d^2y}{dx^2} = -\frac{y - x \frac{dy}{dx}}{y^2}. \text{ Substitute } \frac{dy}{dx} = -\frac{x}{y}: -\frac{y + \frac{x^2}{y}}{y^2} = -\frac{y^2 + x^2}{y^3} = -\frac{25}{y^3}.$$

At $(3,4)$: $-\frac{25}{64}$. The negative sign means the circle is concave down at that point.

$$-\frac{25}{64}$$

Problem 20: Town population growth

$$P'(t) = 1200(0.03)e^{0.03t} = 36e^{0.03t}.$$

$$P'(10) = 36e^{0.3} \approx 36(1.34986) \approx 48.6.$$

In 2030 the population is growing at about 48.6 residents per year.

AP Calculus Chain Worksheet Answer Key: problems 21-24

Problem 21: Growing balloon

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \text{ by the chain rule. At } t = 4: r = 2 + 0.5(4) = 4 \text{ and } \frac{dr}{dt} = 0.5.$$

$$\frac{dV}{dt} = 4\pi(16)(0.5) = 32\pi.$$

$$32\pi \approx 100.5 \text{ in}^3/\text{s}$$

Problem 22: Composites from a table

$$h'(1) = f'(g(1)) \cdot g'(1) = f'(2) \cdot g'(1) = 4 \cdot (-1) = -4.$$

$$k'(3) = g'(f(3)) \cdot f'(3) = g'(2) \cdot f'(3) = 5 \cdot (-3) = -15.$$

$$h'(1) = -4, \quad k'(3) = -15$$

Problem 23: Choosing the procedure

$$\text{Product rule with a chain inside: } f'(x) = \frac{1}{x}\sqrt{x^2+1} + \ln x \cdot \frac{x}{\sqrt{x^2+1}}.$$

$$\text{At } x = 1, \ln 1 = 0, \text{ so the second term vanishes: } f'(1) = \frac{1}{1}\sqrt{2}.$$

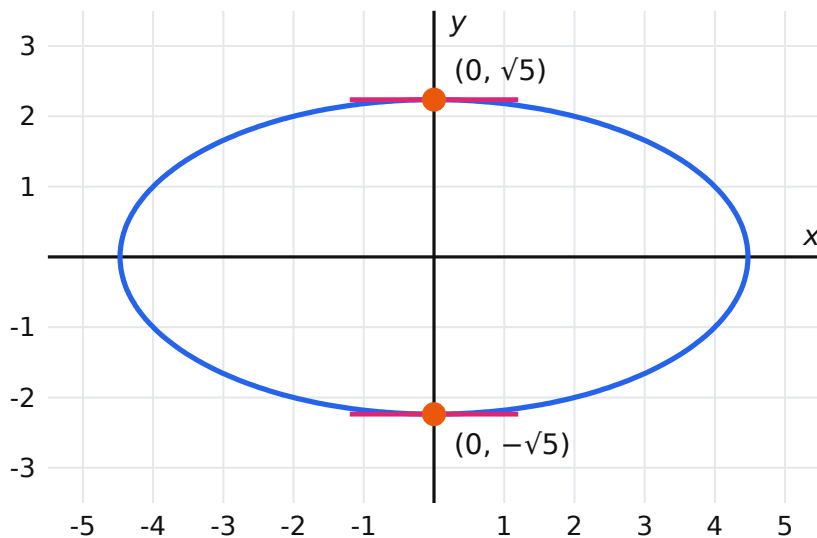
$$f'(1) = \sqrt{2} \approx 1.414$$

Problem 24: Horizontal tangents on an ellipse

$$2x + 8y \frac{dy}{dx} = 0, \text{ so } \frac{dy}{dx} = -\frac{x}{4y}.$$

$$\text{A horizontal tangent needs } \frac{dy}{dx} = 0, \text{ so } x = 0 \text{ (with } y \neq 0). \text{ Then } 4y^2 = 20 \text{ and } y = \pm\sqrt{5}.$$

The points are $(0, \sqrt{5})$ and $(0, -\sqrt{5})$.



Horizontal tangents at the top and bottom of the ellipse, at $y = \pm\sqrt{5} \approx \pm 2.24$.

How to use this ap calculus chain worksheet answer key

Read this ap calculus chain worksheet answer key after trying each problem yourself, and redo any problem you missed.

