

# AP Calculus AB · Chain Rule, Implicit Differentiation and Inverse Functions · Test

Common Core AP Calculus AB Unit 3 · mathovore.fr

Name: \_\_\_\_\_ Date: \_\_\_\_/\_\_\_\_/\_\_\_\_ Score: \_\_\_\_\_

This ap calculus chain implicit test checks chain rule, implicit differentiation and inverse functions in 45 minutes, with 8 questions for a total of 20 points.

This free-response test covers Unit 3 of AP Calculus AB. It has eight questions worth 20 points. Show all work, write derivatives in correct notation, and include units in context problems. A calculator is allowed for decimal approximations only.

## Test information

- **Grade:** AP Calculus AB
- **Chapter:** Chain Rule, Implicit Differentiation and Inverse Functions
- **Common Core:** AP Calculus AB Unit 3
- **Time:** 45 minutes
- **Total:** 20 points (Question 1: 2 pts; Question 2: 3 pts; Question 3: 3 pts; Question 4: 3 pts; Question 5: 2 pts; Question 6: 2 pts; Question 7: 2 pts; Question 8: 3 pts)

## AP Calculus Chain Implicit Test: questions 1-5

### Question 1: Chain rule derivatives (2 points)

- Find  $\frac{d}{dx} \cos(3x^2)$ . (1 point)
- Find  $\frac{d}{dx} e^{\sin x}$ . (1 point)

### Question 2: Composites from a table (3 points)

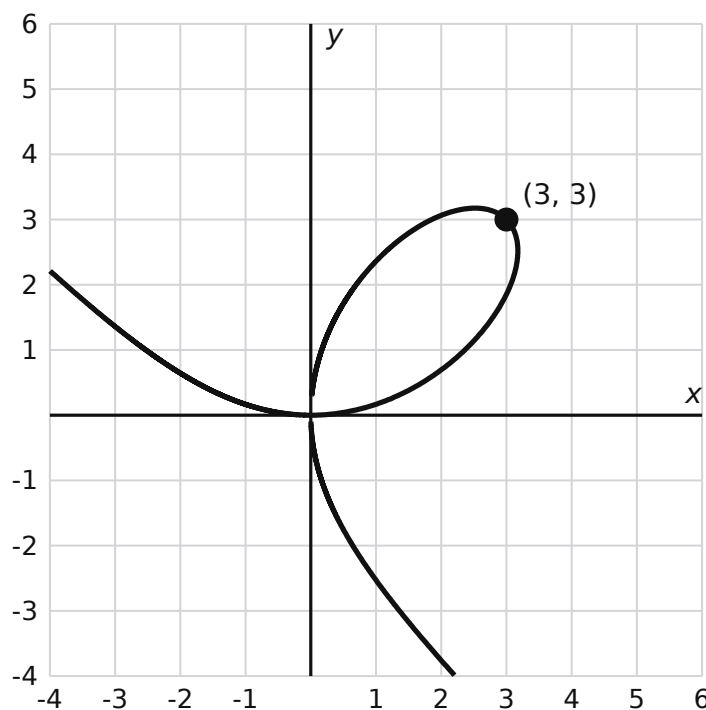
The table gives values of  $f$ ,  $g$  and their derivatives.

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|-----|--------|---------|--------|---------|
| 1   | 2      | 5       | 3      | -1      |
| 2   | 4      | -2      | 1      | 7       |
| 3   | 1      | 3       | 4      | 2       |
| 4   | 3      | 6       | 2      | -3      |

- Let  $h(x) = f(g(x))$ . Find  $h'(1)$ . (1 point)
- Let  $k(x) = g(f(x))$ . Find  $k'(2)$ . (1 point)
- Let  $p(x) = f(x)g(x)$ . Find  $p'(3)$ . (1 point)

### Question 3: Implicit differentiation on a curve (3 points)

The curve  $x^3 + y^3 = 6xy$  is shown.

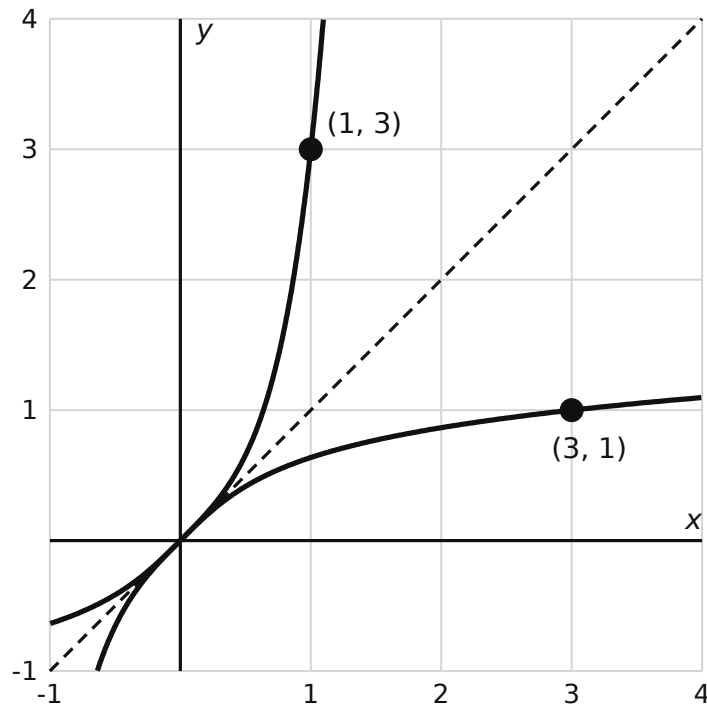


The curve  $x^3 + y^3 = 6xy$ .

- Find  $\frac{dy}{dx}$ . (2 points)
- Write an equation of the tangent line at  $(3,3)$ . (1 point)

### Question 4: Derivative of an inverse (3 points)

Let  $f(x) = x^5 + x^3 + x$  and let  $g$  be its inverse function.



The graph of  $f$  and its reflection across  $y = x$ .

- Show that  $f$  has an inverse function. (1 point)
- Find  $g'(3)$ . (2 points)

### Question 5: Inverse trig derivatives (2 points)

- Find  $\frac{d}{dx} \tan^{-1}(x^2)$ . (1 point)
- Find  $\frac{d}{dx} \sin^{-1}(3x)$ . (1 point)

## AP Calculus Chain Implicit Test: questions 6-8

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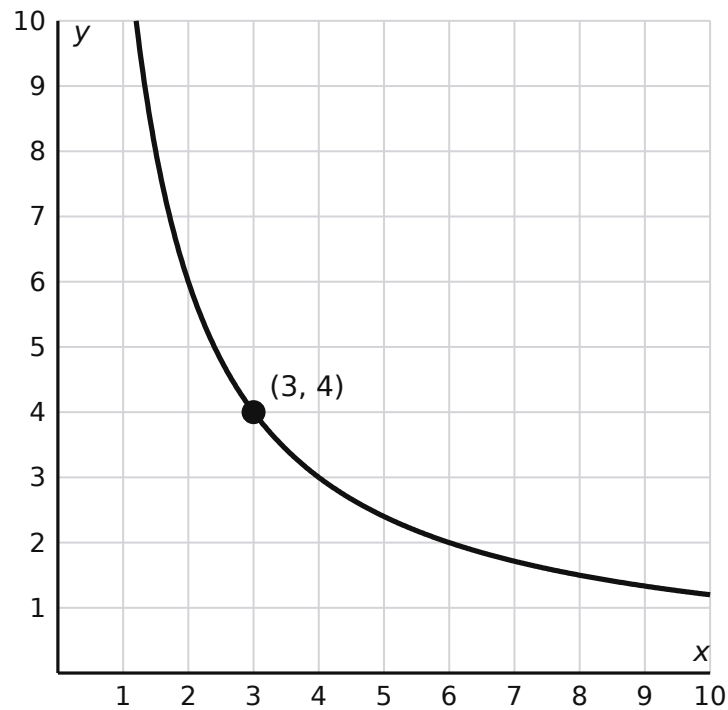
### Question 6: Second derivative of a product (2 points)

Let  $f(x) = xe^{-x}$ .

- Find  $f''(x)$ . (1 point)
- Find all  $x$  for which  $f''(x) = 0$ . (1 point)

### Question 7: Second derivative, implicitly (2 points)

The curve  $xy = 12$  passes through  $(3, 4)$ .



The curve  $xy = 12$  in the first quadrant.

- Find  $\frac{dy}{dx}$  at  $(3, 4)$ . (1 point)
- Find  $\frac{d^2y}{dx^2}$  at  $(3, 4)$ . (1 point)

### Question 8: Bacteria growth rate (3 points)

A culture contains  $B(t) = 2000e^{0.5\sqrt{t}}$  bacteria after  $t$  hours, for  $t > 0$ .

- Find  $B'(t)$ . (2 points)
- Find  $B'(4)$  and state its units. (1 point)

### How to use this ap calculus chain implicit test

Take this ap calculus chain implicit test without notes, then grade it with the test answer key out of 20 points.

