

Algebra 2 · Graphs of Trigonometric Functions

· Lesson

Common Core F-TF.B.5, F-IF.C.7e · mathovore.fr

The graphs of sine and cosine are smooth waves that repeat every 2π and stay between -1 and 1 . In $y = A\sin(B(x-C)) + D$, the amplitude is $|A|$, the period is $\frac{2\pi}{B}$, the phase shift is C , and the midline is $y = D$. These four numbers let you graph any sinusoid and model periodic situations.

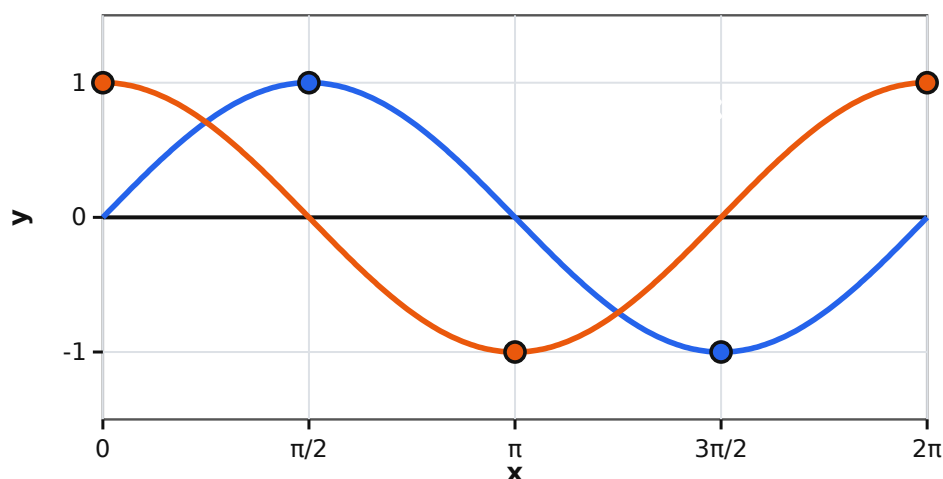
Algebra 2 Common Core: F-TF.B.5 · F-IF.C.7e

Key facts:

- Standards F-TF.B.5 and F-IF.C.7e: graph sine and cosine and use them to model periodic phenomena.
- Parent functions have amplitude 1, period 2π , midline $y = 0$ and range $[-1, 1]$.
- Period is $\frac{2\pi}{B}$: a larger B squeezes the wave horizontally.
- Maximum $= D + |A|$ and minimum $= D - |A|$, so $D = \frac{\max + \min}{2}$.
- To model a situation, match the radius or half-range to A , the center to D and the time per cycle to the period.

What do the graphs of sine and cosine look like?

The graphs of $y = \sin x$ and $y = \cos x$ are smooth waves that repeat every 2π units, with x measured in radians. Both stay between -1 and 1 . The sine wave starts at $(0, 0)$ and rises; the cosine wave starts at its maximum $(0, 1)$. Cosine is just sine shifted left by $\frac{\pi}{2}$.

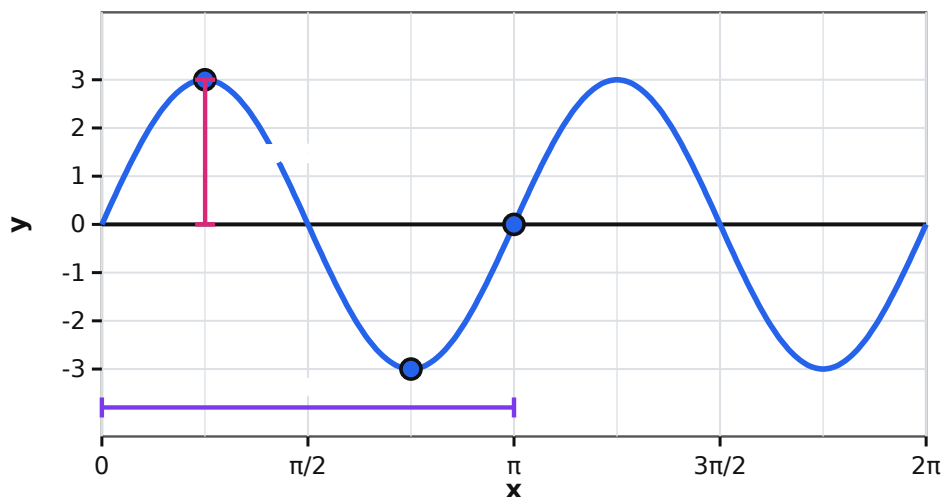


Sine (blue) and cosine (orange) over one full period. Dots mark maxima and minima.

What do A, B, C and D change?

Every sinusoid can be written $y = A\sin(B(x-C)) + D$, or with cosine in place of sine. Each letter transforms the parent graph in one way.

Letter	Feature	How to find it
A	Amplitude	$ A $; a negative A flips the wave
B	Period	$\frac{2\pi}{B}$ for $B > 0$
C	Phase shift	Right by C if $C > 0$, left if C is negative
D	Midline	The horizontal line $y = D$



Graph of $y = 3 \sin 2x$: the amplitude is 3 and one cycle takes only π units.

Vocabulary

Amplitude: half the distance from the maximum to the minimum.

Period: the length of one complete cycle.

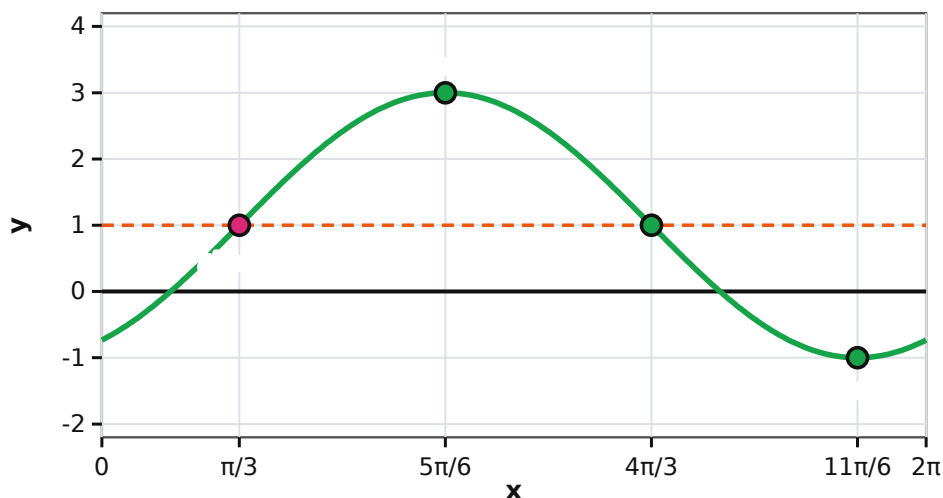
Midline: the horizontal line halfway between the maximum and minimum.

Phase shift: the horizontal shift of the starting point of the cycle.

How do you graph a sinusoid step by step?

Example: graph $y = 2\sin\left(x - \frac{\pi}{3}\right) + 1$.

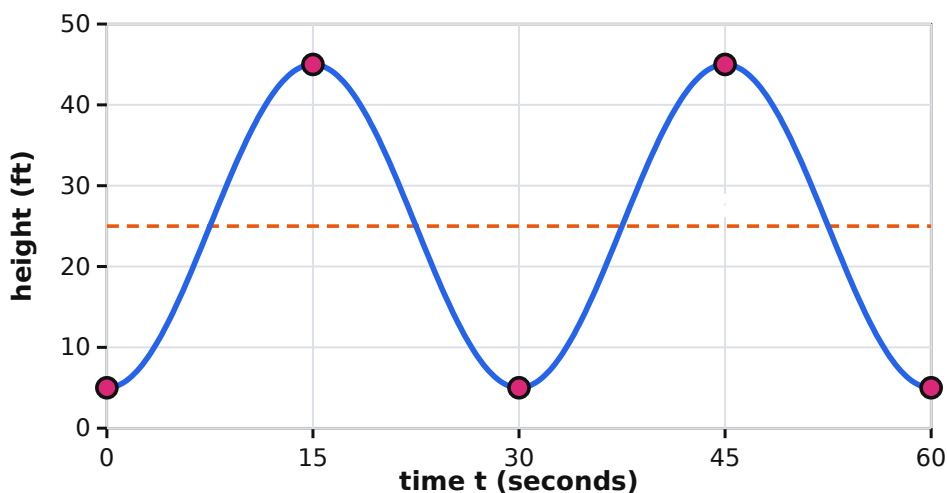
1. Read the features: amplitude 2, period 2π , phase shift $\frac{\pi}{3}$ to the right, midline $y=1$.
2. Draw the dashed midline $y=1$. The maximum is $1+2=3$ and the minimum is $1-2=-1$.
3. Start the cycle at $(\frac{\pi}{3}, 1)$. Quarter-period steps of $\frac{\pi}{2}$ give the maximum at $x=\frac{5\pi}{6}$, the midline at $\frac{4\pi}{3}$, and the minimum at $\frac{11\pi}{6}$.
4. Join the points with a smooth wave and continue it as far as the domain requires.



The key points of one cycle of $y = 2 \sin(x - \pi/3) + 1$.

How do sinusoids model real situations?

Anything that repeats at a steady rate, such as tides, daylight hours or a Ferris wheel, can be modeled by a sinusoid. A wheel with radius 20 ft, center 25 ft above the ground and one turn every 30 seconds has height $h(t) = 25 - 20 \cos(\frac{2\pi t}{30})$. The amplitude is the radius, the midline is the center height, and the period is the time for one turn. A reflected cosine starts at the minimum, which is where a rider boards.



Rider height over two turns of the wheel.

Graphs of Trigonometric Functions: frequently asked questions

What is the difference between the graphs of sine and cosine?

They have the same shape. Cosine starts at its maximum $(0,1)$, while sine starts at the midline $(0,0)$. Shifting the sine graph left $\frac{\pi}{2}$ produces the cosine graph.

How do you find the amplitude of a sinusoid?

Take half the distance between the maximum and the minimum, or read $|A|$ from the equation. For a graph with maximum 7 and minimum -3 , the amplitude is $\frac{7-(-3)}{2}=5$.

How do you find the period from the equation?

Use $\text{period} = \frac{2\pi}{B}$ with $B > 0$. For $y = \cos(4x)$ the period is $\frac{2\pi}{4} = \frac{\pi}{2}$. If B is a fraction like $\frac{1}{2}$, the period is longer: 4π .

What does a negative A do to the graph?

A negative A reflects the wave across its midline. The amplitude is still $|A|$, but a sine wave now falls first, and a cosine wave starts at its minimum instead of its maximum.

Why use cosine for a Ferris wheel model?

A rider boards at the lowest point, so a reflected cosine such as $25 - 20\cos(Bt)$ starts at the minimum when $t = 0$. Any starting position can also be modeled with sine and a phase shift.