

AP Calculus AB · Applications of Integration · Worksheet

Common Core AP Calculus AB Unit 8 · mathovore.fr

Name: _____ Date: ____/____/____ Score: _____

These 24 problems practice AP Calculus AB Unit 8: average value, particle motion, area between curves, volumes with cross sections, discs and washers, and accumulation with rates. They move from one-step integrals to multi-part word problems.

Sketch each region, label limits, and write the integral before evaluating it.

AP Calculus ABCommon Core: AP Calculus AB Unit 8

Problem 1: Average value of a polynomial

Determine the average value of $g(x) = 3x^2 - 2x$ over the interval $[1, 4]$.

Problem 2: Average value of sine

Find the average value of $f(x) = \sin x$ on $[0, \pi]$. Give an exact answer and a decimal rounded to four places.

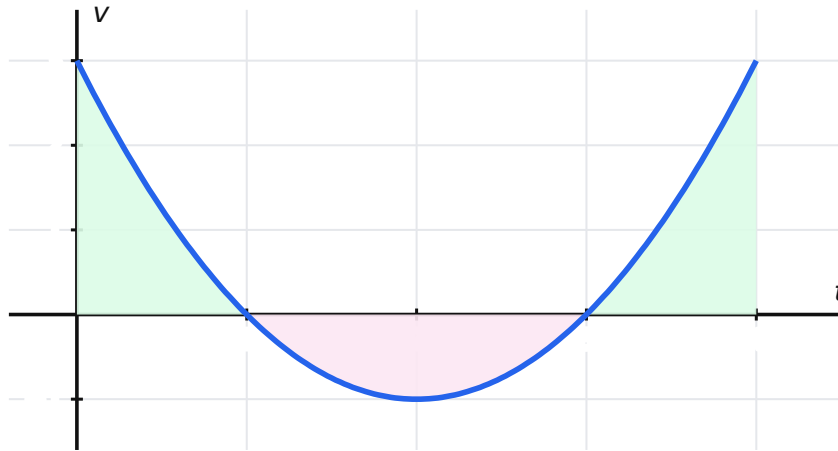
Problem 3: Average value of a quadratic

Find the average value of $f(x) = 3x^2 - 2x + 1$ on $[1, 3]$.

Problem 4: Where the average value occurs

For $f(x) = x^2$ on $[0, 3]$, find the value of c in $(0, 3)$ at which $f(c)$ equals the average value.

Problem 5: Displacement and distance



Velocity $v(t) = t^2 - 4t + 3$.

A particle moves along a line with velocity $v(t) = t^2 - 4t + 3$ for $0 \leq t \leq 4$. Find the displacement and the total distance traveled.

Problem 6: Final position

A particle has velocity $v(t) = 3t^2 - 6t$ and position $s(0) = 2$. Find $s(2)$.

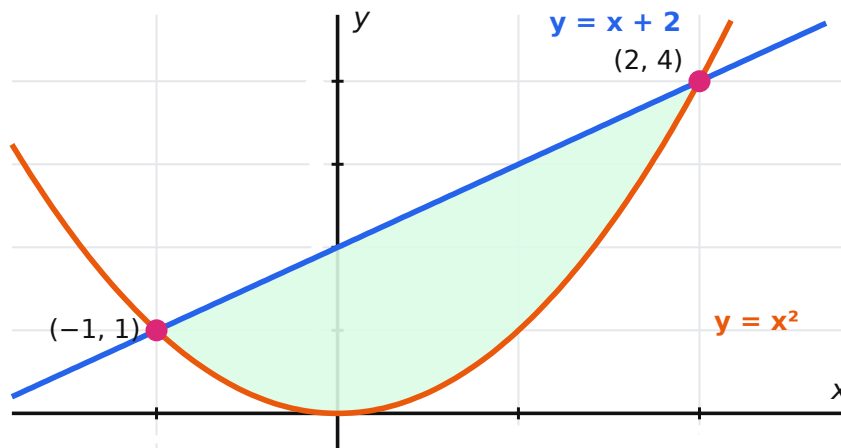
Problem 7: Cosine motion

A particle has velocity $v(t) = \cos t$ for $0 \leq t \leq \pi$. Find the displacement and the total distance.

Problem 8: Area between a parabola and a line

Find the area of the region enclosed by the line $y = 3x + 4$ and the parabola $y = x^2$.

Problem 9: Area with two intersection points



The region between $y = x+2$ and $y = x^2$; each slice has length top minus bottom.

Find the area of the region bounded by $y = x+2$ and $y = x^2$, shown above.

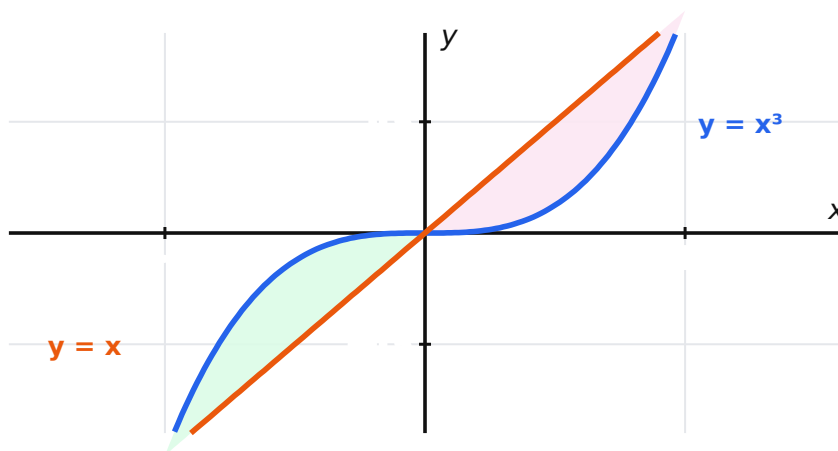
Problem 10: Area between exponentials

Find the area between $y = e^x$ and $y = e^{-x}$ for $0 \leq x \leq \ln 3$.

Problem 11: Area using y

Find the area of the region bounded by $x = y^2$ and $x = 2y$ by integrating with respect to y .

Problem 12: Three intersection points



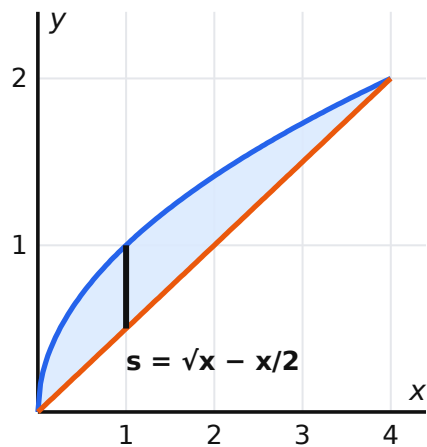
Two regions enclosed by $y = x^3$ and $y = x$.

Find the total area enclosed by $y = x^3$ and $y = x$.

Problem 13: Sine and cosine area

Find the area enclosed by $y = \sin x$ and $y = \cos x$ between consecutive intersections at $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.

Problem 14: Square cross sections



Cross section: square



$$\text{area} = s^2 = (\sqrt{x} - x/2)^2$$

slice at $x = 1$: $s = 0.5$

A square cross section stands on each slice of the base region.

The base of a solid is the region between $y = \sqrt{x}$ and $y = \frac{x}{2}$. Cross sections perpendicular to the x -axis are squares. Find the volume.

Problem 15: Semicircle cross sections

The base of a solid is the region bounded by $y = \sqrt{x}$, the x -axis and $x = 4$. Cross sections perpendicular to the x -axis are semicircles whose diameters lie in the base. Find the volume.

Problem 16: Triangle cross sections

The base of a solid is the region between $y = 6x - x^2$ and the x -axis. Cross sections perpendicular to the x -axis are equilateral triangles with a side in the base. Find the volume.

Problem 17: Disc method about the x-axis

The region bounded by $y = \sqrt{x}$, the x -axis and $x = 4$ is revolved about the x -axis. Find the volume.

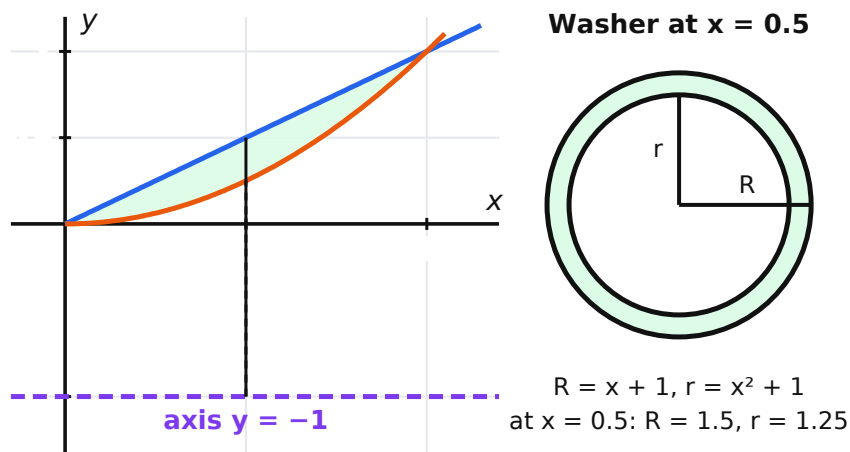
Problem 18: Disc method about the y-axis

The region bounded by $y = x^2$, the y -axis and $y = 4$ is revolved about the y -axis. Find the volume.

Problem 19: Washer about the x-axis

The region between $y = x$ and $y = x^3$ in the first quadrant is revolved about the x -axis. Find the volume.

Problem 20: Washer about a horizontal line



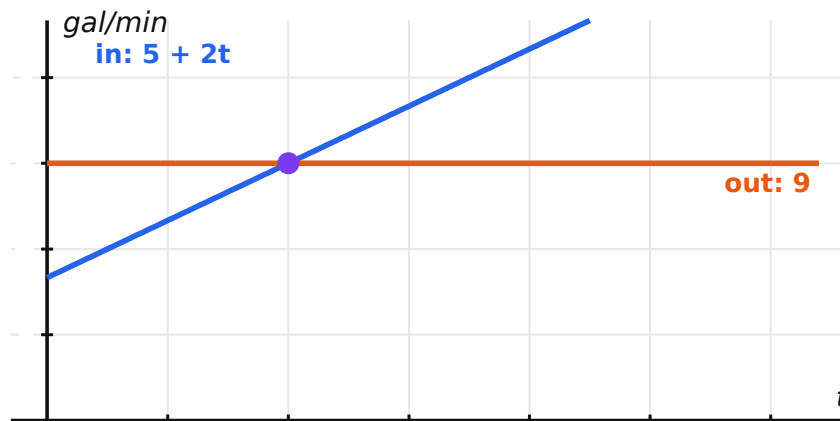
Revolving about $y = -1$: outer radius $x+1$, inner radius x^2+1 .

The region between $y=x$ and $y=x^2$ is revolved about the line $y=-1$. Find the volume.

Problem 21: Rotation about a vertical line

The region bounded by $y=\sqrt{x}$, the x -axis and $x=4$ is revolved about the line $x=4$. Find the volume.

Problem 22: Water in a tank



Inflow and outflow rates for the tank.

A tank holds 100 gallons at $t=0$. Water flows in at $5+2t$ gallons per minute and out at 9 gallons per minute for $0 \leq t \leq 6$. Find the amount at $t=6$, and find when the amount is least.

Problem 23: Oil leak

Oil leaks from a tank at a rate of $12e^{-0.2t}$ gallons per hour. How many gallons leak during the first 5 hours? Round to the nearest hundredth.

Problem 24: Average temperature

The temperature in a greenhouse is $T(t) = 60 + 10\sin\left(\frac{\pi t}{12}\right)$ degrees Fahrenheit, $0 \leq t \leq 12$, where t is in hours after 6 a.m. Find the average temperature on this interval, to the nearest tenth of a degree.