

AP Calculus AB · Analyzing Functions with Derivatives · Worksheet

Common Core AP Calculus AB Unit 5 · mathovore.fr

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This ap calculus analyzing functions worksheet gives 24 problems on analyzing functions with derivatives, ordered from easy to hard, with a separate answer key.

These 24 problems practice AP Calculus AB Unit 5: critical numbers, increasing and decreasing behavior, the first and second derivative tests, concavity and inflection, the Mean Value and Extreme Value Theorems, implicit relations, and optimization.

Problems start with routine derivative analysis and finish with multi-step applied problems. Show a justification, such as a sign change, for every conclusion about extrema or concavity.

AP Calculus AB Common Core: AP Calculus AB Unit 5

AP Calculus Analyzing Functions Worksheet: problems 1-5

Problem 1: Critical numbers

Find all critical numbers of $f(x) = 2x^3 - 3x^2 - 36x + 5$.

Problem 2: Increasing and decreasing

Find the intervals on which $g(x) = x^4 - 8x^2$ is increasing and the intervals on which it is decreasing. Then identify each local extremum.

Problem 3: First Derivative Test

Use the First Derivative Test to classify each critical point of $h(x) = x^3 - 12x + 4$.

Problem 4: Concavity of a quartic

Determine where $f(x) = x^4 - 4x^3$ is concave up and concave down, and find its points of inflection.

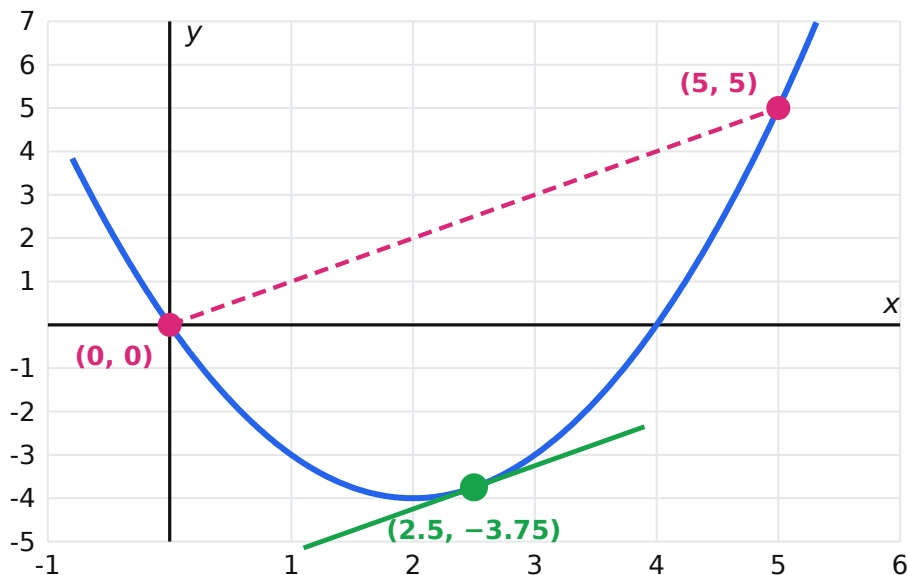
Problem 5: Second Derivative Test

Use the Second Derivative Test to classify the critical points of $p(x) = 2x^3 - 9x^2 + 12x$.

AP Calculus Analyzing Functions Worksheet: problems 6-10

Problem 6: Mean Value Theorem, quadratic

Let $f(x) = x^2 - 4x$ on $[0, 5]$. Verify that the Mean Value Theorem applies and find every c it guarantees.



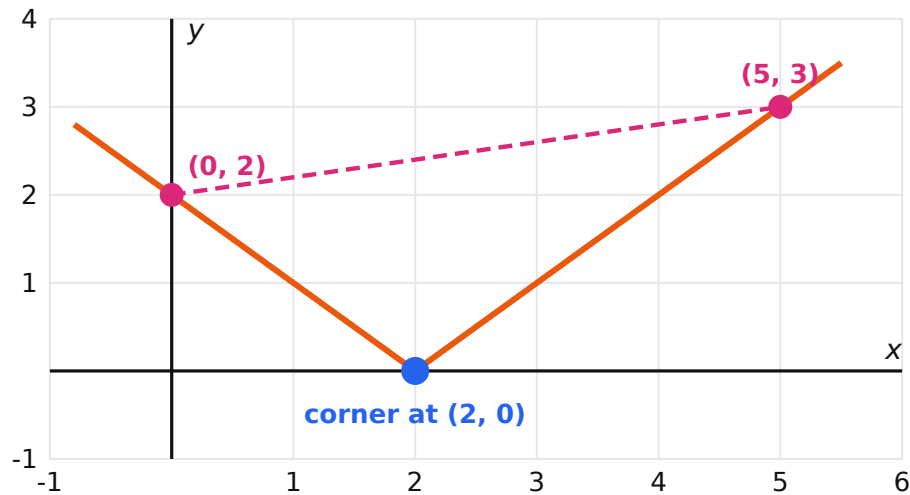
The secant line over $[0, 5]$ and a parallel tangent line.

Problem 7: Mean Value Theorem, shifted square root

Find the value of c guaranteed by the Mean Value Theorem for $f(x) = \sqrt{x+1}$ on $[0, 8]$.

Problem 8: When the theorem fails

Explain why the Mean Value Theorem does not apply to $f(x) = |x-2|$ on $[0, 5]$, and show that no value c satisfies its conclusion.



A V-shaped graph has a corner at $x = 2$.

Problem 9: Candidates Test, cubic

Find the absolute maximum and absolute minimum of $f(x) = 2x^3 - 3x^2 - 12x + 1$ on $[-2, 3]$.

Problem 10: Candidates Test, trigonometric

Find the exact absolute maximum and absolute minimum values of $f(x) = x + 2\cos x$ on $[0, \pi]$.

AP Calculus Analyzing Functions Worksheet: problems 11-15

Problem 11: Points of inflection

Find the points of inflection of $f(x) = x^4 - 8x^3 + 18x^2$. Justify each one.

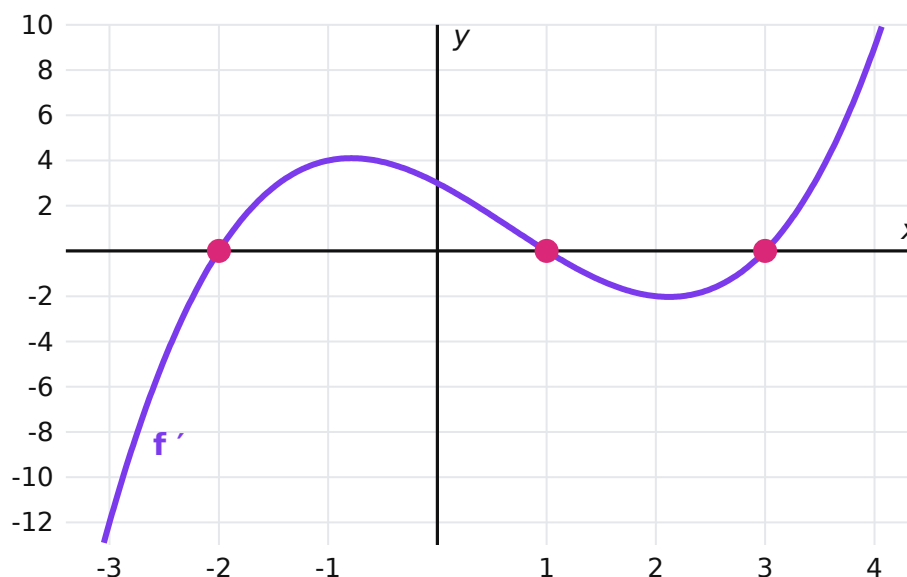
Problem 12: Not an inflection point

For $f(x) = x^4$, show that $f''(0) = 0$ but $(0, 0)$ is not a point of inflection.

Problem 13: Reading the graph of f prime

The graph of f' , where $f'(x) = \frac{1}{2}(x+2)(x-1)(x-3)$, is shown.

- On what intervals is f increasing?
- Identify the x -values of the local maxima and minima of f .
- Find the x -coordinates of the points of inflection of f .



The graph of f' , the derivative of f .

Problem 14: Derivative given in factored form

A function h has $h'(x) = (x-1)^2(x+2)$. Find the x -values of all local extrema of h and explain why $x=1$ is not one of them.

Problem 15: A logarithmic function

Let $f(x) = x - 2\ln x$ for $x > 0$. Find the absolute minimum value of f , and determine its concavity.

AP Calculus Analyzing Functions Worksheet: problems 16-20

Problem 16: Complete function analysis

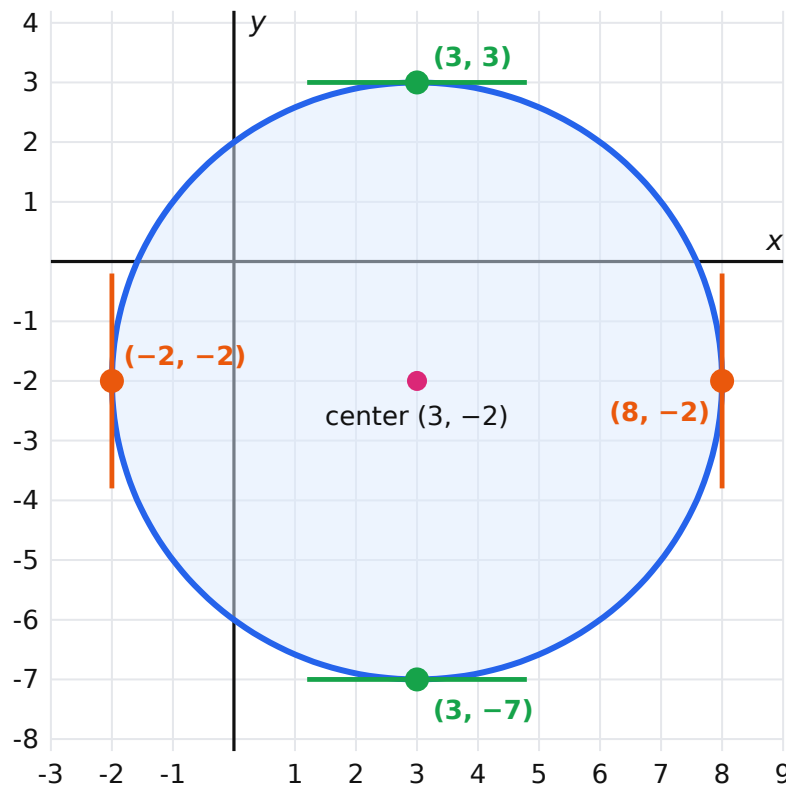
For $f(x) = x^4 - 4x^3$, find the intervals of increase and decrease, the local extrema, the intervals of concavity, and the points of inflection.

Problem 17: Concavity of an implicit curve

For the circle $x^2 + y^2 = 25$, find $\frac{d^2y}{dx^2}$ in terms of y and decide whether the curve is concave up or down at $(3, 4)$.

Problem 18: Tangent lines to a circle

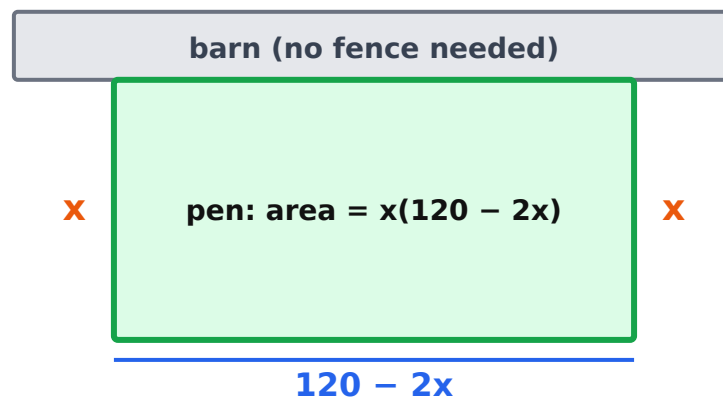
Find every point on the curve $x^2 + y^2 - 6x + 4y = 12$ where the tangent line is horizontal.



Horizontal tangents (green) and vertical tangents (orange) on the circle.

Problem 19: Fencing against a barn

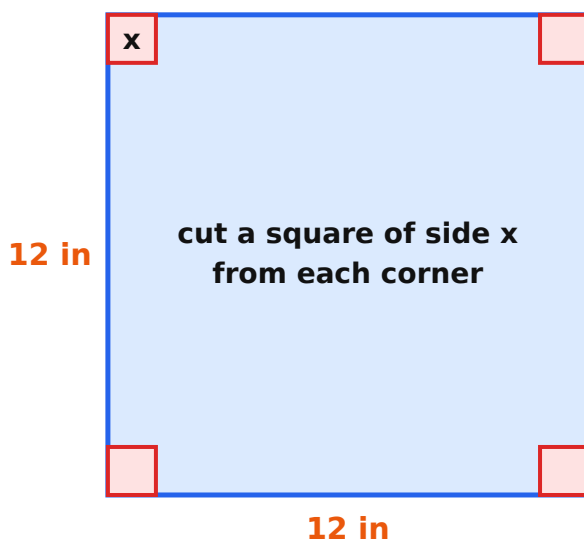
Diego has 120 ft of fencing to build a rectangular pen against a barn wall, so only three sides need fence. Find the dimensions that give the greatest area, and that area.



A pen against a barn uses fence on only three sides.

Problem 20: Open-top box

Maya cuts a square of side x from each corner of a 12 in by 12 in sheet of cardboard and folds up the sides to make an open-top box. Find the value of x that maximizes the volume, and the maximum volume.



Cut the red corner squares, then fold up the flaps.

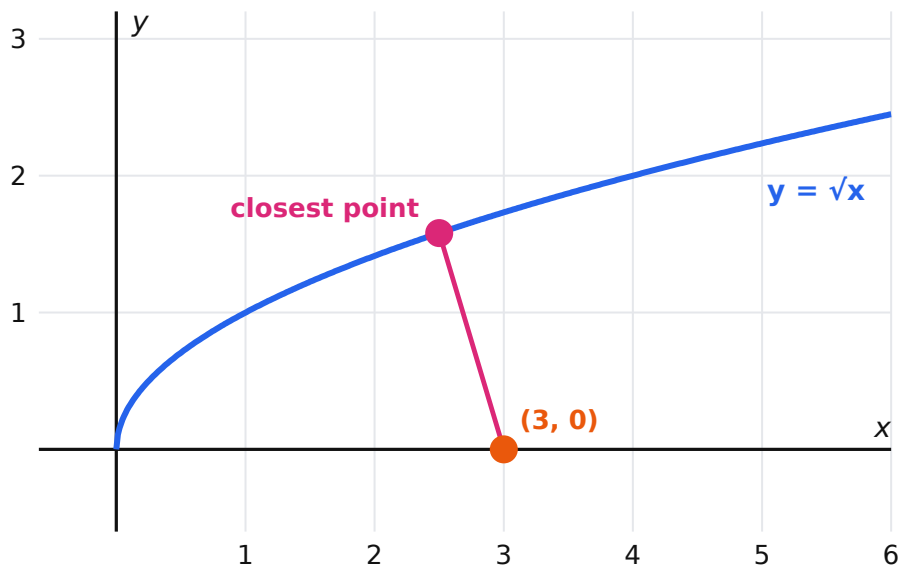
AP Calculus Analyzing Functions Worksheet: problems 21-24

Problem 21: Can of minimum surface area

A closed cylindrical can must hold 16π cubic inches. Find the radius and height that minimize the surface area, and the minimum surface area.

Problem 22: Closest point on a curve

Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$, and the minimum distance.



The shortest segment from $(3, 0)$ to the curve.

Problem 23: Height of a thrown ball

Jamal throws a ball upward. Its height in feet after t seconds is $s(t) = -16t^2 + 64t + 5$. Find the maximum height and the time it occurs, and find the velocity at $t = 3$.

Problem 24: Maximum profit

The daily profit, in dollars, from selling x cupcakes at the school store is $P(x) = -0.02x^2 + 8x - 300$. Find the number of cupcakes that maximizes profit and the maximum profit.

How to use this ap calculus analyzing functions worksheet

Print this ap calculus analyzing functions worksheet as a PDF, solve every problem on paper, then compare with the answer key.

