

Algebra 2 · Graphs of Trigonometric Functions

· Worksheet Answer Key

Common Core F-TF.B.5, F-IF.C.7e · mathovore.fr

Every solution below shows the formula, the substitution and the final value in bold.
Compare each step with your own work, and recheck the sign of A and the grouping inside the sine or cosine whenever your answer is off.

Algebra 2 Common Core: F-TF.B.5 · F-IF.C.7e

Problem 1: Sine at key angles

- a. $\sin 0 = 0$
- b. $\sin \frac{\pi}{2} = 1$ (top of the first hump)
- c. $\sin \pi = 0$
- d. $\sin \frac{3\pi}{2} = -1$ (bottom of the wave)
- e. $\sin 2\pi = 0$ (one full cycle is done)

Problem 2: Cosine at key angles

- a. $\cos 0 = 1$
- b. $\cos \frac{\pi}{2} = 0$
- c. $\cos \pi = -1$
- d. $\cos \frac{3\pi}{2} = 0$
- e. $\cos 2\pi = 1$

Problem 3: Find the amplitude

The amplitude is $|A|$.

- a. $|4| = 4$
- b. $|-3| = 3$
- c. $|\frac{1}{2}| = \frac{1}{2}$
- d. $|2.5| = 2.5$

Problem 4: Find the period

Use $\text{period} = \frac{2\pi}{B}$.

- a. $\frac{2\pi}{2} = \pi$

- b. $\frac{2\pi}{4} = \frac{\pi}{2}$
- c. Here $B = \frac{1}{2}$, so $2\pi \div \frac{1}{2} = 4\pi$
- d. $\frac{2\pi}{\pi} = 2$
- e. $\frac{2\pi}{3} : \frac{2\pi}{3}$

Problem 5: Midline and range

The midline is $y = D$; the range is $[D - |A|, D + |A|]$.

- a. Midline $y = 3$, range $[2, 4]$
- b. Midline $y = -1$, range $[-1 - 2, -1 + 2] = [-3, 1]$
- c. Midline $y = 2$, range $[2 - 4, 2 + 4] = [-2, 6]$

Problem 6: Read the phase shift

- a. Subtracting $\frac{\pi}{4}$: **right** $\frac{\pi}{4}$
- b. Adding $\frac{\pi}{3}$ means $x - (-\frac{\pi}{3})$: **left** $\frac{\pi}{3}$
- c. The factored form $2(x - \frac{\pi}{6})$ shows **right** $\frac{\pi}{6}$ (the shift is not $\frac{\pi}{3}$)
- d. **right** $\frac{3\pi}{2}$

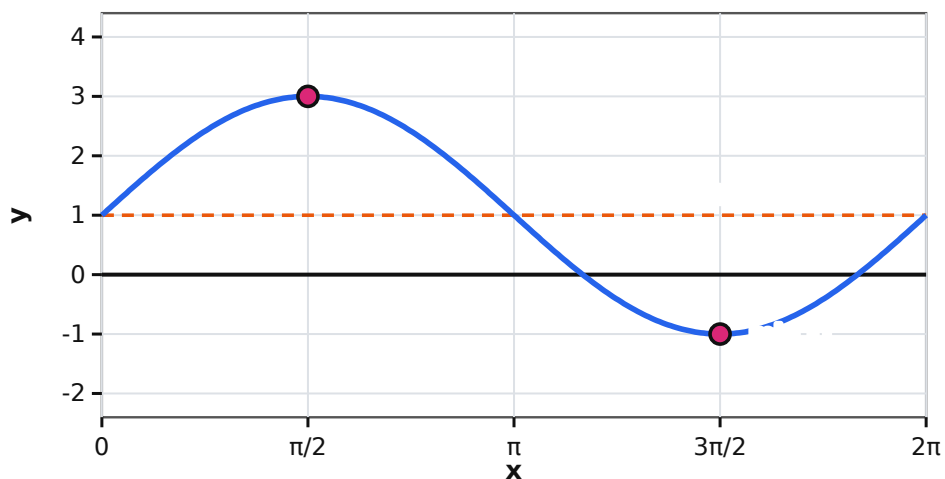
Problem 7: Equation from a cosine graph

The maximum is 3 at $x = 0$, and $\cos 0 = 1$, so $A = 3$. The minimum -3 at $x = \pi$ agrees.

$$A = 3, y = 3\cos x$$

Problem 8: Midline from a graph

Maximum 3, minimum -1 . Midline $= \frac{3 + (-1)}{2} = 1$. Amplitude $= \frac{3 - (-1)}{2} = 2$.



Midline and extreme values of the Exercise 8 graph.

Equation: $y = 2\sin x + 1$

Problem 9: Period from a graph

The graph completes one cycle between $x=0$ and $x=\pi$, so the period is π . Then $B = \frac{2\pi}{\pi} = 2$. The maximum is 2, so $A = 2$.

Period π , $B = 2$, $y = 2\cos(2x)$

Problem 10: Evaluate a sinusoid

- a. $2 \cdot \frac{1}{2} + 1 = 2$
- b. $2 \cdot 1 + 1 = 3$
- c. $\sin \frac{7\pi}{6} = -\frac{1}{2}$, so $2\left(-\frac{1}{2}\right) + 1 = 0$
- d. $2(-1) + 1 = -1$

Problem 11: Maximum and minimum values

- a. Maximum $2+5 = 7$, minimum $2-5 = -3$
- b. Amplitude 2, midline -1 : maximum $-1+2 = 1$, minimum $-1-2 = -3$

Problem 12: Three features at once

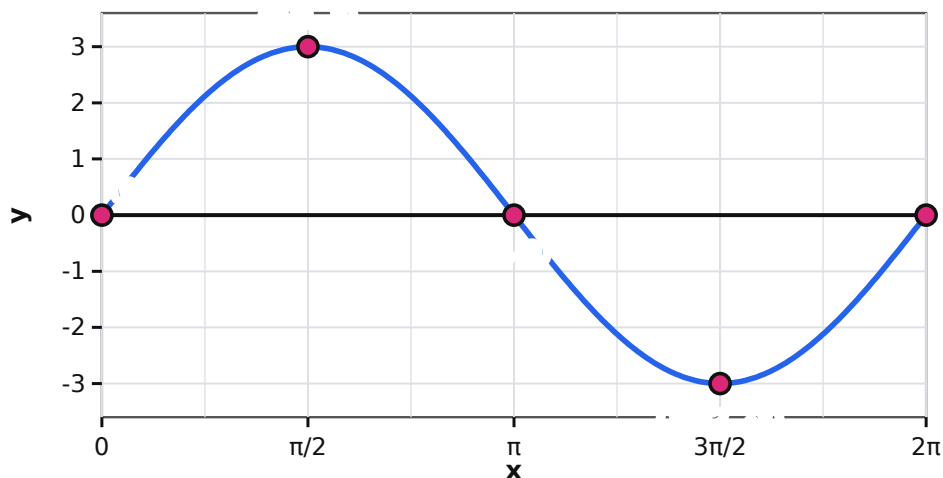
Amplitude $|3| = 3$. Period $\frac{2\pi}{4} = \frac{\pi}{2}$. Midline $y = -2$.

Amplitude 3, period $\frac{\pi}{2}$, midline $y = -2$

Problem 13: Plot key points

Values: $3\sin 0 = 0$, $3\sin \frac{\pi}{2} = 3$, $3\sin \pi = 0$, $3\sin \frac{3\pi}{2} = -3$, $3\sin 2\pi = 0$.

y : 0, 3, 0, -3 , 0



Problem 14: All four features

Amplitude $|-3|=3$. Period $\frac{2\pi}{2}=\pi$. The factor is $x+\frac{\pi}{4}=x-(-\frac{\pi}{4})$, so the shift is $\frac{\pi}{4}$ to the left.
Midline $y=1$.

Maximum $1+3=4$, minimum $1-3=-2$ (the negative sign flips the wave but does not change these values).

Amplitude 3, period π , left $\frac{\pi}{4}$, midline $y=1$, max 4, min -2

Problem 15: Write a sine equation

$A=4$, $D=1$, and $B=\frac{2\pi}{\pi}=2$.

$$y=4\sin(2x)+1$$

Problem 16: Write a cosine equation

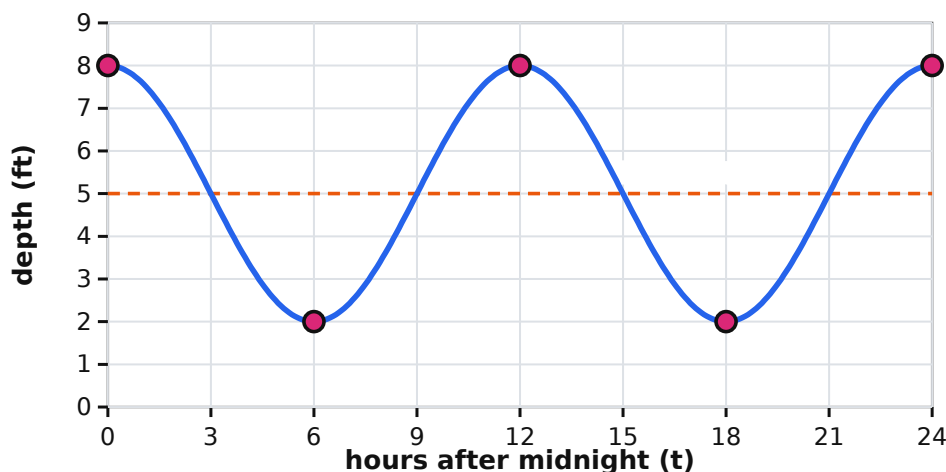
$A=\frac{8-2}{2}=3$, $D=\frac{8+2}{2}=5$, $B=\frac{2\pi}{8}=\frac{\pi}{4}$.

$$y=3\cos\left(\frac{\pi}{4}x\right)+5$$

Check: at $x=0$ the value is $3+5=8$, the maximum.

Problem 17: Tide model

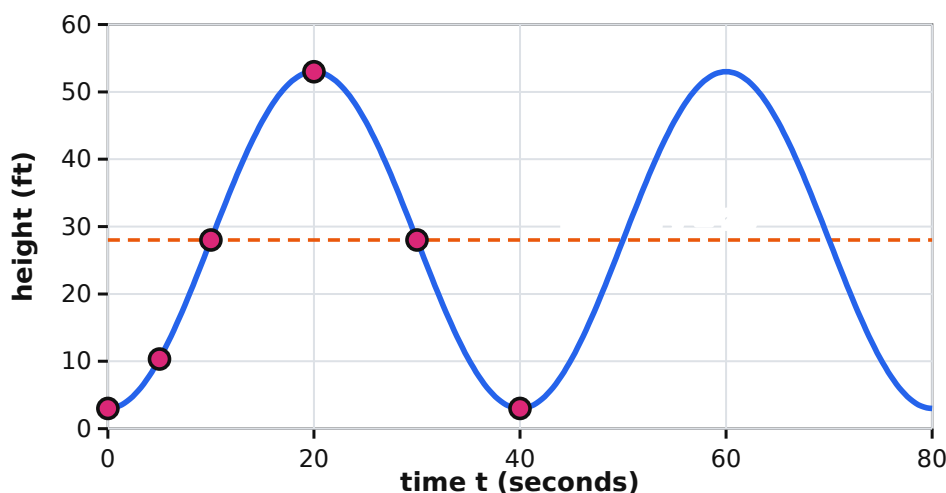
- Amplitude 3, period $\frac{2\pi}{\pi/6}=12$ hours: 12, midline $y=5$
- Highest $5+3=8$ ft, lowest $5-3=2$ ft
- $D(2)=5+3\cos\frac{\pi}{3}=5+1.5=6.5$ ft; $D(4)=5+3\cos\frac{2\pi}{3}=5-1.5=3.5$ ft



Marked highs and lows of the tide.

Problem 18: Ferris wheel

- a. Period $= \frac{2\pi}{\pi/20} = 40$ s
b. Lowest $28-25 = 3$ ft, highest $28+25 = 53$ ft
c. $h(10) = 28 - 25\cos\frac{\pi}{2} = 28 - 0 = 28$ ft; $h(5) = 28 - 25\cos\frac{\pi}{4} \approx 28 - 17.68 = 10.3$ ft



Marked points on the Ferris wheel graph.

Problem 19: Daylight hours

- a. Amplitude 2.5, period 365 days, midline $y = 12$, phase shift 80 days to the right
b. Longest $12 + 2.5 = 14.5$ h, shortest $12 - 2.5 = 9.5$ h
c. $D(125) = 12 + 2.5\sin\frac{2\pi \cdot 45}{365} \approx 12 + 2.5(0.6967) \approx 13.7$ h (calculator in radian mode)

Problem 20: Bobbing buoy

- a. $A = \frac{14-8}{2} = 3$, $D = \frac{14+8}{2} = 11$, $B = \frac{2\pi}{4} = \frac{\pi}{2}$: $h(t) = 3\cos\left(\frac{\pi}{2}t\right) + 11$
b. $h(1.5) = 11 + 3\cos\frac{3\pi}{4} \approx 11 - 2.12 = 8.9$ ft
c. Set $\cos\left(\frac{\pi}{2}t\right) = 0$: $\frac{\pi}{2}t = \frac{\pi}{2}$, so $t = 1$ **second**

Problem 21: Daily temperature

- a. The maximum is $62 + 14 = 76^\circ\text{F}$ when $\frac{\pi(t-9)}{12} = \frac{\pi}{2}$, so $t = 15$ (**3 p.m.**). The minimum is $62 - 14 = 48^\circ\text{F}$ when the angle is $-\frac{\pi}{2}$, so $t = 3$ (**3 a.m.**).
b. $T(12) = 62 + 14\sin\frac{\pi}{4} \approx 62 + 9.90 = 71.9^\circ\text{F}$
c. Solve $\sin\left(\frac{\pi(t-9)}{12}\right) = 0$: the angle is 0 or π , so $t = 9$ and $t = 21$.

Problem 22: Spring and weight

- a. Amplitude 6 in, period $\frac{2\pi}{2\pi/3} = 3$ s
b. $h(1) = 30 + 6\cos\frac{2\pi}{3} = 30 - 3 = 27$ in
c. Solve $30 + 6\cos\frac{2\pi t}{3} = 33$, so $\cos\frac{2\pi t}{3} = \frac{1}{2}$. In one cycle the angle is $\frac{\pi}{3}$ or $\frac{5\pi}{3}$, giving $t = \frac{1}{2}$ or $t = \frac{5}{2}$:

$t = 0.5$ s and $t = 2.5$ s