

AP Calculus AB · Applications of Integration · Test Answer Key

Common Core AP Calculus AB Unit 8 · mathovore.fr

This ap calculus applications test answer key gives the full solution and the scoring guide for each of the 8 test questions on applications of integration.

Full solutions and a scoring guide for the applications of integration test. Points follow AP-style rubrics: one for the setup, one for the limits or integrand, and one for the final value, so partial credit is explained for every question.

AP Calculus ABCommon Core: AP Calculus AB Unit 8

AP Calculus Applications Test Answer Key: questions 1-5

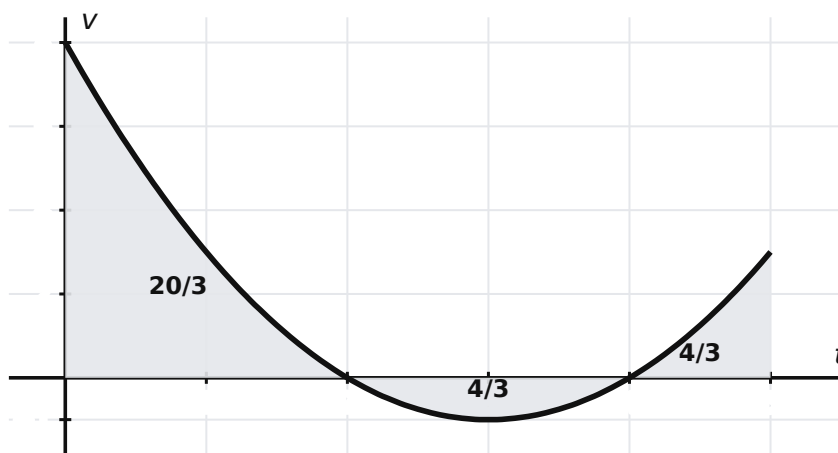
Question 1: Average value (2 points)

- a. $\int_0^2 x^3 dx = 4$, so $f_{\text{avg}} = \frac{4}{2} = 2$.
b. $c^3 = 2$, so $c = 2^{1/3} \approx 1.260$.

(a) 2; (b) $c \approx 1.260$

Scoring: 1 point for the average value 2; 1 point for $c \approx 1.260$.

Question 2: Particle motion (3 points)



Areas of the three regions.

$v = (t-2)(t-4)$. Antiderivative: $\frac{t^3}{3} - 3t^2 + 8t$.

a. $\int_0^5 v \, dt = \frac{125}{3} - 75 + 40 = \frac{20}{3}$, so $s(5) = 1 + \frac{20}{3} = \frac{23}{3}$.

b. $\int_0^2 v \, dt = \frac{20}{3}$, $\int_2^4 v \, dt = -\frac{4}{3}$, $\int_4^5 v \, dt = \frac{4}{3}$. Distance $= \frac{20}{3} + \frac{4}{3} + \frac{4}{3} = \frac{28}{3}$.

(a) $\frac{23}{3}$; (b) $\frac{28}{3}$

Scoring: (a) 1 point. (b) 1 point for splitting at $t=2$ and $t=4$; 1 point for $\frac{28}{3}$.

Question 3: Area between curves (2 points)

Intersections: $6x - x^2 = x^2$ gives $2x(3-x) = 0$, so $x=0$ and $x=3$. The parabola $6x - x^2$ is on top.

$$A = \int_0^3 (6x - 2x^2) \, dx = \left[3x^2 - \frac{2x^3}{3} \right]_0^3 = 27 - 18 = 9.$$

9

Scoring: 1 point for the correct limits and integrand; 1 point for the value 9.

Question 4: Area using y (2 points)

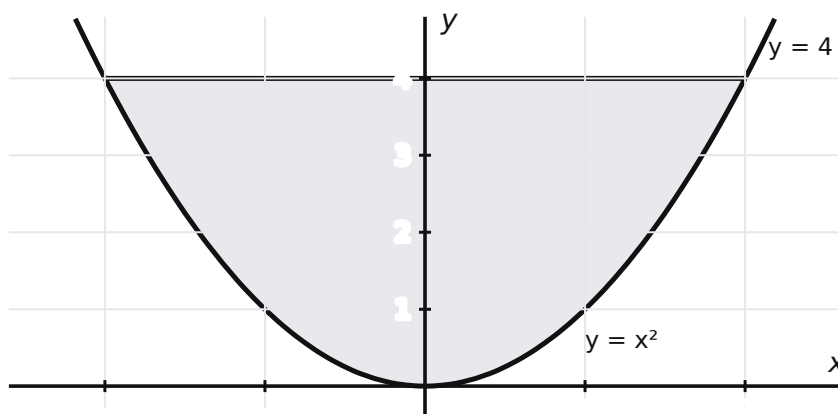
$y^2 = 6 - y$ gives $y^2 + y - 6 = 0$, so $y = -3$ or $y = 2$. The line $x = 6 - y$ is the right curve.

$$A = \int_{-3}^2 (6 - y - y^2) \, dy = \left[6y - \frac{y^2}{2} - \frac{y^3}{3} \right]_{-3}^2 = \frac{22}{3} - \left(-\frac{27}{2} \right) = \frac{125}{6}.$$

$\frac{125}{6}$

Scoring: 1 point for the limits and integrand; 1 point for $\frac{125}{6}$.

Question 5: Semicircle cross sections (3 points)



The base of the solid.

The diameter at x is $4-x^2$, for $-2 \leq x \leq 2$. The area is $\frac{\pi}{2} \left(\frac{4-x^2}{2} \right)^2 = \frac{\pi}{8} (4-x^2)^2$.

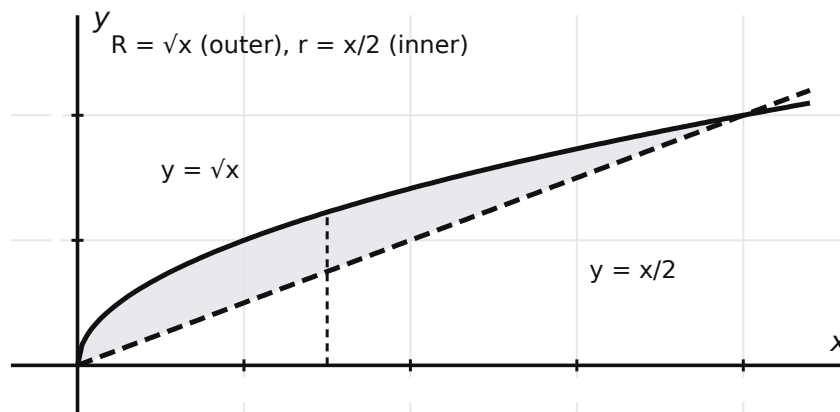
$$V = \frac{\pi}{8} \int_{-2}^2 (4-x^2)^2 dx = \frac{\pi}{8} \cdot \frac{512}{15} = \frac{64\pi}{15} \approx 13.404.$$

$$\frac{64\pi}{15}$$

Scoring: 1 point for the semicircle area $\frac{\pi}{8}(4-x^2)^2$; 1 point for the integral setup with limits; 1 point for the value.

AP Calculus Applications Test Answer Key: questions 6-8

Question 6: Washer method (3 points)



The region to revolve about the x -axis.

Intersections: $\sqrt{x} = \frac{x}{2}$ at $x = 0$ and $x = 4$. Outer radius $R = \sqrt{x}$, inner radius $r = \frac{x}{2}$.

$$V = \pi \int_0^4 \left(x - \frac{x^2}{4} \right) dx = \pi \left(8 - \frac{16}{3} \right) = \frac{8\pi}{3}.$$

$$\frac{8\pi}{3}$$

Scoring: 1 point for the limits; 1 point for $\pi(R^2 - r^2)$ with correct radii; 1 point for the value.

Question 7: Rates of change (3 points)

$$A'(t) = 2t - 6.$$

- $A(6) = 50 + \int_0^6 (2t - 6) dt = 50 + (36 - 36) = 50.$
- $A'(t) = 0$ at $t = 3$; $A' < 0$ before and $A' > 0$ after, so A is least at $t = 3$ (the endpoints are $A(0) = 50$ and $A(8) = 66$).
- $A(3) = 50 + (9 - 18) = 41.$

(a) 50; (b) $t = 3$; (c) 41 hundred people

Scoring: 1 point for each part.

Question 8: Area with three intersections (2 points)

$x^3 - 9x = x(x-3)(x+3)$, so the graph meets the axis at $x = -3, 0, 3$. On $[-3, 0]$ it is above the axis; on $[0, 3]$ below.

$$\int_{-3}^0 (x^3 - 9x) dx = 0 - \left(\frac{81}{4} - \frac{81}{2}\right) = \frac{81}{4}. \quad \int_0^3 (x^3 - 9x) dx = \frac{81}{4} - \frac{81}{2} = -\frac{81}{4}. \quad \text{Area} = \frac{81}{4} + \frac{81}{4} = \frac{81}{2}.$$

$$\frac{81}{2}$$

Scoring: 1 point for the three intersections and the split; 1 point for $\frac{81}{2}$.

Scoring summary

Total
20 points: add the points of every question of this ap calculus applications test answer key.

How to use this ap calculus applications test answer key

Use this ap calculus applications test answer key to score the test fairly and to see which skills need more practice.

