

Algebra 2 · Transformations of Functions · Worksheet Answer Key

Common Core F-BF.B.3, F-IF.C.7b · mathovore.fr

Each solution shows which part of the equation causes which change, and the graphs mark the key points. If an answer differs, check whether a shift inside the function was applied in the wrong direction.

Algebra 2 Common Core: F-BF.B.3 · F-IF.C.7b

Problem 1: Describe the shift

- a. Up 5.
- b. Down 3.
- c. Right 4 (the -4 inside moves the graph right).
- d. Left 1.

Answers: up 5, down 3, right 4, left 1.

Problem 2: Absolute value from a graph

- a. The vertex is $(-2, -3)$.
- b. $h = -2$ and $k = -3$: $y = |x + 2| - 3$. Check $(1, 0)$: $|3| - 3 = 0$.

Problem 3: Transform a table

- a. Add 4 to each output: $(-2, 7), (-1, 3), (0, 6), (1, 9), (2, 4)$.
- b. Move each point right 1: $(-1, 3), (0, -1), (1, 2), (2, 5), (3, 0)$.
- c. Double each output: $(-2, 6), (-1, -2), (0, 4), (1, 10), (2, 0)$.
- d. Change each output's sign: $(-2, -3), (-1, 1), (0, -2), (1, -5), (2, 0)$.

Problem 4: Reflections

- a. Multiplying the output by -1 reflects the graph over the x-axis. Domain: $x \geq 0$.
- b. Replacing x with $-x$ reflects the graph over the y-axis. We need $-x \geq 0$, so the domain is $x \leq 0$.

Problem 5: Stretch or compress

- a. Vertical stretch by a factor of 3.
- b. Vertical compression by a factor of $\frac{1}{3}$.

- c. $|3x|=3|x|$, so it is the same as part (a): a horizontal compression by $\frac{1}{3}$, which looks like a vertical stretch by 3.

Problem 6: Write the equation

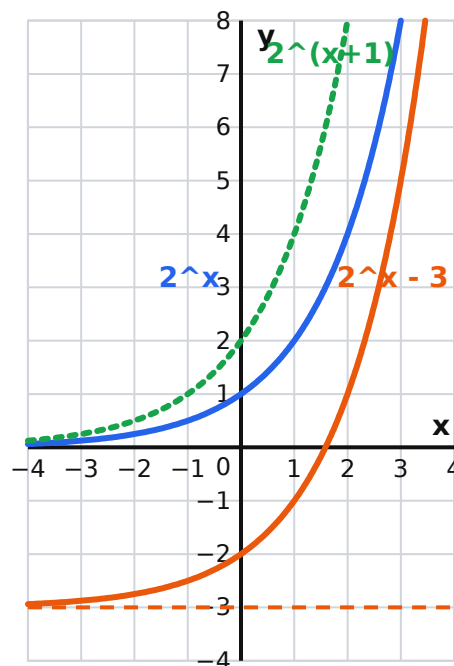
- $y=(x+3)^2-6$
- $y=|x-5|+1$
- $y=\sqrt{x}-4$
- $y=2^x+3$

Problem 7: Several transformations

Factor 2: vertical stretch by 2. The negative sign: reflection over the x-axis. $x-1$: right 1. $+5$: up 5. The vertex is $(1,5)$ and the parabola opens **down**.

Problem 8: Exponential transformations

- The graph moves down 3, so the asymptote is $y=-3$. $g(0)=1-3=-2$: y-intercept $(0,-2)$.
- A horizontal shift does not move the asymptote: $y=0$. $h(0)=2^1=2$: $(0,2)$.
- $2^{x+1}=2^x \cdot 2^1=2 \cdot 2^x$, so shifting left 1 is a vertical stretch by 2.



Three exponential graphs.

Problem 9: Even, odd or neither

- $f(-x)=(-x)^4-3(-x)^2=x^4-3x^2=f(x)$: even.
- $g(-x)=-x^3-x=-(x^3+x)=-g(x)$: odd.

c. $h(-x) = x^2 - x$, which is neither $h(x)$ nor $-h(x)$: **neither**.

Problem 10: Use symmetry

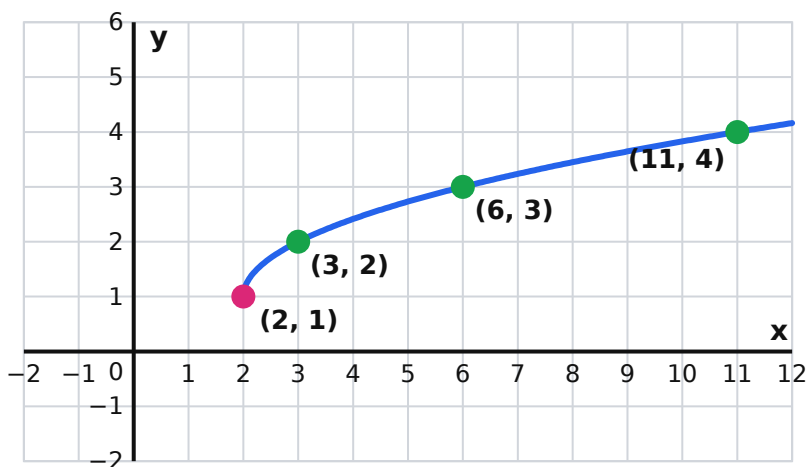
- a. For an odd function $f(-x) = -f(x)$: $f(-1) = -3$, $f(-2) = 5$, $f(-3) = -8$. Also $f(0) = -f(0)$, so $f(0) = 0$.
- b. For an even function $g(-2) = g(2) = 7$.

Problem 11: Parent functions and symmetry

- a. $|-x| = |x|$: **even**.
- b. $\sqrt[3]{-x} = -\sqrt[3]{x}$: **odd**.
- c. $\sqrt{-1}$ is not real, so $f(-x)$ is not defined for positive x : **neither**.
- d. $2^{-1} = \frac{1}{2}$, which is neither 2 nor -2 : **neither**.

Problem 12: Graph a square root function

The graph starts at $(2, 1)$. Choose $x-2$ to be 0, 1, 4 and 9: $(2, 1), (3, 2), (6, 3), (11, 4)$.

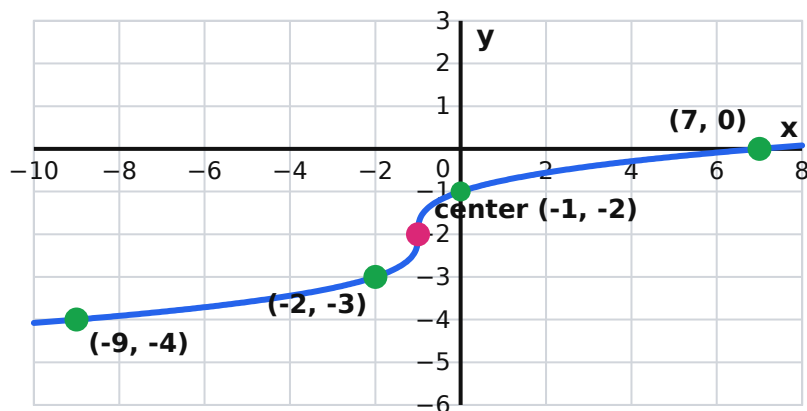


The graph starts at $(2, 1)$ and rises.

Domain: $x \geq 2$. Range: $y \geq 1$.

Problem 13: Graph a cube root function

The center is $(-1, -2)$. Choose $x+1$ to be $-8, -1, 0, 1, 8$: $x = -9, -2, -1, 0, 7$ give $y = -4, -3, -2, -1, 0$.



The graph passes through its center $(-1, -2)$.

Domain: **all real numbers**. Range: **all real numbers**.

Problem 14: Read a square root graph

- It starts at $(-3, 2)$ and goes right and down. Domain: $x \geq -3$. Range: $y \leq 2$.
- $h = -3$, $k = 2$. The point $(1, 0)$ gives $0 = a\sqrt{4} + 2$, so $a = -1$. Equation: $y = -\sqrt{x+3} + 2$. Check $(6, -1)$: $-3 + 2 = -1$.

Problem 15: Read a cube root graph

The graph flattens at its center $(2, -1)$, so $h = 2$ and $k = -1$. Check $(10, 1)$: $\sqrt[3]{8} - 1 = 1$, and $(-6, -3)$: $\sqrt[3]{-8} - 1 = -3$. Equation: $y = \sqrt[3]{x-2} - 1$.

Problem 16: Move the points

- Add 3 to the y-coordinate: $(6, -1)$.
- Add 2 to the x-coordinate: $(8, -4)$.
- Double the y-coordinate: $(6, -8)$.
- Divide the x-coordinate by 3: $(2, -4)$.
- Negate the x-coordinate: $(-6, -4)$.
- Negate the y-coordinate: $(6, 4)$.

Problem 17: Ball toss

- Vertical stretch by 16, reflection over the t-axis, right 2, up 64.
- The vertex is $(2, 64)$: the maximum height is **64 feet** at **2 seconds**.
- $h(0) = -16(4) + 64 = 0$ and $h(4) = -16(4) + 64 = 0$. The ball is in the air **4 seconds**.

Problem 18: Weekly profit

- The vertex is $(15, 450)$: the maximum profit is **\$450** with **15 hoodies**.

- b. $P(10) = -2(25) + 450 = 400$ and $P(20) = -2(25) + 450 = 400$. Both are 5 units from 15, and the parabola is symmetric about $x = 15$.

Problem 19: Gift box

- a. It is shifted up 2.
b. $E(27) = 3 + 2 = 5$ and $E(125) = 5 + 2 = 7$. Edges: **5 in. and 7 in.**
c. $\sqrt[3]{V} + 2 = 9$ gives $\sqrt[3]{V} = 7$ and $V = 343$. The gift volume is **343 cubic inches**.

Problem 20: Skid marks

- a. $\sqrt{144} = 12$, $\sqrt{1296} = 36$, $\sqrt{3600} = 60$. Speeds: **12, 36 and 60 mph**.
b. $s(d) = f(24d)$ for $f(d) = \sqrt{d}$: a horizontal compression by $\frac{1}{24}$.
c. $48 = \sqrt{24d}$ gives $2304 = 24d$ and $d = 96$. The marks are **96 feet** long.

Problem 21: Bacteria colony

- a. $N(0) = 500$ and $N(3) = 500 \cdot 8 = 4000$.
b. $N(t+2) = 500 \cdot 2^{t+2} = 500 \cdot 4 \cdot 2^t = 2000 \cdot 2^t = 4N(t)$.
c. Waiting 2 more hours quadruples the colony at every time, so every height on the left-shifted graph is 4 times the original height.

Problem 22: Thermostat error

- a. It is shifted right 68, so the vertex is at $(68, 0)$.
b. $E(65) = |-3| = 3$ and $E(71) = |3| = 3$.
c. $|T - 68| = 5$ gives $T - 68 = 5$ or $T - 68 = -5$, so $T = 73$ or $T = 63$. Temperatures: **63°F and 73°F**.