

Algebra 2 · Sequences and Series · Worksheet Answer Key

Common Core A-SSE.B.4, F-BF.A.2, F-IF.A.3 · mathovore.fr

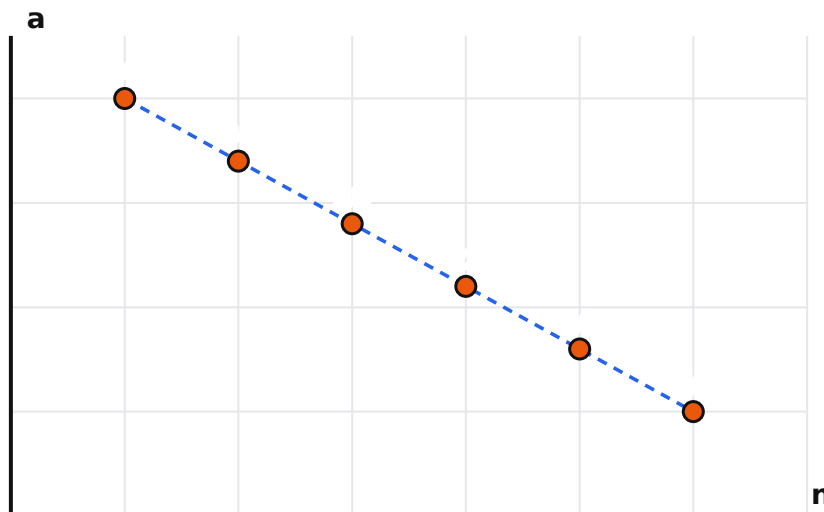
Every solution shows the formula, the substitution and a final answer in bold. Compare your steps as well as your results, and recheck sums by listing a few terms.

Algebra 2 Common Core: A-SSE.B.4 · F-BF.A.2 · F-IF.A.3

Problem 1: Arithmetic or geometric

- a. Differences: $9-5=4$, $13-9=4$, $17-13=4$. **Arithmetic**, $d=4$.
- b. Differences 3, 6, 12 are not equal; ratios $\frac{6}{3}=\frac{12}{6}=\frac{24}{12}=2$. **Geometric**, $r=2$.
- c. Differences are all -3 . **Arithmetic**, $d=-3$.
- d. Ratios: $\frac{27}{81}=\frac{9}{27}=\frac{3}{9}=\frac{1}{3}$. **Geometric**, $r=\frac{1}{3}$.

Problem 2: Reading a graph



The line $a_n = 46 - 6n$ passes through every point.

- a. Each step is -6 : 40, 34, 28, 22, 16. **Arithmetic**, $d=-6$.
- b. $a_n = 40 + (n-1)(-6) = 46 - 6n$. $a_n = 46 - 6n$
- c. $a_{12} = 46 - 6(12) = 46 - 72 = -26$.

Problem 3: Explicit arithmetic rule

$$a_n = 7 + (n-1)(5) = 5n + 2. \quad a_n = 5n + 2$$

$$a_{20} = 5(20) + 2 = 102.$$

Problem 4: Explicit geometric rule

$$a_n = 3 \cdot 2^{n-1}$$

$$a_{10} = 3 \cdot 2^9 = 3 \cdot 512 = 1536.$$

Problem 5: Terms from a recursive rule

- a. Multiply by -2 each time: 4, -8 , 16, -32 , 64.
- b. Subtract 6 each time: 50, 44, 38, 32, 26.

Problem 6: Writing a recursive rule

- a. Differences are all 7: $a_1 = 12$, $a_n = a_{n-1} + 7$.
- b. Ratios are all $\frac{1}{2}$: $a_1 = 80$, $a_n = \frac{1}{2}a_{n-1}$.

Problem 7: How many terms

Solve $8 + (n-1)(5) = 163$: $5(n-1) = 155$, so $n-1 = 31$. $n = 32$ **terms**.

Problem 8: Rule from two terms

From a_2 to a_7 there are 5 steps: $5d = 39 - 9 = 30$, so $d = 6$.

$a_1 = 9 - 6 = 3$. Then $a_n = 3 + 6(n-1) = 6n - 3$. Check: $a_7 = 6(7) - 3 = 39$.

Problem 9: Geometric from two terms

From a_2 to a_5 there are 3 steps: $r^3 = \frac{324}{12} = 27$, so $r = 3$.

$a_1 = \frac{12}{3} = 4$. Then $a_8 = 4 \cdot 3^7 = 4 \cdot 2187 = 8748$.

Problem 10: Evaluating sigma notation

Substitute $k = 1, 2, 3, 4, 5, 6$: $3 + 5 + 7 + 9 + 11 + 13 = 48$.

Check with the formula: $\frac{6(3+13)}{2} = 48$.

Problem 11: Geometric sigma

Terms: 2, 6, 18, 54. Sum: 80.

Check: $\frac{2(1-3^4)}{1-3} = \frac{2(-80)}{-2} = 80$.

Problem 12: Writing sigma notation

- a. Arithmetic with $a_1 = 6$, $d = 4$: the rule is $4k + 2$. $\sum_{k=1}^5 (4k + 2)$.
b. Geometric with $a_1 = 3$, $r = 2$: the rule is $3 \cdot 2^{k-1}$. $\sum_{k=1}^6 3 \cdot 2^{k-1}$.

Problem 13: Arithmetic series sum

Number of terms: $4 + 5(n-1) = 99$ gives $n = 20$.

$$S_{20} = \frac{20(4+99)}{2} = 10 \cdot 103 = 1030.$$

Problem 14: Geometric series sum

$$S_8 = \frac{5(1-2^8)}{1-2} = \frac{5(-255)}{-1} = 1275.$$

Problem 15: Ratio less than 1

$$S_6 = \frac{81\left(1 - \left(\frac{1}{3}\right)^6\right)}{1 - \frac{1}{3}} = \frac{81 \cdot \frac{728}{729}}{\frac{2}{3}} = \frac{\frac{728}{9}}{\frac{2}{3}} = \frac{364}{3}, \text{ or about } 121.33.$$

Check by adding: $81 + 27 + 9 + 3 + 1 + \frac{1}{3} = 121\frac{1}{3}$.

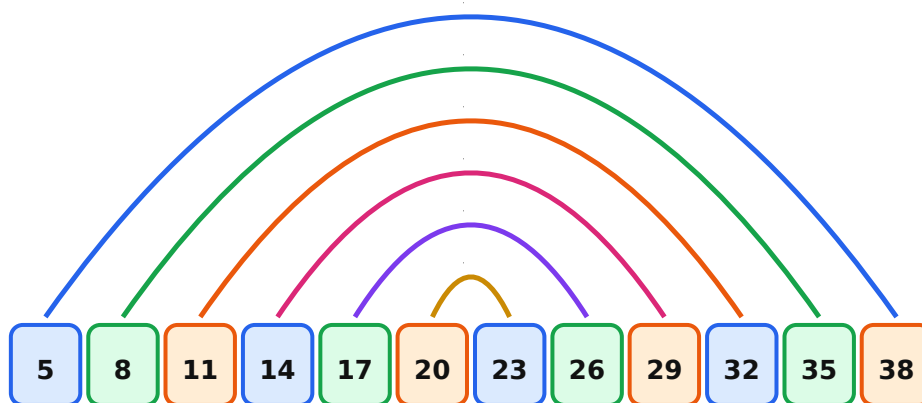
Problem 16: Finding the number of terms

$$S_n = \frac{n}{2}(2a_1 + (n-1)d) = \frac{n}{2}(4 + 3n - 3) = \frac{n(3n+1)}{2}.$$

Set equal to 155: $3n^2 + n = 310$, so $3n^2 + n - 310 = 0$, which factors as $(3n+31)(n-10) = 0$.

$n = -\frac{31}{3}$ is impossible, so $n = 10$ **terms**. Check: $a_{10} = 29$ and $\frac{10(2+29)}{2} = 155$.

Problem 17: Weekly savings



6 pairs, each adding to 43

Six pairs of 43 give 258.

a. $a_{12} = 5 + 11(3) = 38$. **\$38.**

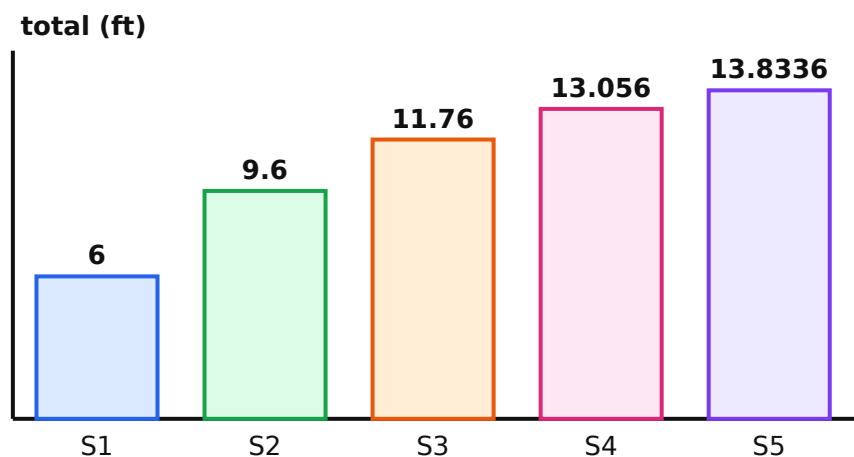
b. $S_{12} = \frac{12(5+38)}{2} = 6 \cdot 43 = 258$. **\$258.**

Problem 18: Theater seating

a. $a_{18} = 14 + 17(2) = 48$. **48 seats.**

b. $S_{18} = \frac{18(14+48)}{2} = 9 \cdot 62 = 558$. **558 seats.**

Problem 19: Bouncing ball



Running total of the bounce heights.

a. Bounce 1 is $10(0.6) = 6$ feet, with ratio 0.6: $h_n = 6(0.6)^{n-1}$.

b. $h_5 = 6(0.6)^4 = 6(0.1296) = \mathbf{0.7776 \text{ ft.}}$

c. $S_5 = \frac{6(1-0.6^5)}{1-0.6} = \frac{6(0.92224)}{0.4} = \mathbf{13.8336 \text{ ft.}}$

Problem 20: Chain email

- a. Geometric with $a_1 = 3$, $r = 3$: $a_7 = 3 \cdot 3^6 = 2187$. **2,187 people.**
b. $S_7 = \frac{3(1-3^7)}{1-3} = \frac{3(-2186)}{-2} = 3279$. **3,279 people.**

Problem 21: Raises over ten years

Salaries form a geometric sequence with $a_1 = 42000$ and $r = 1.03$.

- a. $a_{10} = 42000(1.03)^9 \approx \$54,800.47$. **\$54,800.47.**
b. $S_{10} = \frac{42000(1.03^{10}-1)}{0.03} \approx \$481,482.93$. **\$481,482.93** earned in 10 years.

Problem 22: Savings account

The last deposit earns no interest and the first earns interest for 11 months. The values form a geometric series with $a_1 = 200$, $r = 1.005$, $n = 12$.

$$S_{12} = \frac{200(1.005^{12}-1)}{0.005} \approx \$2,467.11. \quad \mathbf{\$2,467.11.}$$

Problem 23: Car loan

The values form a geometric series with $a_1 = \frac{300}{1.005}$, $r = \frac{1}{1.005}$, $n = 36$.

- a. $P = 300 \cdot \frac{1-1.005^{-36}}{0.005} \approx \$9,861.30$. **\$9,861.30.**
b. Total paid: $300 \cdot 36 = \$10,800$. Interest: $10800 - 9861.30 \approx \$938.70$. **\$938.70.**

Problem 24: Mortgage payment

Payment k is worth $M(1.005)^{-k}$ today. The sum of 180 of these is $M \cdot \frac{1-1.005^{-180}}{0.005}$, and it equals 150,000.

- a. $M = \frac{150000(0.005)}{1-1.005^{-180}} \approx \$1,265.79$. **\$1,265.79 per month.**
b. Total paid: $1265.79 \cdot 180 = \$227,842.20$. Interest: **\$77,842.20.**