

AP Calculus AB · Analyzing Functions with Derivatives · Test

Common Core AP Calculus AB Unit 5 · mathovore.fr

Name: _____ Date: ____/____/____ Score: _____

This ap calculus analyzing functions test checks analyzing functions with derivatives in 45 minutes, with 8 questions for a total of 20 points.

This 45-minute test covers AP Calculus AB Unit 5 in free-response style. Questions test critical numbers, the Mean Value Theorem, reading the graph of a derivative, absolute extrema, implicit curves and optimization. Justify each conclusion with a sign change, a theorem or a derivative test. The test is worth 20 points.

Test information

- **Grade:** AP Calculus AB
- **Chapter:** Analyzing Functions with Derivatives
- **Common Core:** AP Calculus AB Unit 5
- **Time:** 45 minutes
- **Total:** 20 points (Question 1: 2 pts; Question 2: 2 pts; Question 3: 2 pts; Question 4: 3 pts; Question 5: 3 pts; Question 6: 3 pts; Question 7: 2 pts; Question 8: 3 pts)

AP Calculus Analyzing Functions Test: questions 1-5

Question 1: Critical numbers and monotonicity (2 points)

Let $f(x) = x^3 - 3x^2 - 24x + 5$.

- Find all critical numbers of f . (1 point)
- Find the intervals on which f is increasing and decreasing, and classify each local extremum with a justification. (1 point)

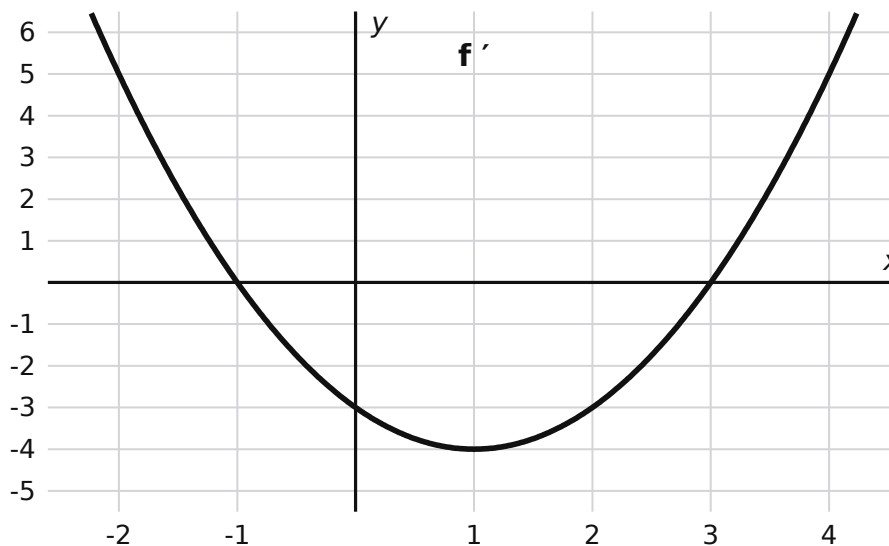
Question 2: Mean Value Theorem (2 points)

Let $f(x) = x^3 - x$ on the interval $[0, 2]$.

- Explain why the Mean Value Theorem applies. (1 point)
- Find the value of c in $(0, 2)$ guaranteed by the theorem. (1 point)

Question 3: Reading the graph of f' prime (2 points)

The graph of f' , the derivative of a twice-differentiable function f , is shown for $-2 \leq x \leq 4$.



The graph of f' on $[-2, 4]$.

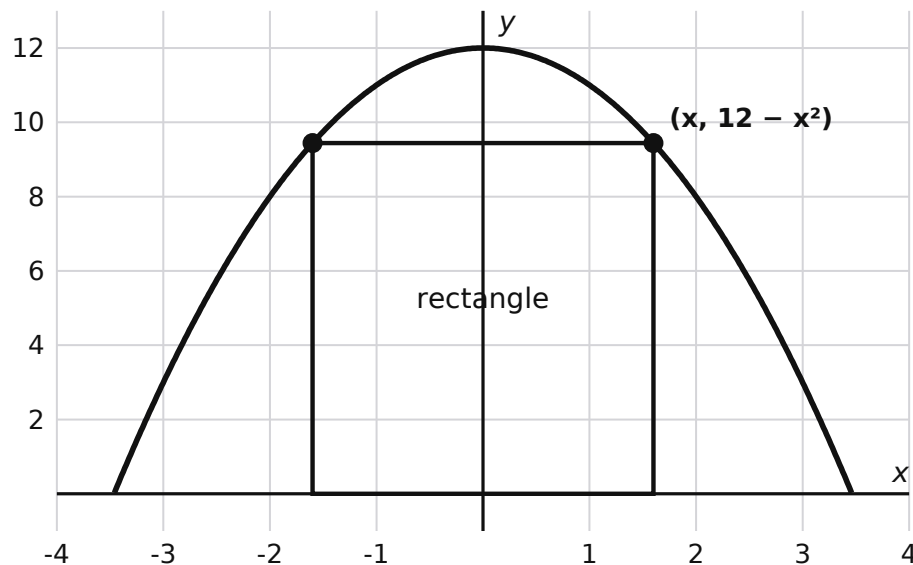
- Find the x -values of the local maximum and local minimum of f . Justify your answer. (1 point)
- Find the x -coordinate of the point of inflection of f . Justify your answer. (1 point)

Question 4: Candidates Test (3 points)

Find the absolute maximum value and the absolute minimum value of $f(x) = x^4 - 8x^2 + 3$ on the interval $[-1, 3]$. Show the values that support your answer.

Question 5: Inscribed rectangle (3 points)

A rectangle has its base on the x -axis and its upper two vertices on the parabola $y = 12 - x^2$, symmetric about the y -axis. Find the dimensions and area of the rectangle of greatest area. Justify that your answer is a maximum.



A rectangle inscribed under the parabola $y = 12 - x^2$.

AP Calculus Analyzing Functions Test: questions 6-8

Question 6: Implicit curve (3 points)

Consider the curve $x^2 - xy + y^2 = 3$, which passes through the point $(1, 2)$.

- Find $\frac{dy}{dx}$. (1 point)
- Show that the tangent line at $(1, 2)$ is horizontal. (1 point)
- Find $\frac{d^2y}{dx^2}$ at $(1, 2)$ and decide whether the curve has a local maximum or minimum there. (1 point)

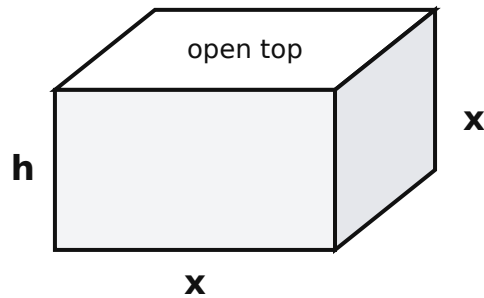
Question 7: Factored derivative (2 points)

A function g is differentiable for all real numbers and $g'(x) = (x-1)(x-4)^2$.

- On what intervals is g increasing? (1 point)
- Does g have a local extremum at $x = 4$? Justify your answer. (1 point)

Question 8: Open-top storage bin (3 points)

Noah builds an open-top storage bin with a square base of side x feet and height h feet. The volume must be 32 cubic feet. Find the value of x that minimizes the total surface area (base plus four sides), the dimensions of the bin, and the minimum surface area. Justify that it is a minimum.



An open-top bin with a square base.

How to use this ap calculus analyzing functions test

Take this ap calculus analyzing functions test without notes, then grade it with the test answer key out of 20 points.

