

AP Calculus AB · Chain Rule, Implicit Differentiation and Inverse Functions · Test Answer Key

Common Core AP Calculus AB Unit 3 · mathovore.fr

This ap calculus chain test answer key gives the full solution and the scoring guide for each of the 8 test questions on chain rule, implicit differentiation and inverse functions.

Worked solutions and a scoring guide for the Unit 3 test. Points are awarded for each correct step, so partial credit applies. The total is 20 points.

AP Calculus AB Common Core: AP Calculus AB Unit 3

AP Calculus Chain Test Answer Key: questions 1-5

Question 1: Chain rule derivatives (2 points)

- a. $-6x\sin(3x^2)$
- b. $\cos x e^{\sin x}$

Scoring: 1 point for each correct derivative, including the inner derivative factor.

Question 2: Composites from a table (3 points)

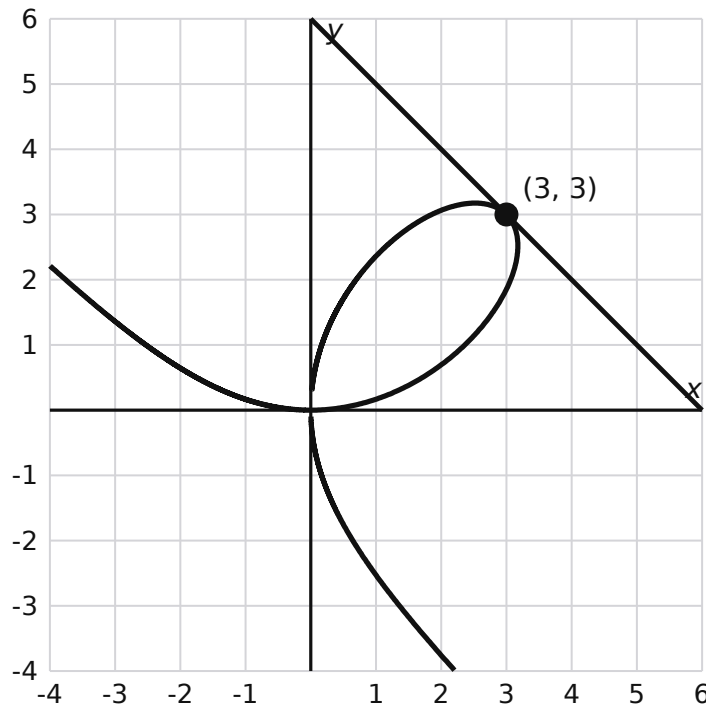
- a. $h'(1) = f'(g(1)) g'(1) = f'(3) g'(1) = 3(-1) = -3$
- b. $k'(2) = g'(f(2)) f'(2) = g'(4) f'(2) = (-3)(-2) = 6$
- c. $p'(3) = f'(3)g(3) + f(3)g'(3) = 3(4) + 1(2) = 14$

Scoring: 1 point per part. In (a) and (b), award the point only if the inner value is evaluated first.

Question 3: Implicit differentiation on a curve (3 points)

- a. $3x^2 + 3y^2 \frac{dy}{dx} = 6y + 6x \frac{dy}{dx}$. Collect: $(3y^2 - 6x) \frac{dy}{dx} = 6y - 3x^2$. $\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$
- b. At (3,3): $\frac{6-9}{9-6} = -1$. $y-3 = -(x-3)$, or $y = -x+6$.

Scoring: (a) 1 point for differentiating both sides correctly (product rule on $6xy$, chain factor on y^3), 1 point for solving for $\frac{dy}{dx}$. (b) 1 point for slope and equation.

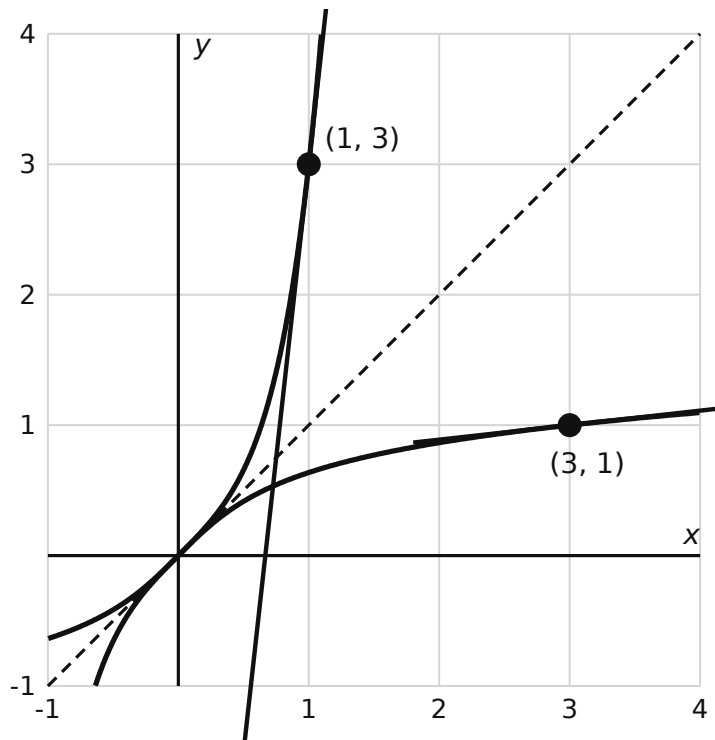


The curve $x^3 + y^3 = 6xy$. The tangent line at $(3, 3)$ is drawn.

Question 4: Derivative of an inverse (3 points)

- $f'(x) = 5x^4 + 3x^2 + 1 \geq 1 > 0$ for all x , so f is increasing and one-to-one. **So f has an inverse.**
- $f(1) = 1 + 1 + 1 = 3$, so $g(3) = 1$. $f'(1) = 5 + 3 + 1 = 9$. $g'(3) = \frac{1}{9}$

Scoring: (a) 1 point for showing the derivative is positive for every x . (b) 1 point for finding $f(1) = 3$, 1 point for the reciprocal $1/9$.



The graph of f and its reflection across $y = x$. Tangent slopes: 9 at $(1, 3)$ and $1/9$ at $(3, 1)$.

Question 5: Inverse trig derivatives (2 points)

- $\frac{2x}{1+x^4}$
- $\frac{3}{\sqrt{1-9x^2}}$

Scoring: 1 point each. No credit if the inner derivative factor is missing.

AP Calculus Chain Test Answer Key: questions 6-8

Question 6: Second derivative of a product (2 points)

- $f'(x) = e^{-x} - xe^{-x} = e^{-x}(1-x)$; $f''(x) = -e^{-x}(1-x) - e^{-x} = e^{-x}(x-2)$. $f''(x) = (x-2)e^{-x}$
- $e^{-x} > 0$ always, so $x = 2$.

Scoring: (a) 1 point for the correct simplified second derivative. (b) 1 point for $x = 2$.

Question 7: Second derivative, implicitly (2 points)

- $y + x \frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{y}{x}$. At $(3, 4)$: $-\frac{4}{3}$.
- $\frac{d^2y}{dx^2} = -\frac{x \frac{dy}{dx} - y}{x^2} = -\frac{-y - y}{x^2} = \frac{2y}{x^2}$. At $(3, 4)$: $\frac{8}{9}$.

Scoring: 1 point for each part. Check: $y = 12x^{-1}$ gives $y'' = 24x^{-3} = \frac{24}{27} = \frac{8}{9}$.

Question 8: Bacteria growth rate (3 points)

- $B'(t) = 2000e^{0.5\sqrt{t}} \cdot 0.5 \cdot \frac{1}{2\sqrt{t}} = \frac{500e^{0.5\sqrt{t}}}{\sqrt{t}}$

b. $B'(4) = \frac{500e^1}{2} = 250e \approx 679.6$. **About 680 bacteria per hour** at $t = 4$.

Scoring: (a) 1 point for the chain on the exponent, 1 point for the derivative of the square root.
(b) 1 point for the value with units.

Scoring summary

Total
20 points: add the points of every question of this ap calculus chain test answer key.

How to use this ap calculus chain test answer key

Use this ap calculus chain test answer key to score the test fairly and to see which skills need more practice.

