

Algebra 2 · Transformations of Functions · Lesson

Common Core F-BF.B.3, F-IF.C.7b · mathovore.fr

A function transformation moves or reshapes the graph of a parent function. Adding k outside shifts it up or down, adding k inside shifts it left or right, multiplying by a stretches or reflects it vertically, and multiplying x by k changes its width. Even functions are symmetric about the y -axis; odd functions about the origin.

Algebra 2 Common Core: F-BF.B.3 · F-IF.C.7b

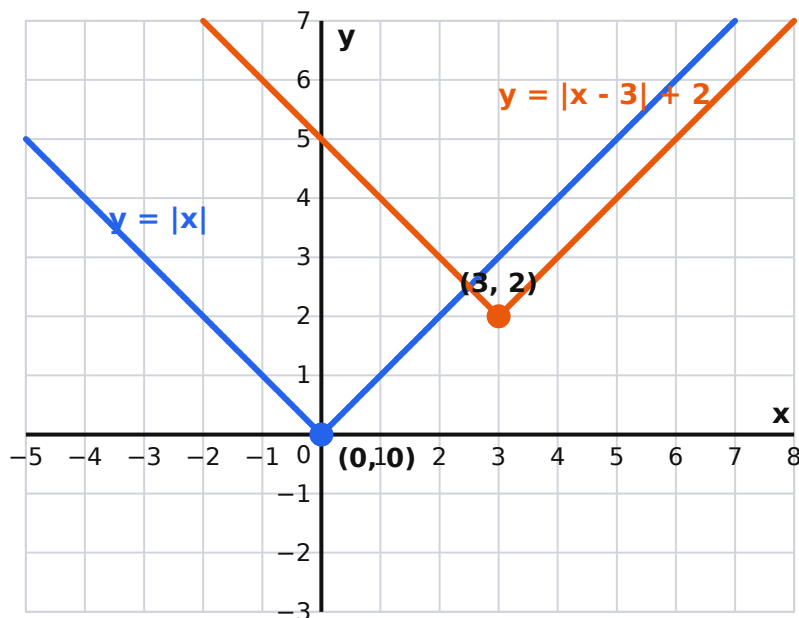
Key facts:

- Standards F-BF.B.3 and F-IF.C.7b: identify the effect of transformations and graph square root, cube root and other basic functions.
- $f(x)+k$ shifts vertically; $f(x+k)$ shifts horizontally in the opposite direction of the sign.
- $a \cdot f(x)$ stretches or compresses vertically; $f(kx)$ compresses or stretches horizontally by $\frac{1}{|k|}$; a negative factor reflects.
- Even: $f(-x) = f(x)$. Odd: $f(-x) = -f(x)$. Test by substituting $-x$.
- $y = \sqrt{x-h}+k$ starts at (h,k) ; $y = \sqrt[3]{x-h}+k$ is centered at (h,k) .

What is a function transformation?

A **transformation** changes the graph of a parent function $y = f(x)$ by shifting, stretching, compressing or reflecting it. Each change comes from one small edit to the equation.

New function	Effect on the graph of f
$f(x)+k$	Up k units (down if $k < 0$)
$f(x+k)$	Left k units (right if $k < 0$)
$a \cdot f(x)$	Vertical stretch by $ a $ if $ a > 1$, compression if $0 < a < 1$; reflection over the x -axis if $a < 0$
$f(kx)$	Horizontal compression by $\frac{1}{ k }$ if $ k > 1$, stretch if $0 < k < 1$; reflection over the y -axis if $k < 0$



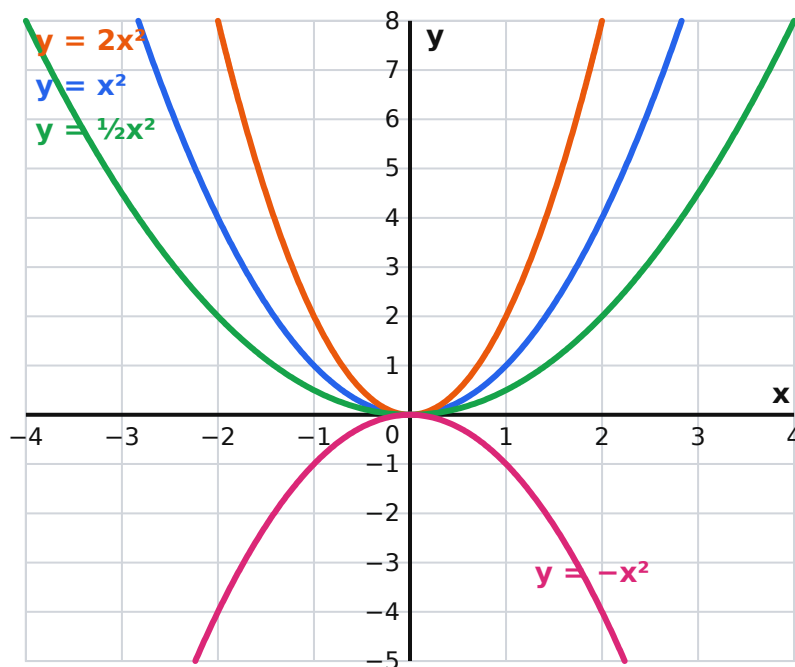
The orange graph of $y = |x - 3| + 2$ is the blue parent $y = |x|$ moved right 3 and up 2.

How do shifts work?

A number added outside the function moves the graph vertically. A number added inside moves it horizontally, in the direction opposite to the sign. For $y = |x - 3| + 2$, the vertex moves from $(0,0)$ to $(3,2)$: right 3 because of $x - 3$, up 2 because of $+2$.

How do stretches, compressions and reflections work?

Multiplying the output by a multiplies every y-coordinate by a . For $f(x) = x^2$, the point $(2,4)$ moves to $(2,8)$ on $y = 2x^2$ and to $(2,-4)$ on $y = -x^2$. Multiplying the input by k divides every x-coordinate by k , so $f(2x)$ is half as wide as $f(x)$.



A factor of 2 stretches $y = x^2$ vertically, a factor of $\frac{1}{2}$ compresses it, and a negative factor reflects it over the x-axis.

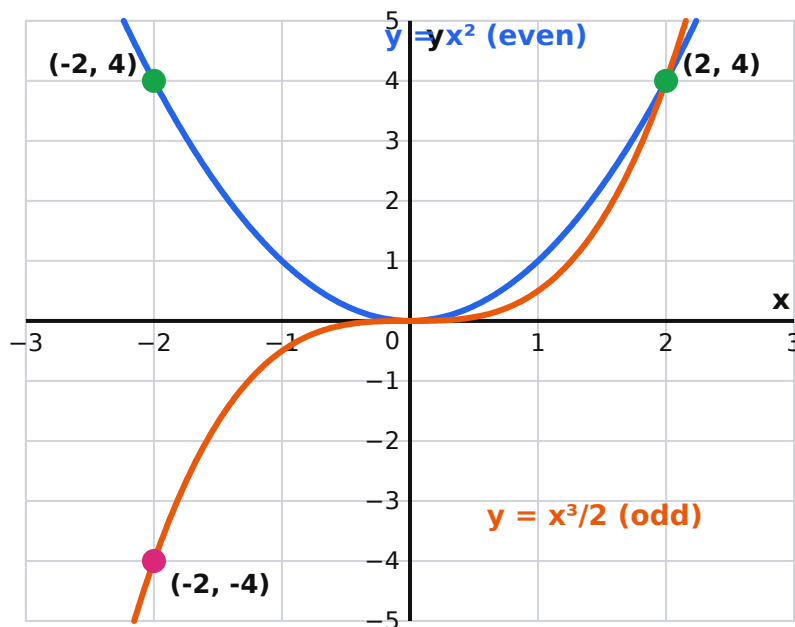
What are even and odd functions?

Symmetry tests

Even: $f(-x) = f(x)$ for every x . The graph is symmetric about the y-axis, as with x^2 and $|x|$.

Odd: $f(-x) = -f(x)$ for every x . The graph is symmetric about the origin, as with x^3 and $\sqrt[3]{x}$.

Most functions are neither. For $h(x) = x^2 + x$, $h(-x) = x^2 - x$, which is not $h(x)$ and not $-h(x)$.

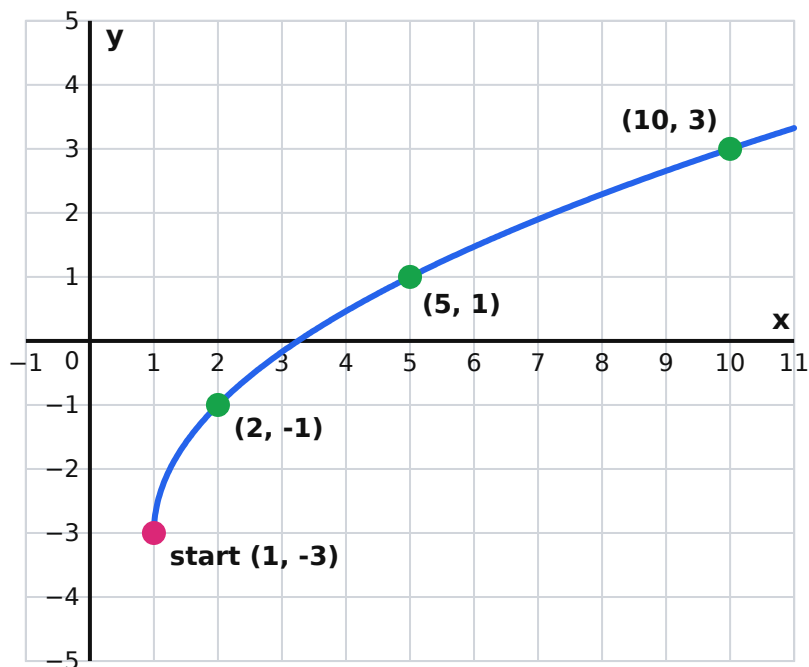


Even functions mirror across the y-axis; odd functions look the same after a half-turn about the origin.

How do you graph a square root or cube root function?

1. Find the starting point (square root) or center (cube root) from the shifts.
2. Choose x-values that make the radicand a perfect square or cube.
3. Apply the stretch and reflection to the y-values, then plot and connect.

Worked example. Graph $y = 2\sqrt{x-1} - 3$. It starts at $(1, -3)$. Then $x = 2$ gives $2(1) - 3 = -1$, $x = 5$ gives $2(2) - 3 = 1$ and $x = 10$ gives $2(3) - 3 = 3$. The domain is $x \geq 1$ and the range is $y \geq -3$.



The graph of $y = 2\sqrt{x-1} - 3$ starts at $(1, -3)$ and rises to the right.

Transformations of Functions: frequently asked questions

Why does $f(x + 3)$ shift the graph left?

The output $f(x+3)$ equals the old output at an x -value 3 larger. Each point of the original graph therefore appears 3 units farther left. For example, $|x+3|$ has its vertex at $x = -3$.

What is the difference between $2f(x)$ and $f(2x)$?

The factor $2f(x)$ doubles every height, a vertical stretch. The factor $f(2x)$ halves every x -coordinate, a horizontal compression. For $f(x) = x^2$, $f(2x) = 4x^2$, which happens to equal $4f(x)$.

How do you test whether a function is even or odd?

Compute $f(-x)$ and simplify. If it equals $f(x)$, the function is even. If it equals $-f(x)$, it is odd. If neither, the function is neither. For $f(x) = x^3 + x$, $f(-x) = -x^3 - x = -f(x)$, so it is odd.

What are the domain and range of a square root function?

For $y = a\sqrt{x-h} + k$, the radicand must be nonnegative, so the domain is $x \geq h$. The range is $y \geq k$ when $a > 0$ and $y \leq k$ when $a < 0$. A cube root function has domain and range of all real numbers.

How does a transformation change an exponential function?

Adding k to 2^x moves the horizontal asymptote to $y = k$. Shifting left 1 gives $2^{x+1} = 2 \cdot 2^x$, which is also a vertical stretch by 2, so the two descriptions give the same graph.