

Algebra 2 · Radians and the Unit Circle · Lesson

Common Core F-TF.A.1, F-TF.A.2, F-TF.C.8 · mathovore.fr

A radian measures an angle by the arc it cuts off divided by the radius, so a full circle is 2π radians. On the unit circle, the point at angle θ has coordinates $(\cos \theta, \sin \theta)$, which defines sine, cosine and tangent for any angle. Every such point satisfies $\sin^2 \theta + \cos^2 \theta = 1$.

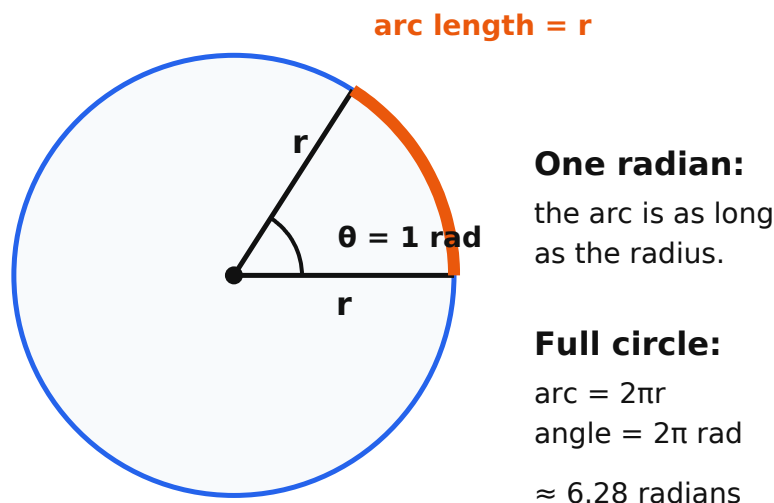
Algebra 2 Common Core: F-TF.A.1 · F-TF.A.2 · F-TF.C.8

Key facts:

- $\theta = \frac{s}{r}$, so $180^\circ = \pi$ radians and $360^\circ = 2\pi$ radians.
- Arc length is $s = r\theta$ when θ is in radians.
- The unit circle point at angle θ is $(\cos \theta, \sin \theta)$.
- Use the reference angle and the quadrant signs to evaluate any angle.
- Pythagorean identity: $\sin^2 \theta + \cos^2 \theta = 1$.

What is a radian?

A **radian** is the measure of a central angle whose arc is exactly as long as the radius. For any circle, the angle in radians is $\theta = \frac{s}{r}$, where s is the arc length and r is the radius. A full turn is 2π radians, so $180^\circ = \pi$ radians.



The arc of length r creates an angle of 1 radian.

Example: a sector with radius 10 inches and central angle $\frac{3\pi}{5}$ has arc length $s = 10 \cdot \frac{3\pi}{5} = 6\pi$ inches, about 18.85 inches. Because radians are a ratio of two lengths, they have no units, and formulas like $s = r\theta$ work without any conversion factor.

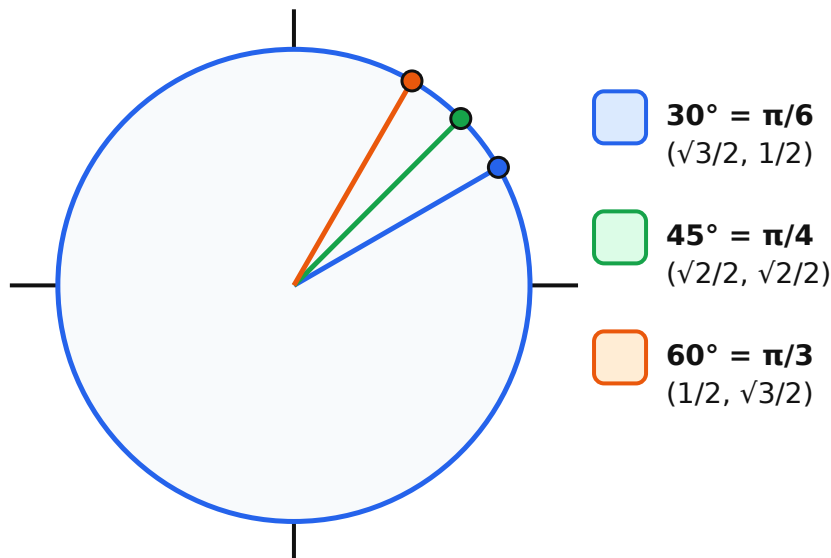
Converting and measuring arcs

Degrees to radians: multiply by $\frac{\pi}{180}$. Radians to degrees: multiply by $\frac{180}{\pi}$. Arc length: $s = r\theta$ with θ in radians.

Degrees	30°	45°	60°	90°	180°	360°
Radians	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	2π

What is the unit circle?

The **unit circle** has radius 1 and its center at the origin. The terminal side of an angle θ in standard position meets the circle at the point $(\cos\theta, \sin\theta)$. Then $\tan\theta = \frac{\sin\theta}{\cos\theta}$, which is undefined where $\cos\theta = 0$. This works for every real angle, positive or negative.



Exact coordinates for the three special angles in Quadrant I.

Negative angles turn clockwise, and angles larger than 2π wrap around the circle again. For example, $-\frac{\pi}{3}$ and $\frac{5\pi}{3}$ end at the same point, so they have the same sine and cosine. On the axes, the points are $(1,0)$, $(0,1)$, $(-1,0)$ and $(0,-1)$ for 0 , $\frac{\pi}{2}$, π and $\frac{3\pi}{2}$.

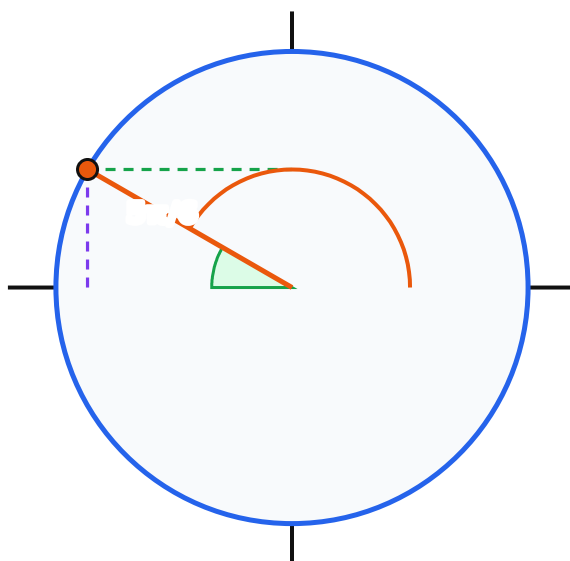
How do you find sine and cosine of any angle?

Find the **reference angle**, the acute angle between the terminal side and the x-axis, then attach signs by quadrant.

Quadrant II sin + cos – tan –	Quadrant I sin + cos + tan +
Quadrant III sin – cos – tan +	Quadrant IV sin – cos + tan –

Signs of sine, cosine and tangent in each quadrant.

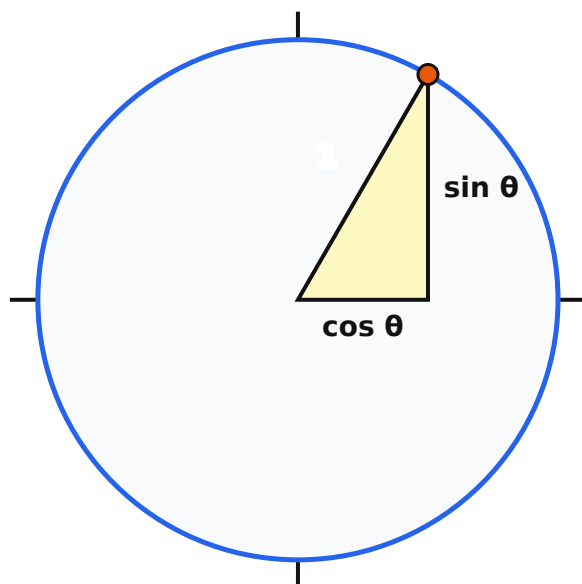
Example: $\theta = \frac{5\pi}{6}$ is in Quadrant II with reference angle $\frac{\pi}{6}$. So $\sin\theta = \frac{1}{2}$, $\cos\theta = -\frac{\sqrt{3}}{2}$, and $\tan\theta = -\frac{\sqrt{3}}{3}$.



Reflecting the Quadrant I point across the y-axis gives the point for $\frac{5\pi}{6}$.

What is the Pythagorean identity?

Every point $(\cos\theta, \sin\theta)$ is on a circle of radius 1, so $\sin^2\theta + \cos^2\theta = 1$ for all angles.



Right triangle with
hypotenuse 1:

$$(\cos \theta)^2 + (\sin \theta)^2$$

from the Pythagorean
theorem.

The legs $\cos\theta$ and $\sin\theta$ and hypotenuse 1.

Worked example: $\sin\theta = \frac{3}{5}$ and θ is in Quadrant II. Find $\cos\theta$ and $\tan\theta$.

1. $\cos^2\theta = 1 - \left(\frac{3}{5}\right)^2 = \frac{16}{25}$.
2. Cosine is negative in Quadrant II, so $\cos\theta = -\frac{4}{5}$.
3. $\tan\theta = \frac{3/5}{-4/5} = -\frac{3}{4}$.

Radians and the Unit Circle: frequently asked questions

Why do we use radians instead of degrees?

Radians tie an angle directly to arc length through $s = r\theta$, with no conversion factor. This makes formulas in algebra and calculus simpler, and it is the standard unit for trigonometric functions of real numbers.

How do I convert between degrees and radians?

Multiply degrees by $\frac{\pi}{180}$ to get radians, and multiply radians by $\frac{180}{\pi}$ to get degrees. For example, $60^\circ = \frac{\pi}{3}$ radians.

What are the coordinates on the unit circle?

The point at angle θ is $(\cos\theta, \sin\theta)$. The x-coordinate is the cosine and the y-coordinate is the sine of the angle.

Why is tangent undefined at $\frac{\pi}{2}$?

Tangent is sine divided by cosine. At $\frac{\pi}{2}$ the point is $(0,1)$, so the cosine is 0 and the division is impossible.

How do I find cosine when I only know sine?

Use $\cos^2\theta = 1 - \sin^2\theta$ and take the square root. Pick the sign from the quadrant: cosine is positive in Quadrants I and IV and negative in II and III.