

AP Calculus AB · Analyzing Functions with Derivatives · Lesson

Common Core AP Calculus AB Unit 5 · mathovore.fr

This ap calculus analyzing functions lesson explains analyzing functions with derivatives in one printable page, with definitions, a method and a worked example.

Derivatives describe the shape of a graph. The sign of the first derivative shows where a function increases or decreases, the sign of the second derivative shows concavity, and critical numbers, inflection points and the Mean Value Theorem turn those signs into precise conclusions and optimization answers.

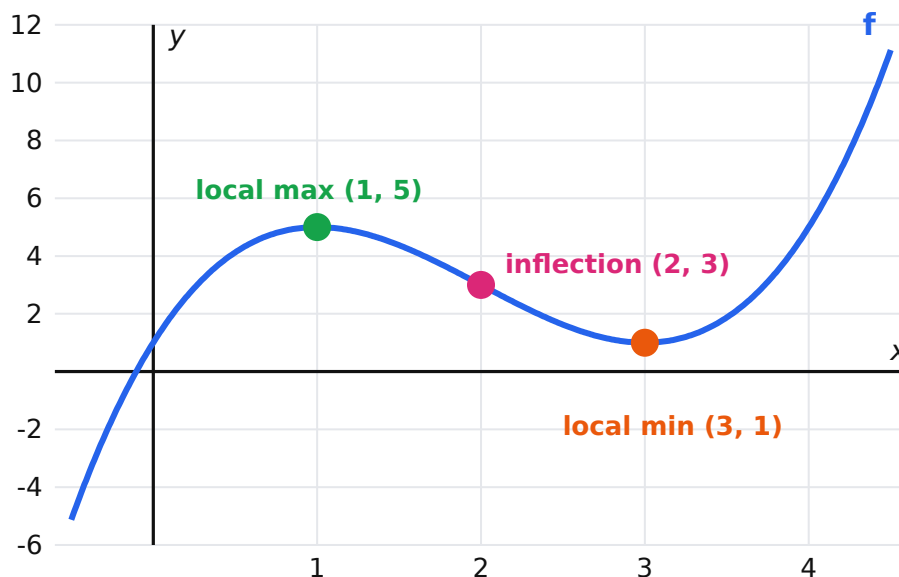
AP Calculus ABCommon Core: AP Calculus AB Unit 5

AP Calculus Analyzing Functions Lesson: key facts

- AP Calculus AB Unit 5: use f' and f'' to describe increasing, decreasing, concavity, extrema and inflection points.
- Candidates Test: on $[a,b]$, compare f at critical numbers and at both endpoints.
- Mean Value Theorem: continuous on $[a,b]$ and differentiable on (a,b) guarantees $f'(c) = \frac{f(b)-f(a)}{b-a}$ for some c .
- Second Derivative Test: if $f'(c) = 0$, then $f''(c) < 0$ means a local maximum and $f''(c) > 0$ means a local minimum.

What do the derivatives tell us about a graph?

The first derivative reports direction and the second reports bending. Where $f'(x) > 0$, the function **increases**; where $f'(x) < 0$, it **decreases**. Where $f''(x) > 0$, the graph is **concave up** (the slopes are growing); where $f''(x) < 0$, it is **concave down**. We follow one function all the way through: $f(x) = x^3 - 6x^2 + 9x + 1$, with $f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$ and $f''(x) = 6x - 12$.



The graph of f rises to a local maximum, falls through an inflection point, and bottoms out at a local minimum.

Critical number

A number c in the domain of f with $f'(c)=0$ or $f'(c)$ undefined. A local extremum can only occur at a critical number.

How do the first and second derivative tests work?

The **First Derivative Test** looks at the sign of f' on both sides of a critical number c : a change from $+$ to $-$ gives a local maximum, a change from $-$ to $+$ gives a local minimum, and no change gives neither. The **Second Derivative Test** needs only $f'(c)=0$: if $f''(c)<0$ there is a local maximum, if $f''(c)>0$ a local minimum, and if $f''(c)=0$ the test fails. A **point of inflection** is where the concavity changes. For our function, $f''(1)=-6$ and $f''(3)=6$, and f'' changes sign at $x=2$.

$$f'(x) = 3(x-1)(x-3)$$



$$f''(x) = 6(x-2)$$



Sign charts for f' and f'' summarize where f rises, falls, and bends.

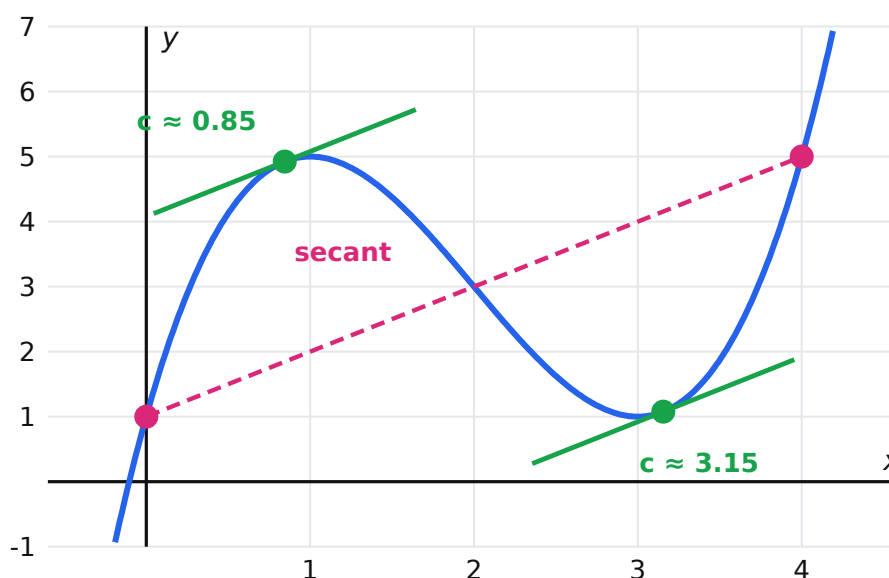
What do the Extreme Value and Mean Value Theorems guarantee?

Two guarantees

Extreme Value Theorem: a function continuous on a closed interval $[a,b]$ has both an absolute maximum and an absolute minimum there.

Mean Value Theorem: if f is continuous on $[a,b]$ and differentiable on (a,b) , some c in (a,b) has $f'(c) = \frac{f(b)-f(a)}{b-a}$.

On $[0,4]$ our function has $f(0)=1$ and $f(4)=5$, so the average rate of change is $\frac{5-1}{4}=1$. Solving $3x^2-12x+9=1$ gives $c=2\pm\frac{2\sqrt{3}}{3}$, and both values lie in $(0,4)$.



Two tangent lines (green) are parallel to the secant line (pink); the Mean Value Theorem guarantees at least one.

How do you find absolute extrema and solve optimization problems?

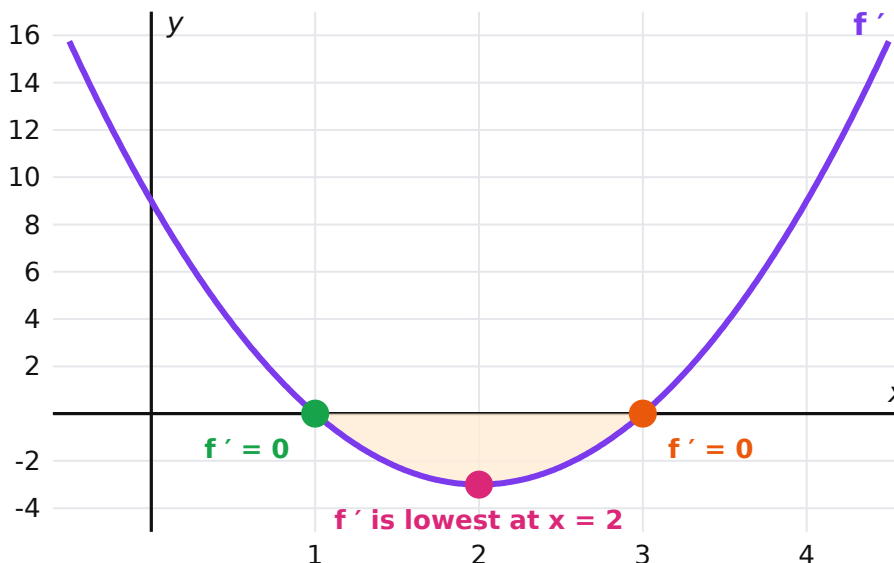
Use the **Candidates Test** on a closed interval:

1. Find every critical number inside the interval.
2. Evaluate f at those numbers and at both endpoints.
3. The largest value is the absolute maximum and the smallest is the absolute minimum.

On $[-1,4]$ the candidates are $x=-1,1,3,4$:

x	-1	1	3	4
$f(x)$	-15	5	1	5

The absolute minimum is -15 at $x = -1$; the absolute maximum is 5 , reached at both $x = 1$ and $x = 4$. Optimization works the same way: write the quantity as a function of one variable, state its domain, differentiate, and justify with the Candidates Test or a derivative test. Finally, the graph of f' tells the whole story of f , because its zeros are critical numbers and its lowest point marks the inflection.



The graph of f' crosses zero at 1 and 3 (the critical numbers of f) and bottoms out at $x = 2$, where f has its inflection point.

Analyzing Functions with Derivatives: frequently asked questions

What is the difference between a critical number and a point of inflection?

A critical number is where f' is zero or undefined and is a candidate for a local extremum. A point of inflection is where the concavity changes, which happens where f'' changes sign.

If f' of c equals 0, must f have a maximum or minimum at c ?

No. For $f(x) = x^3$ we have $f'(0) = 0$, yet f' is positive on both sides, so there is no extremum. You need a sign change in f' or the sign of f'' .

When can I use the Mean Value Theorem?

Only when the function is continuous on the closed interval and differentiable on the open interval. On the exam, state both conditions before you claim that a value c exists.

Why must the Candidates Test include the endpoints?

An absolute extremum on a closed interval can occur at an endpoint where the derivative is not zero. Skipping the endpoints can miss the true maximum or minimum.

How should I justify an answer on the free-response section?

Name the test and cite the evidence: for example, " f' changes from positive to negative at $x=1$, so f has a local maximum there." A bare number without a reason earns no justification point.

How to use this ap calculus analyzing functions lesson

Use this ap calculus analyzing functions lesson before the worksheet, then check what you learned with the quiz.

