

AP Calculus AB · Analyzing Functions with Derivatives · Test Answer Key

Common Core AP Calculus AB Unit 5 · mathovore.fr

This ap calculus analyzing test answer key gives the full solution and the scoring guide for each of the 8 test questions on analyzing functions with derivatives.

Worked solutions and scoring guides for the Unit 5 test. Each answer shows the reasoning an AP reader looks for: the derivative, the sign analysis or the named theorem, and a clear conclusion. Award points exactly as the scoring note describes, and total them out of 20.

AP Calculus ABCommon Core: AP Calculus AB Unit 5

AP Calculus Analyzing Test Answer Key: questions 1-5

Question 1: Critical numbers and monotonicity (2 points)

(a) $f'(x) = 3x^2 - 6x - 24 = 3(x-4)(x+2)$, so the critical numbers are $x = -2$ and $x = 4$.

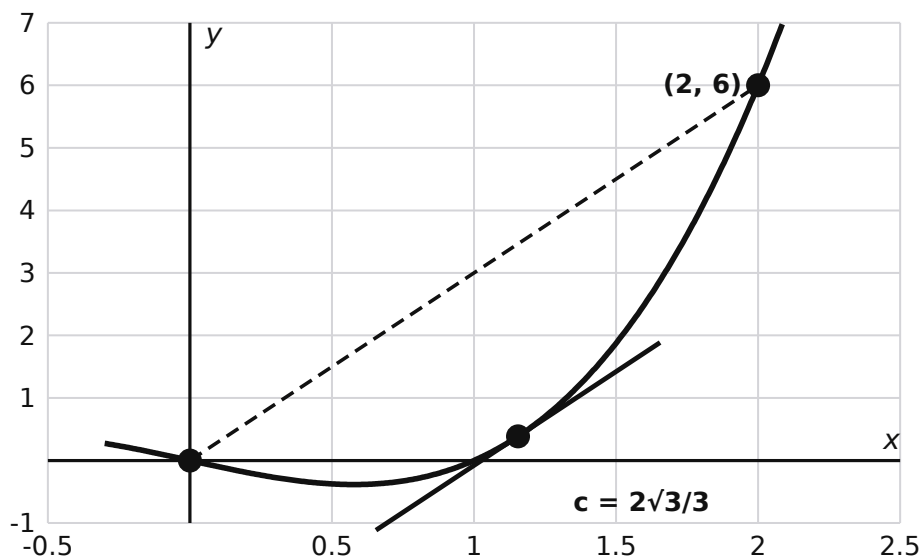
(b) $f' > 0$ on $(-\infty, -2)$ and $(4, \infty)$; $f' < 0$ on $(-2, 4)$. So f is increasing on $(-\infty, -2)$ and $(4, \infty)$ and decreasing on $(-2, 4)$. f' changes from $+$ to $-$ at $x = -2$ (local maximum $f(-2) = 33$) and from $-$ to $+$ at $x = 4$ (local minimum $f(4) = -75$).

Scoring: (a) 1 point for both critical numbers. (b) 1 point for the correct intervals and the classification with the sign-change reason.

Question 2: Mean Value Theorem (2 points)

(a) f is a polynomial, so it is continuous on $[0, 2]$ and differentiable on $(0, 2)$.

(b) $\frac{f(2) - f(0)}{2 - 0} = \frac{6 - 0}{2} = 3$. Solving $3c^2 - 1 = 3$ gives $c^2 = \frac{4}{3}$, so $c = \frac{2\sqrt{3}}{3} \approx 1.155$ (the negative root is outside the interval).



The tangent at $x = c$ is parallel to the secant line.

Scoring: (a) 1 point for naming both continuity and differentiability. (b) 1 point for the correct value of c with the root in $(0, 2)$ chosen.

Question 3: Reading the graph of f prime (2 points)

(a) $f'(x) = x^2 - 2x - 3 = (x - 3)(x + 1)$. f' changes from positive to negative at $x = -1$, so f has a local maximum at $x = -1$; it changes from negative to positive at $x = 3$, so f has a local minimum at $x = 3$.

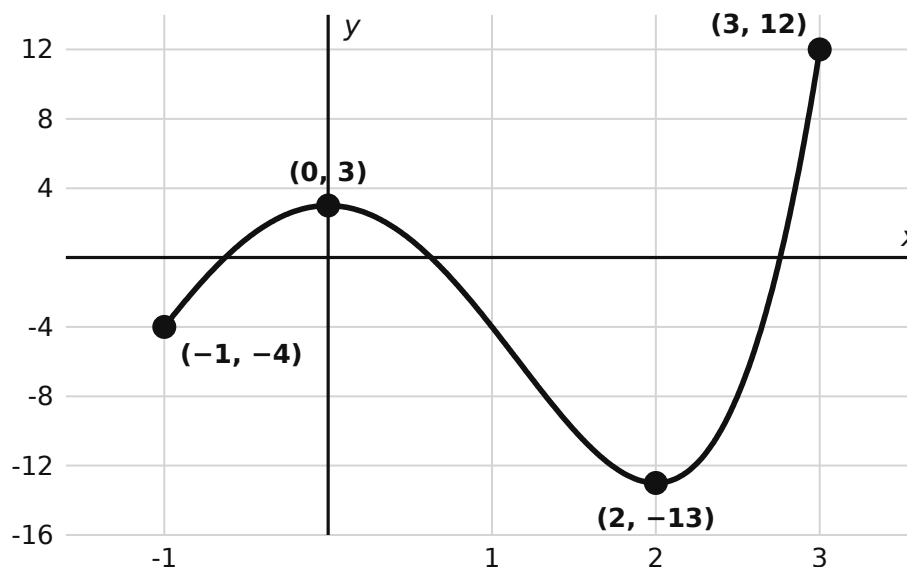
(b) f' is decreasing for $x < 1$ and increasing for $x > 1$, so f'' changes from negative to positive at $x = 1$. The inflection point is at $x = 1$.

Scoring: (a) 1 point for $x = -1$ (maximum) and $x = 3$ (minimum) with the sign-change reason. (b) 1 point for $x = 1$ with the reason that f' changes from decreasing to increasing.

Question 4: Candidates Test (3 points)

$f'(x) = 4x^3 - 16x = 4x(x - 2)(x + 2)$. The critical numbers in $[-1, 3]$ are $x = 0$ and $x = 2$ ($x = -2$ is outside).

$f(-1) = -4$, $f(0) = 3$, $f(2) = -13$, $f(3) = 12$.



Candidates for the absolute extrema on $[-1, 3]$.

The absolute maximum is 12 (at $x=3$) and the absolute minimum is -13 (at $x=2$).

Scoring: 1 point for the critical numbers in the interval; 1 point for the four function values; 1 point for the correct maximum and minimum.

Question 5: Inscribed rectangle (3 points)

With upper right vertex $(x, 12-x^2)$, the width is $2x$ and the height is $12-x^2$, so $A(x) = 2x(12-x^2) = 24x - 2x^3$ for $0 < x < 2\sqrt{3}$.

$A'(x) = 24 - 6x^2 = 0$ gives $x = 2$ (positive root). $A''(x) = -12x$, so $A''(2) = -24 < 0$: a maximum. (Also, $A \rightarrow 0$ at both ends of the domain.)

Width = 4, height = $12 - 4 = 8$, area = 32.

Scoring: 1 point for $A(x)$ and its derivative; 1 point for $x = 2$; 1 point for the justification of the maximum and the dimensions 4×8 with area 32.

AP Calculus Analyzing Test Answer Key: questions 6-8

Question 6: Implicit curve (3 points)

(a) Differentiate: $2x - y - x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$, so $\frac{dy}{dx} = \frac{y-2x}{2y-x}$.

(b) At $(1, 2)$: $\frac{dy}{dx} = \frac{2-2}{4-1} = 0$, so the tangent is horizontal.

(c) $\frac{d^2y}{dx^2} = \frac{\left(\frac{dy}{dx} - 2\right)(2y-x) - (y-2x)\left(2\frac{dy}{dx} - 1\right)}{(2y-x)^2}$. At $(1, 2)$ with $\frac{dy}{dx} = 0$: $\frac{(-2)(3) - 0}{9} = -\frac{2}{3}$. Since it is negative, the

curve is concave down and has a local maximum at $(1,2)$.

Scoring: (a) 1 point for the correct derivative. (b) 1 point for substituting and getting 0. (c) 1 point for $-\frac{2}{3}$ and the conclusion (local maximum).

Question 7: Factored derivative (2 points)

(a) The factor $(x-4)^2$ is nonnegative, so the sign of g' matches the sign of $x-1$ (for $x \neq 4$). $g' > 0$ for $x > 1$ with $x \neq 4$, and since g is continuous, g is increasing on $(1, \infty)$.

(b) No. $g'(4) = 0$, but g' is positive on both sides of $x = 4$, so there is no sign change and no local extremum.

Scoring: (a) 1 point for $(1, \infty)$. (b) 1 point for "no" with the reason that g' does not change sign.

Question 8: Open-top storage bin (3 points)

$x^2h = 32$ gives $h = \frac{32}{x^2}$. The surface area is $S = x^2 + 4xh = x^2 + \frac{128}{x}$ for $x > 0$.

$S'(x) = 2x - \frac{128}{x^2} = 0$ gives $x^3 = 64$, so $x = 4$. $S''(x) = 2 + \frac{256}{x^3} > 0$, so $x = 4$ gives a minimum.

$h = \frac{32}{16} = 2$. $S(4) = 16 + 32 = 48$. The bin is 4 ft by 4 ft by 2 ft, with surface area 48 square feet.

Scoring: 1 point for $S(x) = x^2 + \frac{128}{x}$; 1 point for S' and $x = 4$; 1 point for the minimum justification, $h = 2$ and $S = 48$.

Scoring summary

Total
20 points: add the points of every question of this ap calculus analyzing test answer key.

How to use this ap calculus analyzing test answer key

Use this ap calculus analyzing test answer key to score the test fairly and to see which skills need more practice.

