

Algebra 2 · Sequences and Series · Lesson

Common Core A-SSE.B.4, F-BF.A.2, F-IF.A.3 · mathovore.fr

This algebra 2 sequences series lesson explains sequences and series in one printable page, with definitions, a method and a worked example.

A sequence is an ordered list of numbers and a series is the sum of its terms. An arithmetic sequence adds a constant difference d each step, while a geometric sequence multiplies by a constant ratio r . Explicit formulas find any term directly; recursive formulas build each term from the previous one.

Algebra 2 Common Core: A-SSE.B.4 · F-BF.A.2 · F-IF.A.3

Algebra 2 Sequences Series Lesson: key facts

- Arithmetic: explicit $a_n = a_1 + (n-1)d$, recursive $a_n = a_{n-1} + d$.
- Geometric: explicit $a_n = a_1 r^{n-1}$, recursive $a_n = r \cdot a_{n-1}$.
- Arithmetic series: $S_n = \frac{n(a_1 + a_n)}{2}$.
- Geometric series: $S_n = \frac{a_1(1-r^n)}{1-r}$ for $r \neq 1$.
- Sigma notation $\sum_{k=1}^n$ is shorthand for adding terms from $k=1$ to $k=n$.

What are arithmetic and geometric sequences?

A **sequence** is an ordered list of numbers a_1, a_2, a_3 and so on. In an **arithmetic sequence** every term is found by adding the same **common difference** d . In a **geometric sequence** every term is found by multiplying by the same **common ratio** r .

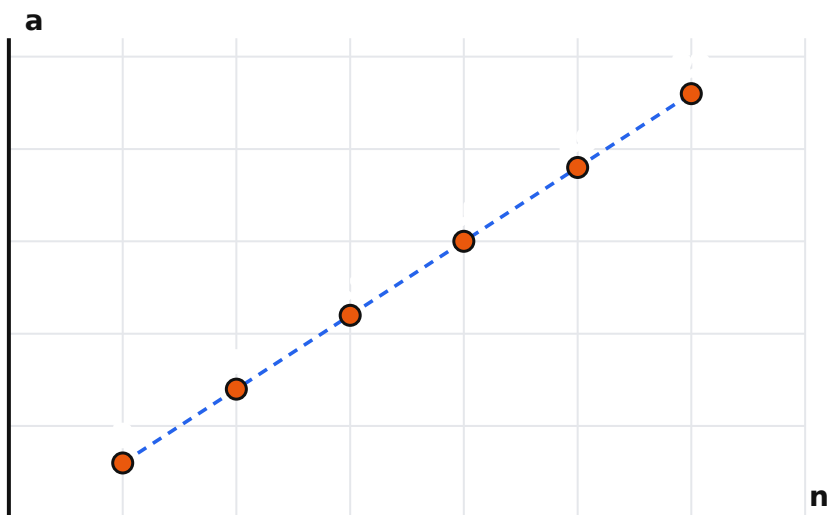
Two ways to describe a sequence

Explicit formula: gives a_n directly from n , so any term can be found without the earlier ones.

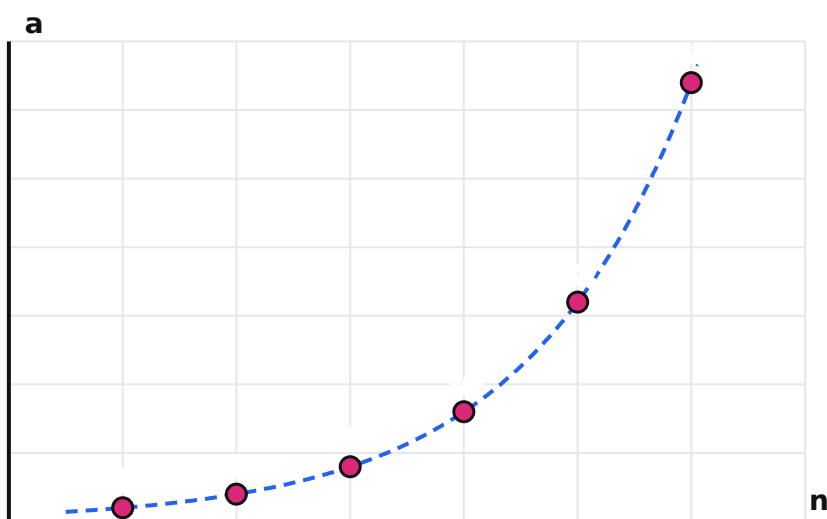
Recursive formula: gives the first term and a rule that builds each term from the one before it.

	Arithmetic	Geometric
Explicit	$a_n = a_1 + (n-1)d$	$a_n = a_1 \cdot r^{n-1}$

	Arithmetic	Geometric
Recursive	$a_n = a_{n-1} + d$	$a_n = r \cdot a_{n-1}$
Sum of n terms	$S_n = \frac{n(a_1 + a_n)}{2}$	$S_n = \frac{a_1(1 - r^n)}{1 - r}, r \neq 1$



Arithmetic: the points lie on a line, rising by 4 each step.

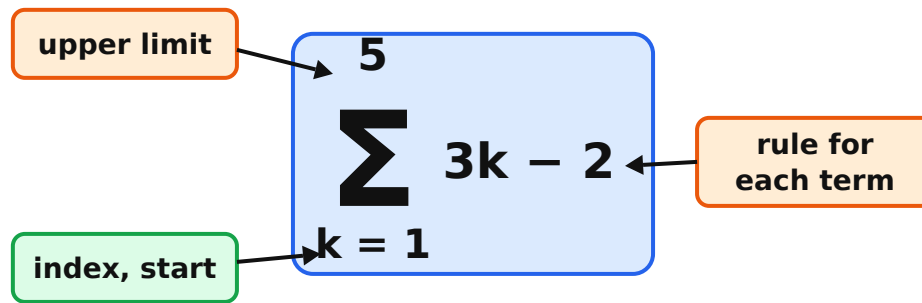


Geometric: the points follow an exponential curve, doubling each step.

How does sigma notation describe a series?

A **series** is the sum of the terms of a sequence. The Greek letter sigma abbreviates the sum:

$\sum_{k=1}^5 (3k-2)$ means substitute $k=1, 2, 3, 4, 5$ into $3k-2$ and add the results.

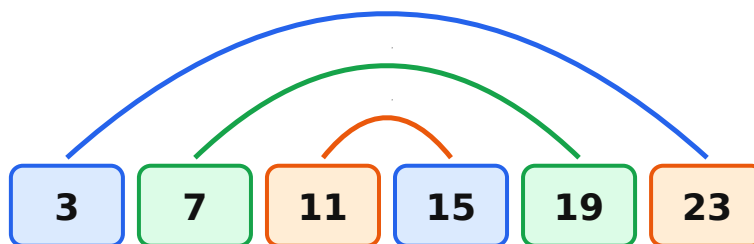


$$= 1 + 4 + 7 + 10 + 13 = 35$$

Reading sigma notation from the lower limit up to the upper limit.

How do you add the terms of a series quickly?

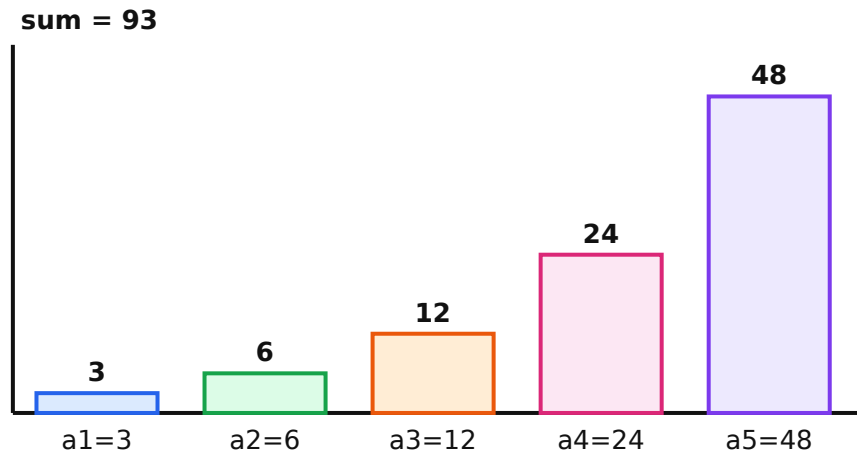
For an arithmetic series, pair the first and last terms, the second and second-to-last, and so on. Every pair has the same sum, so $S_n = \frac{n(a_1 + a_n)}{2}$. For $3+7+11+15+19+23$ there are 3 pairs of 26, giving 78.



3 pairs, each adding to 26

Pairing terms from both ends of an arithmetic series.

For a geometric series, multiplying the sum by r and subtracting leaves only the first and last terms, which gives the formula in the table. For $3+6+12+24+48$: $S_5 = \frac{3(1-2^5)}{1-2} = 93$.



Each bar is twice the one before; together they total 93.

How are series used for savings and loans?

Money that grows by a fixed percent each period forms a geometric sequence. Worked example: Maya deposits \$100 at the end of each month into an account paying 0.5% per month. How much is in the account after 24 months?

1. The deposit made in month 24 earns nothing, so it is worth \$100. The first deposit grows for 23 months.
2. Reading backward gives a geometric series with $a_1 = 100$, $r = 1.005$, $n = 24$.
3. $S_{24} = \frac{100(1.005^{24} - 1)}{0.005} \approx \$2,543.20$.

A mortgage payment works the same way: the loan amount equals the sum of the payments, each divided by 1.005 for every month it is delayed.

Sequences and Series: frequently asked questions

How do I tell whether a sequence is arithmetic or geometric?

Subtract consecutive terms. If the differences are all equal, the sequence is arithmetic. If not, divide consecutive terms. If the quotients are all equal, it is geometric. Some sequences are neither.

What is the difference between explicit and recursive formulas?

An explicit formula such as $a_n = 5n + 2$ lets you compute any term directly. A recursive formula such as $a_n = a_{n-1} + 5$ needs the previous term and a starting value, so you must state a_1 .

What does the index in sigma notation do?

The index, usually k , takes every whole-number value from the lower limit to the upper limit.

You substitute each value into the expression and add all the results.

Why does the geometric sum formula fail when $r = 1$?

When $r = 1$ the denominator $1 - r$ is zero. In that case every term equals a_1 , so the sum is simply $n \cdot a_1$.

How do savings and mortgage problems use series?

Each deposit or payment is worth a different amount because of interest, and those values form a geometric sequence. The geometric sum formula adds them all at once instead of one by one.

How to use this algebra 2 sequences series lesson

Use this algebra 2 sequences series lesson before the worksheet, then check what you learned with the quiz.

