

AP Calculus AB · Chain Rule, Implicit Differentiation and Inverse Functions · Lesson

Common Core AP Calculus AB Unit 3 · mathovore.fr

This ap calculus chain implicit lesson explains chain rule, implicit differentiation and inverse functions in one printable page, with definitions, a method and a worked example.

The chain rule differentiates a composite function: the derivative of $f(g(x))$ is $f'(g(x))$ times $g'(x)$. Implicit differentiation finds dy/dx when y is tangled with x , and the inverse rule gives $(f^{-1})'(b) = 1/f'(a)$. Together with higher-order derivatives, they form the core of AP Calculus AB Unit 3.

AP Calculus AB Common Core: AP Calculus AB Unit 3

AP Calculus Chain Implicit Lesson: key facts

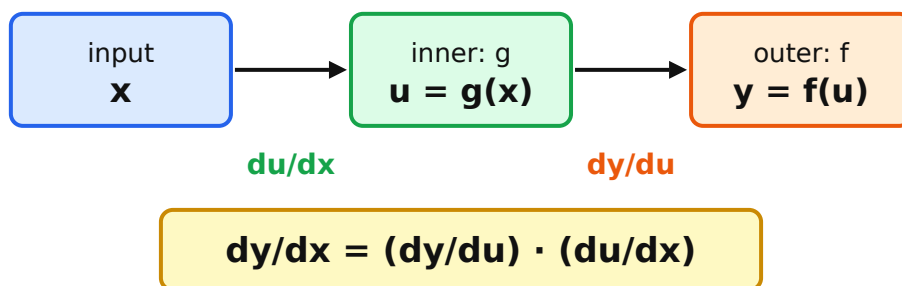
- Chain rule: outer derivative evaluated at the inside, times the inner derivative.
- Implicit differentiation: every y -term gets a factor of dy/dx from the chain rule.
- Inverse functions: if $f(a) = b$, then $(f^{-1})'(b) = 1/f'(a)$.
- The second derivative f'' is the derivative of f' and measures how the slope changes.

What is the chain rule?

The **chain rule** differentiates a function that sits inside another function. It is the most heavily tested derivative rule on the AP Calculus AB exam, because nearly every real function is a composite.

Chain Rule

If $y = f(g(x))$, then $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$. Differentiate the outer function, leave the inside alone, then multiply by the derivative of the inside.



The derivative of a composite multiplies the outer rate by the inner rate.

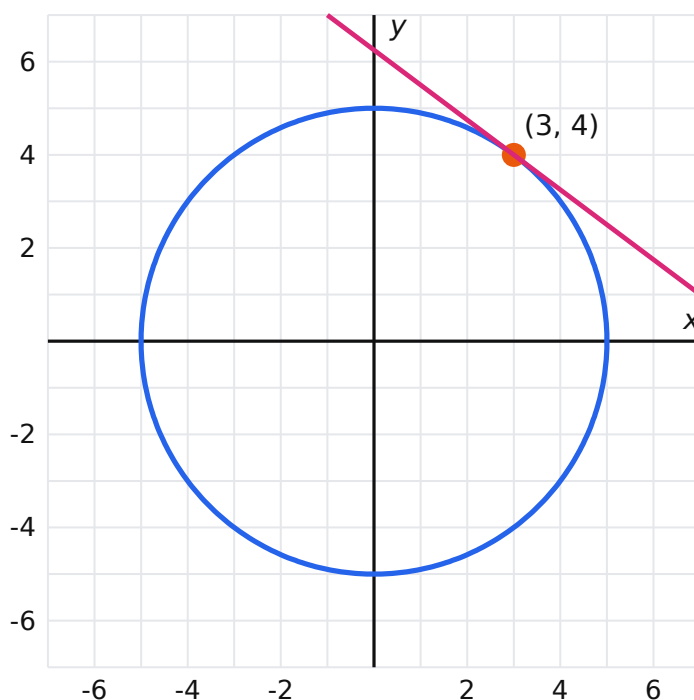
Example. For $y = (3x^2 + 1)^4$, the outer function is u^4 and the inner function is $3x^2 + 1$. Then $\frac{dy}{dx} = 4(3x^2 + 1)^3 \cdot 6x = 24x(3x^2 + 1)^3$.

How does implicit differentiation work?

When y is tangled with x in an equation such as $x^2 + y^2 = 25$, treat y as a function of x and differentiate both sides.

1. Differentiate every term with respect to x . Each y term gets a factor $\frac{dy}{dx}$ from the chain rule.
2. Move all terms containing $\frac{dy}{dx}$ to one side.
3. Factor out $\frac{dy}{dx}$ and divide.

For the circle: $2x + 2y \frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{x}{y}$. At $(3, 4)$ the slope is $-\frac{3}{4}$.



The tangent at (3, 4) is perpendicular to the radius, which has slope $4/3$, so the tangent slope is $-3/4$.

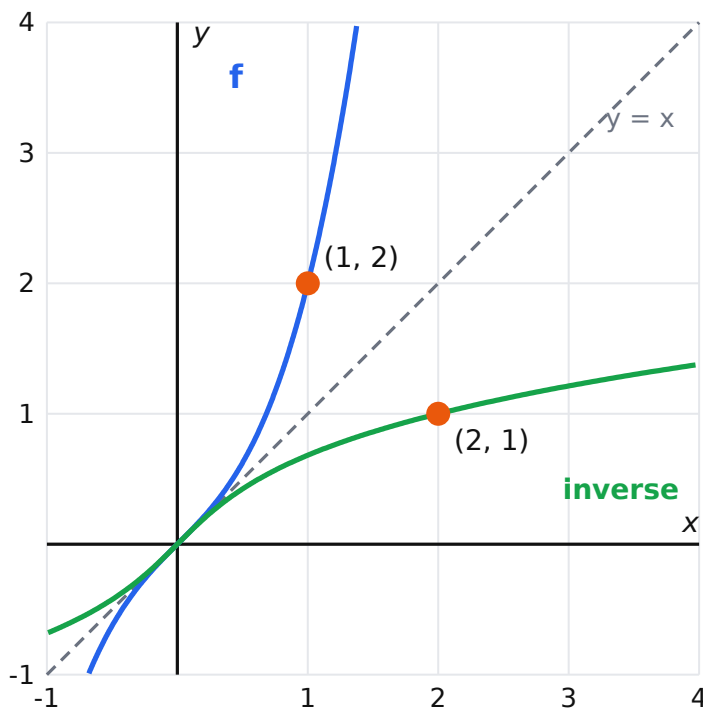
How do you differentiate an inverse function?

Derivative of an Inverse

If $f(a) = b$ and $f'(a) \neq 0$, then $(f^{-1})'(b) = \frac{1}{f'(a)}$.

For $f(x) = x^3 + x$: $f(1) = 2$ and $f'(1) = 4$, so $(f^{-1})'(2) = \frac{1}{4}$. The inverse trig derivatives follow from this rule:

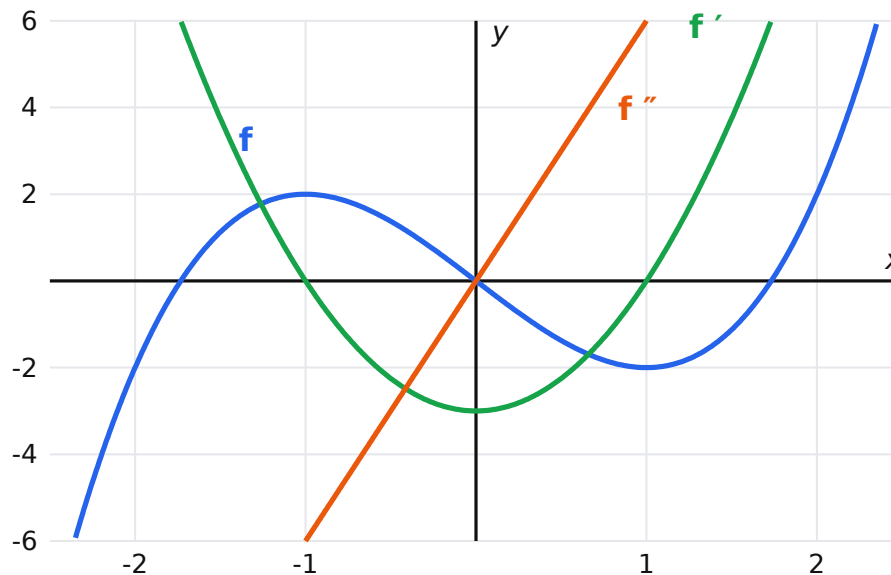
Function	Derivative
$\frac{d}{dx} \sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx} \cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx} \tan^{-1} x$	$\frac{1}{1+x^2}$



An inverse is the mirror image of f across the line $y = x$. The slopes at mirrored points are reciprocals.

What are higher-order derivatives?

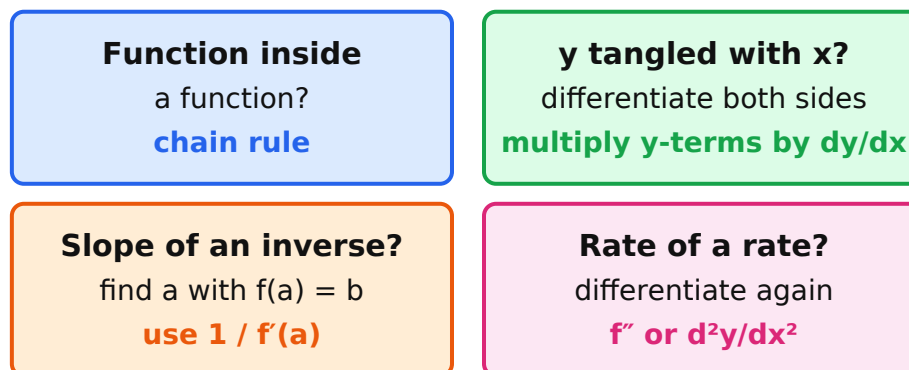
Differentiating f' gives the **second derivative** $f''(x)$, also written $\frac{d^2 y}{dx^2}$. It measures how fast the slope itself changes. For $f(x) = x^3 - 3x$, $f'(x) = 3x^2 - 3$ and $f''(x) = 6x$.



$f(x) = x^3 - 3x$ with $f'(x) = 3x^2 - 3$ and $f''(x) = 6x$. Each derivative is the slope function of the one before.

How do you choose which derivative procedure to use?

Read the structure first. Products need the product rule, quotients need the quotient rule, and anything nested needs the chain rule. Many problems need two or three at once.



Match the situation to the tool before differentiating.

Chain Rule, Implicit Differentiation and Inverse Functions: frequently asked questions

What is the chain rule in calculus?

The chain rule gives the derivative of a function inside another function. For $y = f(g(x))$, the derivative is $f'(g(x))$ times $g'(x)$. In words: differentiate the outside, keep the inside, then multiply by the derivative of the inside.

When should I use implicit differentiation?

Use it when an equation links x and y so that solving for y is hard or impossible, such as $x^3 + y^3 = 6xy$. Differentiate both sides with respect to x , attach dy/dx to every y -term, and solve for dy/dx .

How do you find the derivative of an inverse function?

Find the input a with $f(a) = b$, then take 1 divided by $f'(a)$. You never need a formula for the inverse. The slopes at mirrored points are reciprocals.

What does the second derivative tell you?

The second derivative is the rate of change of the slope. When f'' is positive the graph is concave up, and when it is negative the graph is concave down.

How do I know which derivative rule to use?

Look at the outermost operation. A sum splits into pieces, a product needs the product rule, a quotient needs the quotient rule, and a function inside another function needs the chain rule.

How to use this ap calculus chain implicit lesson

Use this ap calculus chain implicit lesson before the worksheet, then check what you learned with the quiz.

